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GROUP - D

RAILWAY RECRUITMENT BOARD

Quantitative Aptitude - II

SIERRA INNOVATIONS

Contents

NUMBER SYSTEM

Introduction

Numbers

Real Numbers
- Which Can be represented on number line

eg: $-1, 0, 1, 2.3, 5/7$

Imergenary Numbers
- Which Can be not represented On Number line

eg: i, $a+ib$

$i = i0 + a = \sqrt{-1}$

Such number no existin real

Rational Number
- Which Can be written in form of p/q $(q \neq 0)$

eg: $-2/3$, $5/7$, 4, 1, $\sqrt{16}$

Irrational Number
- Which Can not be written in form of p/q.

eg: $\sqrt{5}$, $\sqrt{23}$, π, e

$\pi \Rightarrow$ Pie $= 3.1457\ldots\ldots$

$e \Rightarrow 2.73828\ldots\ldots$

Fraction/Decimal
- Which are in Form of p/q or having Decimal

Integer
- A Complete Number Which is not a fraction or decimal.

eg: $-3, -71, -0, 1, 5, 243$

[It Can be Positive, negative or Zero]

- **Whole Numbers**: Integers Starting from 0.
- **Natural Numbers**: Integers Starting from 1.
- **Prime Numbers**: The number which is divisible by 1 & no. itself is called a Prime number.

  eg: 2, 3, 5, 7, 11, 13 etc.

  1 is not a Prime number

  There are 25 Prime number b/w 1 to 100
- **Composite Number**: The number which have more than two factors are called Composite numbers.

  eg: 4, 6, 12, 21, 28 etc.

  The numbers which are not prime are Composite Number

Co-Prime Number: Numbers having their HCF is 1 are termed as Co-prime Numbers.

eg: 14 & 15.

Even Number: Rational number which are the multiple of 2 is called as even numbers.

eg: 2, 4, 6, 48, 92 _____ etc.

Odd Number: Rational Numbers which are not multiple of 2 are Odd. Number.

eg: 1, 3, 5, 91, 103, 249 _____

even Numbers ending digit is 2, 4, 6, 8, 0 &
Odd Numbers ending digit is 1, 3, 5, 7, 9

Properties of Odd And even Numbers:

- even + even = Even
- ODD + ODD = Even
- Even + ODD = ODD
- Even + Even _ _ _ _ _ _ _ + n times = Even (always)
- Odd + Odd _ _ _ _ _ _ _ _ odd numbers of times = ODD
- ODD + ODD _ _ _ _ _ _ _ _ even number of times = Even
- Even x Even = Even
- Eved x odd = Even
- Odd x odd = Odd
- Even x (Even / Odd) = Even

Decimal

Terminating
- Decimal Vanishes after Some places

These No's Can be represented by– $1.\overline{23}$

Non-terminating

Eg: 1.23456789.....
1.11111.....

(Decimal never Vanishes)

Recurring
Digit of Decimal is repeat after Some decimal Places

Eg: 1.23232323.....

These are rational Numbers

Non Recurring
- Digit of Decimal does not repeat

Eg: 1.233759257.

These are Irrational Number

Converting Recurring in P/q Form:

(Solving the –(Bar) Problems)

Eg: $x = 0.\overline{7}$, Convert x into P/q form.

Soln ⇒ $x = 0.7777\ldots\ldots$ –①

$10x = 7.7777\ldots\ldots$ –②

$9x = 7.0000$

$x = 7/9$

- if (-) on one tight = Multiply by 10
- if (-) on two digit → multiply by 100

Tricks

Type-1

(a) $x = 0.\overline{8}$

$x = \frac{8}{9}$ → As many digits Contain ('-'), Write 9 as many times;-

(b) $x = 0.\overline{78}$

$x = \frac{78}{99} = \frac{26}{33}$ Ans

Type -II

(c) $x = 0.3\overline{84}$

$= \frac{384-3}{990}$ → Number After Decimal − Number not Contain bar
→ I as many digit in (-), & 0 as many times not Contain (-),

$= \frac{381}{990} = \frac{127}{330}$ Ans

$x = 5.2\overline{48}$

$= \frac{5248-52}{9900} = \frac{5196}{9900} \quad \frac{1732}{3300}$ Ans

Type-III

(a) $2.6\overline{5}$

$\Rightarrow 2 + 0.6\overline{5}$

$= 2 + \frac{65-6}{90}$ (Same as type II)

$= 2 + \frac{59}{90} = \frac{239}{90}$ Ans

(b) $5.\overline{95}$

$= 5 + 0.\overline{95}$

$= 5 + \frac{95}{99}$

$= \frac{590}{99}$

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Divisibility Rules :=

NUMBER	RULE	EXAMPLE
2	Last digit is divisible by 2, or last digit is 0, 2 4, 6, 8.	Eg: 2348 1948
3	Sum of digit is divisible by 3.	Eg: 1071 1+0+7+1=9
4	Last two digit of number is divisible by 4	1432 9284
5	Last digit is 5 or 0	2335, 1990
6	Number is divisible by 2 and 3 each	132 → divisible by 2 1+3+2 → divisible 3
7	• Multiply last digit by 5 • Add the above number • If remaing digits divisible by 7, then number is divided by 7	Eg: 343 (i) 3×5 = 15 34-15 = 49 ↓ divisible by 7.
8	Last 3 digit are divisible by 8	8032 → 32 ↓ Divisible by 8.
9.	Sum of digits is divisible by 9	1071 → 1+0+7+1=9 ↓ divisible by 9
11.	• Defrence of Sum of digit at odd Places & Sum of digit at even places.	• 1331 (3+1) − (3+1) = 0 • 11718520 (1+7+8+2) − (1+1+5+0) = 11

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(2) If $3H2680$, is divisible by 11, then the Value of H is :

Soln: (Sum of Odd Place digit) − (Sum of Even Place digit)

$= (3+2+8) - (H+6+0)$

$= 13 - 6 - H$

$= 7 - H$ (Either 0 or divisible by 11)

$= 7 - H = 0$

$\boxed{H = 7}$ Ans.

Cyclicity:

Unit digit is repeated after some time of an enponent.

$2^1 = 2$ $2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 32$ $2^6 = 64$	$3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$ $3^5 = 243$ $3^6 = 729$	$4^1 = 4$ $4^2 = 16$ $4^3 = 64$ $4^4 = 216$	$7^1 = 7$ $7^2 = 49$ $7^3 = 343$ $7^4 = 2401$ $7^5 = 16807$
Cyclicity = 4	Cyclicity = 4	Cyclicity = 2	Cyclicity = 4
$8^1 = 8$ $8^2 = 64$ $8^3 = 512$ $8^4 = 4096$ $8^5 = 32768$	$9^1 = 9$ $9^2 = 81$ $9^3 = 729$ $9^4 = 6561$		
Cyclicity = 4	Cyclicity = 2		

Eg: $(2)^{423}$, Find the digit at units Place

Soln (a) divide the power by 4

In Exams divide in mind, not in Pen-Paper.

$$4\overline{)423}\,(105 \quad \text{Remainder} = 3$$

$$\begin{array}{r} 4 \\ \hline 23 \\ 20 \\ \hline 3 \end{array}$$

$2^3 = 8$ Ans

Unit digit and ten's digit Concept-

★ 1 2 3 4
- → Unit digit
- → Ten's digit

Eg: **Type-I**

(a) $29 \times 45 = 9 \times 5 = 45$ Unit digit = 5

(b) $18 \times 18 \times 18 + 3$

$8 \times 8 \times 8 + 3$

64×8

$32 + 3 = 35 \Rightarrow = 5$

Type = II

(a) (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

↓

Cyclicity Concept

(b) (0, 1, 5, 6)

↓

If these number are at unit Place unit digit of multiplication is also a same number.

Eg; (a) 35×35 (b) 36×96

$1225 \rightarrow$ Same $= 3456 \rightarrow$ Same

Helping Hand :-

(a) Divide the Power by 4.

(b) Remainder of division is 0, 1, 2, 3...

(c) Remainder $\Rightarrow 1 = n^1$ is unit digit
Remainder $\Rightarrow 2 = n^2$ is unit digit
Remainder $\Rightarrow 3 = n^3$ is unit digit
Remainder $\Rightarrow 0 = n^4$ is unit digit

If n^4 is 2 or 3 digit number, then unit digit of that number, will be the unit digit of Original Exponent.

Solved Examples

① What least number must be added to 1056, so that Sum is Completely divisible by 23?

Soln ⇒
$$23\overline{)1056}\,(45$$
$$\underline{92}$$
$$136$$
$$\underline{115}$$
$$21$$

then Number added is = 23-21
= 2 Ans

② The largest + 4 digit number exactly divisible by 88 is-

(a) 9944 (b) 9768 (c) 9988 (d) 8888

Soln ⇒ Largest 4 digit Number = 9999

$$88\overline{)9999}\,(113$$
$$\underline{88}$$
$$119$$
$$\underline{88}$$
$$319$$
$$\underline{264}$$
$$55 \longrightarrow \text{Sub tract From the 4 digit Largest+ number.}$$

= 9999−55 = 9944 Ans

③ If the number 517 H 324 is Completely divisible by 3, then the Smallest Whole no. in place of H will be.

(a) 0 (b) 1 (c) 2 (d) None

5+1+7+H+3+2+4
=22+H

If number is divisible by 3, then Sum of digit is also divisible by 3.

If 2 is used in place of H, then number is divisible by 3 (i.e-24)

④ Which one of the following no. is divisible by 11?

(a) 235641 (b) 245642 (c) 315624 (d) 415624

Soln⇒ (a) 235641
$(2+5+4) - (3+6+1) = 1$ (not divisible by 11)

(b) 245642
$(2+5+4) - (4+6+2) = 1$ (not divisible by 11)

(c) 315624
$(3+5+2) - (1+6+4) = -1$ (not divisible by 11)

(d) 415624
$(4+5+2) - (1+6+4) = 0$ (divisible by 11)

If a number is divisible by 11, the Difference of Sum of digit at odd places & Sum of digit at even places is either 0 Or divisible by 11.

⑤ Which on the following number is divisible by 24 -

Soln⇒ (a) 35718 (b) 63810 (c) 63810 (c) 537804 (d) 3125736

	③	⑧
35718	3+5+7+1+8 = 24 ✓	718 ✗
63810	6+3+8+1+0 = 18 ✓	810 ✗
537804	5+3+7+8+0+4 = 27 ✓	804 ✗
3125736	3+1+2+5+7+3+6 = 27 ✓	736 ✓ ✓

If a no. is divisible by another number then it must be divisible by it's prime factors.

Unit digit Concept:

⑥ The digit at unit's place of the Product-

$81 \times 82 \times 83 \ldots\ldots \times 89$ is

(a) 0 (b) 2 (c) 6 (d) 8

Soln⇒ $81 \times 82 \times 83 \times 84 \times 85 \ldots\ldots \times 89$

$1 \times 2 \times 3 \times 20 \ldots\ldots \times 6 \times 7 \times 8 \times 9$

$= 0$

If we multiply a number by 0, the result at unit place is always zero.

⑦ The digit in unit's place of the Product $(2153)^{167}$ is:

(a) 1 (b) 3 (c) 7 (d) 9

Soln⇒ $215\underline{3}$ ⟶ Let base is 3

ⓑ $\frac{167}{4}$ ⇒ Remainder is 3

ⓒ $3^3 = 27$ → unit digit is 7

⑧ Unit digit in $(264)^{102} + (264)^{103}$ is -

ⓐ 0 (b) 4 (c) 6 (d) 8

Soln⇒ $(264)^{102} + (264)^{103}$

$= 6 + 4$

$= 10$

Unit digit = 0

If Base is 4, then

(a)⇒ Unit digit of even power is always 6

(b)⇒ Unit digit of odd Power is always 4. because Cyclicity is 2

(9) Unit digit of $(169)^{537} + (94)^{394}$ is.

(a) (b) (c) (d)

Soln ⇒ $(169)^{537} + (94)^{394}$

$= 9 + 6$

$= 15$

= unit digit is 5 Ans

If the Base is 9
(a) Unit digit of ODD Power is always 9.
(b) unit digit of even Power is always 1.
because Cyclicity is 2.

(10) The digit in the unit place of
$[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + 259 + (73)$ is —

(a) 1 (b) 4 (c) 5 (d) 6

Soln ⇒ $(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + 259 + (73)^{51}$

$1 + 1 - 6 + 5 - 6 + 9 + 7$

Unit digit of base 1, 5, 6, is always Same

$\frac{51}{3}$ = Remainder 3
$3^3 = 27$

$= 23 - 12 = 11$ Ans

(11) Unit digital in expression of $(2137)^{754}$ is —

(a) 1 (b) 3 (c) 7 (d) 9

Soln ⇒ $(2137)^{754}$ → Base is 7

$\frac{754}{4}$ Remainder = 2

$7^2 = 49$ → unit digit is 9 ✓

(12) Find the unit's digit of $(358)^{64} - (253)^{36}$.

(a) 5 (b) 4 (c) 7 (d) 9

Soln ⇒ $(358)^{64} - (253)^{36}$

$\frac{64}{8}$ $\frac{36}{3}$

0 → Remainder ← 0 → 3^4

$8^4 = 64 \times 64 = 16 - 1$ ⇒ 6 − 1 = 5 Ans

Solved Examples

1- What least number must be added to 1056, so that sum is completely divisible by 23?

(a) 2 (b) 2 (c) 18 (d) 21

sol.

$$23\overline{)1056}\,(45$$

92

136

115

21

then number added is

$= 23-21$

$= 2.$

2- The largest 4 digit number exactly divisible by 88 is-

(a) 9944 (b) 9768 (c) 9988 (d) 8888

sol.

Largest 4 digit Number = 9999

$$88\overline{)9999}\,(113$$

88

119

88

319

264

55 → Subtract from the 4 digit Largest number

$= 9999 - 55$

$= 9944.$

3- If the number 517x324 is completely divisible by 3, them the smallest whole no. in place of x will be-

(a) 0 (b) 1 (c) 2 (d) None

sol.

$5+1+7+x+3+2+4$

$= 22 + x$

If number indivisible by 3 them sum of digit is also divisible by 3.

If 2 is used in place of x, then number is divisible by 3 (i.e. 24)

4- Which one of the following no. is divisible by 11?

(a) 235641 (b) 245642 (c) 315624 (d) 415624

sol. (a) 235641

$(2+5+4)-(3+6+1) = 1$ (not divisible by 11)

(b) 245642

$(2+5+4)-(4+6+2) = -1$ (not divisible by 11)

(c) 315624

$(3+5+2)-(1+6+4) = -1$ (not divisible by 11)

(d) 415624

$(4+5+2)-(1+6+4) = 0$ (divisible by 11)

If a number is divisible by 11, the Difference of Sum of digit of digit at odd places & Sum of digits of even place is either 0 or divisible by 11.

5- Which on of the following no. is divisible by 24?

(a) 35718 (b) 63810 (c) 537804 (d) 3125736

sol. 35718 — ③ $3+5+7+1+8 = 24$ ✓ — ⑧ 718 ✗

63810 — $6+3+7+8+1+0 = 18$ — 810 ✗

537804 — $5+3+7+8+0+4 = 27$ — 804 ✗

✓ 3125736 — $3+1+2+5+7+3+6 = 27$ — 736 ✓

If a no. is divisible by another number, then it must be divisible by its prime factors

6- The digit at unit's place of the product $81 \times 82 \times 83 \ldots \times 89$

(a) 0 (b) 2 (c) 6 (d) 8

sol. $81 \times 82 \times 83 \ldots\ldots \times 89$ is

(a) 0 (b) 2 (c) 6 (d) 8

Soln) $81 \times 82 \times 83 \times 84 \times 85 \ldots\ldots \times 89$

$1 \times 2 \times 3 \times 20 \ldots\ldots \times 6 \times 7 \times 8 \times 9$

$= 0$

If we multiple a number of 0, the result at unit place is always zero.