Toppersnotes
Unleash the topper in you

RRB-NTPC

CBT-I ,CBT-II

QUANTITATIVE APTITUDE - II

SIERRA
INNOVATIONS
Private Limited

Contents

Toppersnotes
Unleash the topper in you

NUMBER SYSTEM

Introduction

Numbers

Real Numbers

- Which Can be represented on number line

eg: −1, 0, 1, 2.3, 5/7

Imergenary Numbers

- Which Can be not represented On Number line

eg: i, $a+ib$

$\boxed{i = i0 + a = \sqrt{-1}}$

Such number no exist in real

Rational Number

- Which Can be written in form of p/q $(q \neq 0)$

eg: $-2/3$, $5/7$, 4, 1, $\sqrt{16}$

Irrational Number

- Which Can not be written in form of p/q.

eg: $\sqrt{5}$, $\sqrt{23}$, π, e

$\pi \Rightarrow$ Pie = 3.1457------

$e \Rightarrow$ 2.73828---------

Fraction/Decimal

- Which are in form of p/q or having Decimal

Integer

- A Complete Number Which is not a fraction or decimal.

eg: −3, −71, −0, 1, 5, 243

[It Can be Positive, negative or zero]

- Whole Numbers: Integers Starting from 0.
- Natural Numbers: Integers Starting from 1.
- Prime Numbers: The number which is divisible by 1 & no. itself is Called a Prime number.

eg: 2, 3, 5, 7, 11, 13 etc.

1 is not a Prime number

There are 25 Prime number b/w 1 to 100

- Composite Number: The number which have more than two factors are Called Composite numbers.

eg: 4, 6, 12, 21, 28 etc.

The numbers which are not prime are Composite Number

Co-Prime Number: Numbers having their HCF is 1 are termed as Co-prime Numbers.

eg: 14 & 15.

Even Number: Rational number which are the multiple of 2 is Called as even numbers.

eg: 2, 4, 6, 48, 92 ..... etc.

Odd Number: Rational Numbers which are not multiple of 2 are Odd Number.

eg: 1, 3, 5, 91, 103, 249 .....

even Numbers ending digit is 2, 4, 6, 8, 0 & Odd Numbers ending digit is 1, 3, 5, 7, 9

Properties of Odd and even Numbers:

- even + even = Even
- ODD + ODD = Even
- Even + ODD = ODD
- Even + Even ---------- + n times = Even (always)
- Odd + Odd ---------- odd numbers of times = ODD
- ODD + ODD ---------- even number of times = Even
- Even x Even = Even
- Eved x odd = Even
- Odd x odd = Odd
- Even x (Even / Odd) = Even

Decimal

Terminating

- Decimal Vanishes after some places

These No's can be represented by–
$1.\overline{23}$

Non-terminating

Eg: 1.23456789......
1.11111 ......

(Decimal never Vanishes)

Recurring

Digit of Decimal is repeat after some decimal places

Eg: 1.23232323......

These are rational Numbers

Non Recurring

- Digit of Decimal does not repeat

Eg: 1.233759257.

These are Irrational Number

—×—×—×—×—×—×—×—×—×—×—×—

Converting Recurring in P/q Form:

(Solving the – (Bar) Problems)

Eg: $x = 0.\overline{7}$, Convert x into P/q form.

Solⁿ ⇒ $x = 0.7777$ ...... –(1)

$10x = 7.7777$ ...... –(2)

$9x = 7.0000$

$\boxed{x = 7/9}$

– if (–) on one digit = Multiply by 10
– if (–) on two digit → multiply by 100

Tricks

Type-1

(a) $x = 0.\overline{8}$

$x = \frac{8}{9}$ → As many digits Contain ('−'), Write 9 as many times;-

(b) $x = 0.\overline{78}$

$x = \frac{\cancel{78}}{\cancel{99}} = \frac{26}{33}$ Ans

Type-II

(c) $x = 0.3\overline{84}$

$= \frac{384-3}{990}$ → Number After Decimal − Number not Contain bar
→ 9 as many digit in (−), & 0 as many times not Contain (−),

$= \frac{381}{990} = \frac{127}{330}$ Ans

$x = 52\overline{48}$

$= \frac{5248-52}{9900} = \frac{5196}{9900}$ $\frac{1732}{3300}$ Ans

Type-III

(a) $2.6\overline{5}$

$\Rightarrow 2 + 0.6\overline{5}$

$= 2 + \frac{65-6}{90}$ (Same as type II)

$= 2 + \frac{59}{90} = \frac{239}{90}$ Ans

(b) $5.\overline{95}$

$= 5 + 0.\overline{95}$

$= 5 + \frac{95}{99}$

$= \frac{590}{99}$

Divisibility Rules :-

NUMBER	RULE	EXAMPLE
2	Last digit is divisible by 2, or last digit is 0,2 4,6,8.	Eg: 2348 1948
3	Sum of digit is divisible by 3.	Eg: 1071 1+0+7+1=9
4	last two digit of number is divisible by 4	14<u>32</u> 92<u>84</u>
5	Last digit is 5 or 0	2335, 1990
6	Number is divisible by 2 and 3 each	132 → divisible by 2 1+3+2 → divisible 3
7	• Multiply last digit by 5 • Add the above number • if remaing digits divisible by 7, then number is divided by 7	Eg: 343 (i) 3×5 = 15 34−15 = 49 ↓ divisible by 7.
8	Last 3 digit are divisible by 8	8032 → 32 ↓ Divisible by 8.
9.	Sum of digits is divisible by 9	1071 → 1+0+7+1 = 9 ↓ divisible by 9
11.	• Defrence of Sum of digit at odd Places & Sum of digit at even places.	• <u>1</u>3<u>3</u>1 (3+1) − (3+1) = 0 • 11718520 (1+7+8+2) −(1+1+5+0) = 11

② If $3x2680$, is divisible by 11, then the Value of x is :

Solⁿ: (Sum of Odd Place digit) − (Sum of Even Place digit)

$= (3+2+8) - (x+6+0)$

$= 13-6-x$

$= 7-x$ (Either 0 or divisible by 11)

$= 7-x=0$

$\boxed{x=7}$ Ans.

Cyclicity:

Unit digit is repeated after Some time of an exponent.

$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$7^1 = 7$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$7^2 = 49$
$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$7^3 = 343$
$2^4 = 16$	$3^4 = 81$	$4^4 = 216$	$7^4 = 2401$
$2^5 = 32$	$3^5 = 243$	Cyclicity=2	$7^5 = 16807$
$2^6 = 64$	$3^6 = 729$		Cyclicity =4
Cyclicity =4	Cyclicity = 4		

$8^1 = 8$	$9^1 = 9$
$8^2 = 64$	$9^2 = 81$
$8^3 = 512$	$9^3 = 729$
$8^4 = 4096$	$9^4 = 6561$
$8^5 = 32768$	Cyclicity=2
Cyclicity = 4	

Eg: $(2)^{423}$, Find the digit at units Place

Soln (a) divide the power by 4

In Exams divide in Mind, not in Pen-Paper.

$$4\overline{)423}(105$$
$$\begin{array}{r} 4 \\ \hline 23 \\ 20 \\ \hline 3 \end{array}$$

Remainder = 3

$2^3 = 8$ Ans

Unit digit and ten's digit Concept-

★ 1 2 3 4
→ Unit digit (4)
→ Ten's digit (3)

Eg: $\boxed{\text{Type-I}}$

(a) $29 \times 45 = 9 \times 5 = 4\textcircled{5}$ Unit digit $= 5$

(b) $18 \times 18 \times 18 + 3$

$8 \times 8 \times 8 + 3$

64×8

$32 + 3 = 35 \Rightarrow = 5$

$\boxed{\text{Type = II}}$

(a) (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
↓
Cyclicity Concept

(b) (0, 1, 5, 6)
↓
If there number are at unit place unit digit of multiplication is also a same number.

Eg; (a) 35×35 (b) 36×96

1225 → Same $= 3456 \rightarrow$ Same

Helping Hand ÷

(a) Divide the Power by 4.

(b) Remainder of division is 0, 1, 2, 3 . . .

(c) Remainder ⇒ 1 = x^1 is unit digit
Remainder ⇒ 2 = x^2 is unit digit
Remainder ⇒ 3 = x^3 is unit digit
Remainder ⇒ 0 = x^4 is unit digit

If x^4 is 2 or 3 digit number, then unit digit of that number, will be the unit digit of original Exponent.

Solved Examples

① What least number must be added to 1056, So that Sum is Completely divisible by 23?

Soln ⇒

```
23 ) 1056 ( 45
     92
     ---
      136
      115
      ---
       21
```

then Number added is $= 23 - 21$
$= 2$ Ans

② The largest 4 digit number exactly divisible by 88 is –

(a) 9944 (b) 9768 (C) 9988 (d) 8888

Soln ⇒ Largest 4 digit Number = 9999

```
88 ) 9999 ( 113
     88
     ---
     119
      88
     ---
      319
      264
      ---
       55
```

55 → Subtract from the 4 digit largest number.

$= 9999 - 55 = 9944$ Ans

③ If the number 517 * 324 is Completely divisible by 3, then the Smallest whole no. in place of * will be.

(a) 0 (b) 1 (c) 2 (d) None

$5 + 1 + 7 + * + 3 + 2 + 4$
$= 22 + *$

If number is divisible by 3, then sum of digit is also divisible by 3.

If 2 is used in place of *, then number is divisible by 3 (i.e. 24)

④ Which one of the following no. is divisible by 11?

(a) 235641 (b) 245642 (c) 315624 (d) 415624

Soln ⇒ (a) 235641
$(2+5+4) - (3+6+1) = 1$ (not divisible by 11)

(b) 245642
$(2+5+4) - (4+6+2) = 1$ (not divisible by 11)

(c) 315624
$(3+5+2) - (1+6+4) = -1$ (not divisible by 11)

(d) 415624
$(4+5+2) - (1+6+4) = 0$ (divisible by 11)

> If a number is divisible by 11, the Difference of sum of digit at odd places & sum of digit at even places is either 0 or divisible by 11.

⑤ Which on the following number is divisible by 24 –

Soln ⇒ (a) 35718 (b) 63810 (c) 63810 (c) 537804 (d) 3125736

	③	⑧
35718	$3+5+7+1+8 = 24$ ✓	718 ✗
63810	$6+3+8+1+0 = 18$ ✓	810 ✗
537804	$5+3+7+8+0+4 = 27$ ✓	804 ✗
3125736	$3+1+2+5+7+3+6 = 27$ ✓	736 ✓ ✓

> If a no. is divisible by another number then it must be divisible by it's prime factors.

Unit digit Concept:

⑥ the digit at unit's place of the Product-

$81 \times 82 \times 83 \ldots\ldots \times 89$ is

(a) 0 (b) 2 (c) 6 (d) 8

Soln ⇒ $81 \times 82 \times 83 \times 84 \times 85 \ldots \times 89$

$1 \times 2 \times 3 \times 20 \ldots \times 6 \times 7 \times 8 \times 9$

$= 0$

If we multiply a number by 0, the result at unit place is always zero.

⑦ The digit in unit's Place of the Product $(2153)^{167}$ is:

(a) 1 (b) 3 (c) 7 (d) 9

Soln ⇒ 215<u>3</u> ⟶ Let base is 3

ⓑ $\frac{167}{4}$ ⇒ Remainder is 3

ⓒ $3^3 = 27$ → unit digit is 7

⑧ Unit digit in $(264)^{102} + (264)^{103}$ is –

ⓐ 0 (b) 4 (c) 6 (d) 8

Soln ⇒ $(264)^{102} + (264)^{103}$

$= 6 + 4$

$= 10$

unit digit $= 0$

If Base is 4, then
(a) ⇒ Unit digit of even power is always 6
(b) ⇒ Unit digit of odd Power is always 4.
because Cyclicity is 2

9. Unit digit of $(169)^{537} + (94)^{394}$ is.

(a) (b) (c) (d)

Soln ⇒ $(169)^{537} + (94)^{394}$

$\downarrow \quad\quad \downarrow$

$= 9 + 6$

$= 15$

= unit digit is 5 Ans

If the Base is 9
(a) Unit digit of ODD Power is always 9.
(b) unit digit of even Power is always 1.
because Cyclicity is 2.

10. The digit in the unit place of
$[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^{4} + 259 + (73)$ is —

(a) 1 (b) 4 (c) 5 (d) 6

Soln ⇒ $(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^{4} + 259 + (73)^{51}$

$1 + 1 - 6 + 5 - 6 + 9 + 7$

$\boxed{\text{Unit digit of base 1, 5, 6, is always Same}}$

$\boxed{\frac{51}{3} = \text{Remainder } 3 \quad 3^3 = 27}$

$= 23 - 12 = 11$ Ans

11. Unit digital in expression of $(2137)^{754}$ is —

(a) 1 (b) 3 (c) 7 (d) 9

Soln ⇒ $(2137)^{754} \rightarrow$ Base is 7

$\frac{754}{4}$ Remainder = 2

$7^2 = 49 \longrightarrow$ unit digit is 9 ✓

12. Find the unit's digit of $(358)^{64} - (253)^{36}$.

(a) 5 (b) 4 (c) 7 (d) 9

Soln ⇒ $(358)^{64} - (253)^{36}$

$\frac{64}{8} \quad\quad \frac{36}{3}$

$0 \longrightarrow$ Remainder $\leftarrow$ 0 $\longrightarrow 3^4$

$8^4 = 64 \times 64 = 16 - 1$

$\Rightarrow 6 - 1$

$= 5$ Ans

solved Examples

1- what Least Number Must be added to 1056, so that sum is completely divisible by 23?

(a) 2 (b) 2 (c) 18 (d) 21

sol.

23) 1056 (45
92
136
115
21

then number added is

= 23-21
= 2.

2- The largest 4 digit number Exactly divisible by 88 is-

(a) 9944 (b) 9768 (c) 9988 (d) 8888

sol.

Largest 4 digit Number = 9999

88) 9999 (113
88
119
88
319
264
55 ⟶ Sub tract from the 4 digit Largest number

= 9999 − 55
= 9944.

3- If the number 517x324 is completely divisible by 3, them the smallest whole no. in place of x will be-

(a) 0 (b) 1 (c) 2 (d) None

sol.

5 + 1 + 7 + x + 3 + 2 + 4
= 22 + x

If number in divisible by 3 them sum of digit is also divisible by 3.

If 2 is used in place of x, then number is divisible by 3 (i.e. 24)

4- which one of the following no. is divisible by 11?

(a) 235641 (b) 245642 (c) 315624 (d) 415624

sol. (a) 235641

$(2+5+4)-(3+6+1) = 1$ (not divisible by 11)

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(c) 315624

$(3+5+2)-(1+6+4) = -1$ (not divisible by 11)

(d) 415624

$(4+5+2)-(1+6+4) = 0$ (divisible by 11)

If a number is divisible by 11, the Difference of Sum of digit of digit at odd places & Sum of digits of even place is either 0 or divisible by 11.

5- which on of the following no. is divisible by 24?

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✓ 3125736	$3+1+2+5+7+3+6 = 27$	736 ✓

If a no. is divisible by another number, then it must be divisible by its Prime factors

6- The digit at unit's place of the product $81 \times 82 \times 83 \ldots \times 89$

(a) 0 (b) 2 (c) 6 (d) 8

sol. $81 \times 82 \times 83 \text{------} \times 89$ is

(a) 0 (b) 2 (c) 6 (d) 8

Soln ⇒ $81 \times 82 \times 83 \times 84 \times 85 \text{----} \times 89$

$1 \times 2 \times 3 \times 20 \text{----} \times 6 \times 7 \times 8 \times 9$

$= 0$

If we multiple a number of 0, the result at unit place is always zero.

7- The digit in unit's place of the product $(2153)^{167}$ is:

(a) 1 (b) 3 (c) 7 (d) 9

sol.

(a) 215$\underline{3}$ ⟶ Let base is 3

(b) $\frac{167}{4}$ → Remainder is 3

(c) $3^3 = 27 \Rightarrow$ Unit digit is 7

8- unit's digit in $(264)^{102} + (264)^{103}$ is:

(a) 0 (b) 4 (c) 6 (d) 8

sol.

$(264)^{102} + (264)^{103}$

(even) ↓ ↓ (odd)

$= 6 + 4$

$= 10$ = Unit digit = 0

If Base is 4, then
(a) Unit digit of even Power is always 6
(b) Unit digit of odd Power is always 4
because Cyclicity is 2

9- unit's digit of $(169)^{537} + (94)^{394}$ is:

(a) 2 (b) 5 (c) 7 (d) 1

sol. $(169)^{537} + (94)^{394}$

$= 9 + 6$

$= 15$ = unit digit is 5

If the Base is 9
(a) Unit digit of ODD Power is always 9.
(b) Unit digit of even Power is always 1.
because Cyclicity is 2.