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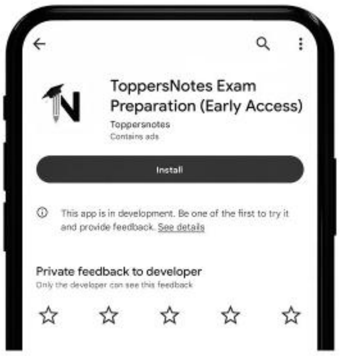

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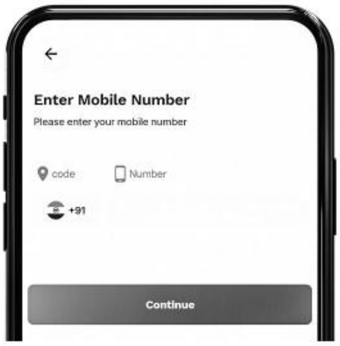

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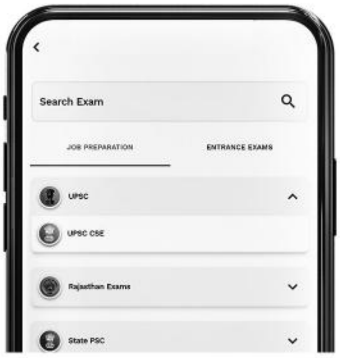

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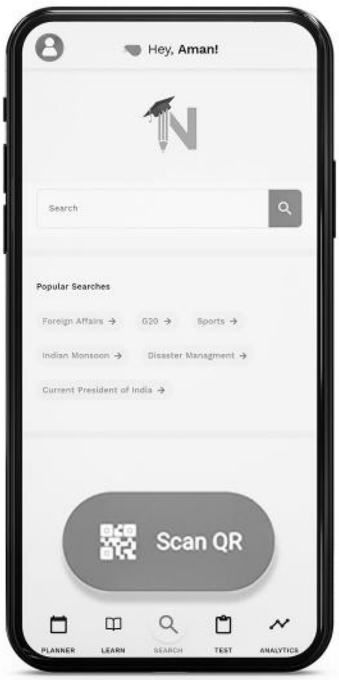

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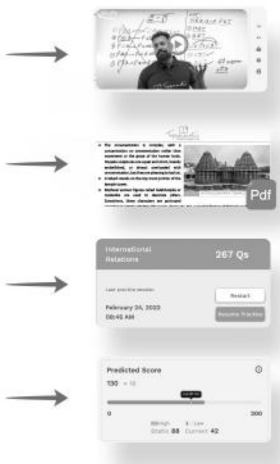

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CHAPTER 1

Moving Charges and Magnetism

CHAPTER OUTLINE

INTRODUCTION

- We learned about the electric force and electric field in previous chapters on electrostatics and current electricity.
- The magnetic force and magnetic field are two more key properties connected with moving charges.
- A magnetic field is created when current flows through a conductor, and any charge travelling through this field will experience a magnetic force that depends on the velocity (both magnitude and direction) as well as some feature of the field. In this chapter, we will look in depth at the properties and laws that regulate the magnetic field and magnetic force.
- Magnetic fields and forces have several industrial and medical applications.
- A common example is the employment of an electromagnet to lift heavy metal items. CD and DVD players, computer hard drives, loud speakers, headphones, televisions, and telephones all employ magnets.
- Magnets are everywhere around us. Magnets are utilised in everything from doorbells to autos to security alarm systems to hospitals.

LORENTZ FORCE: DEFINITION OF MAGNETIC FIELD B

- If an electric field and a magnetic field existence in a region, the force acting on a point charge in the region depends on both the charge's position and velocity.
- The force will be composed of two components: the electric force F_e and the magnetic force F_M.
- The power Fe is not affected by the charge's motion, but only by its position, whereas FM is affected by both the charge's velocity and position (see Fig. 21.1). F_e is magnitude is qE, and its direction is along E (q is positive).
- To determine the size and direction of Fm, we created a vector ***B*** known as magnetic flux density or magnetic induction, which characterises the magnetic field at a certain position.

- Experiments indicate that the force F_m is proportional to the magnitude of charge q, velocity v of the charge, and density B, with this force always perpendicular to the v and B vectors. Furthermore, if the charge moves along the direction of B at a point, the magnetic force acting on it is zero.
- We can summarize all these experimental results with the following vector equation:

  $\vec{F}_m = q\vec{v} \times \vec{B}$

  That is, the force F_m on the point charge is equal to the

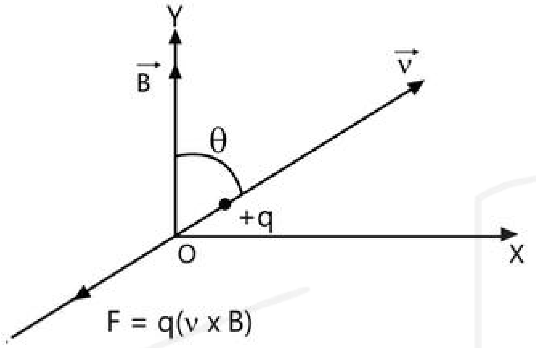


Figure: Magnetic Force on a Moving Charge

charge q times the cross product of its velocity v and the field B (all measured in the same reference frame). Using formula for the magnitude of cross product, we can write the magnitude of F_m as

$F_m = |q|vB\sin\theta$

where θ is the angle between the velocity v and magnetic field B.

- If angle θ is 90^0, then the above relation for magnetic force can be used to define the magnitude of magnetic flux density B as,

  $B = \frac{F_m}{|q|v_\perp}$

  where $v_\perp$ is the velocity component perpendicular to vector B.
- Thus, the total electromagnetic force acting on charge q is given as,

  $\vec{F} = \vec{F}_e + \vec{F}_m$

  or

  $\vec{F} = q\vec{E} + q\left[\vec{v} \times \vec{B}\right]$

  This is called Lorentz force.
- The unit of B is Tesla abbreviated as.
  If q =1C, v =1m/s, sinθ =1 ; θ = 90^0 and Fm = 1N , then B =1 T = 1Weber- m^{-2} .
- Thus 1 Tesla is defined as the unit of magnetic field strength in S.I units which when acting on 1C of charge moving with a velocity of 1m/s at right angles to the magnetic field exerts a force of 1N in a direction perpendicular to that of field and velocity vectors. C.G.S. units of magnetic field strength or magnetic induction is 1 gauss or 1 oersted. 1 gauss = 1 oersted = $10^{-4.}$

RELATION BETWEEN ELECTRIC AND MAGNETIC FIELD

- Assume that in a specific inertial reference frame K, the electric field is zero and the magnetic field has a finite non-zero value. A point charge travelling with velocity v in the frame K feels a magnetic force and its velocity changes as a result.
- Assume we have a frame K' moving with respect to frame K at a constant velocity v. The point charge is initially at rest in frame K', therefore the magnetic force on it is zero. However, while its velocity varies in the K frame, it also changes in the K' frame, implying that it experiences a force in the K' frame.
- Assume we have a frame K' moving with respect to frame K at a constant velocity v. The point charge is initially at rest in frame K', therefore the magnetic force on it is zero. However, while its velocity varies in the K frame, it also changes in the K' frame, implying that it experiences a force in the K' frame.
- As a result, the electric and magnetic fields are interconnected. We define electromagnetic field as a single physical entity. The frame of reference determines whether the electromagnetic field appears as an electric or magnetic field. If we confine ourselves to a specific reference frame, we can treat electric and magnetic fields as distinct things. In the general situation, a field that is constant in one reference frame is observed to fluctuate in another.

MAGNETIC FIELD LINES

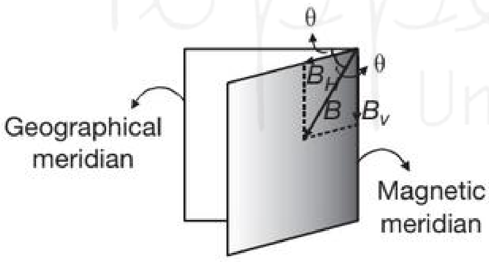
Magnetic field lines are used to represent the magnetic field in a given region. The rules for constructing magnetic field lines are as follows:

(a) The direction of the magnetic flux density vector B at a point is given by the direction of the tangent to a magnetic field line at that position.

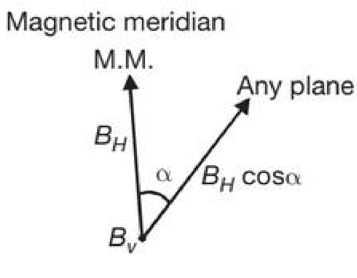
(b) At a given position, the density of magnetic field lines is proportional to the magnitude of vector B. The magnetic field is greater where the field lines are closer together.

EARTH'S MAGNETIC FIELD

- A magnetic field can be found anywhere near the earth's surface. The line of the Earth's magnetic field coincides with the magnetic north-south direction at that location, i.e. the plane going through the geomagnetic poles.
- The Magnetic Meridian is the name given to this plane. This plane is slightly inclined to the plane known as the geographic meridian, which runs through the geographic poles.



- The declination at a point is the angle formed by the magnetic meridian and the geographic meridian at that location.
- The magnetic poles of the Earth are located opposite to the geographic poles, i.e. near the North Pole, the magnetic south pole is located, and vice versa.

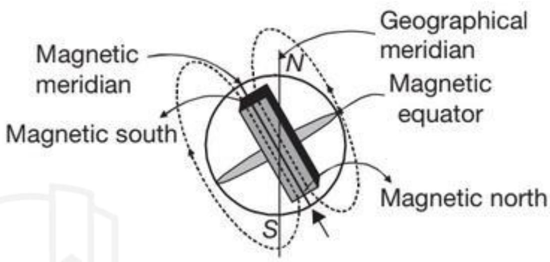


- The magnetic field vector of the earth at every place in the magnetic meridian plane is normally inclined to the horizontal at that point by an angle known as the magnetic dip at that site. If the earth's magnetic field at that point is B and the dip is θ

  B_V = the vertical component of in the magnetic meridian plane = Bsinθ

  B_H = the horizontal component of in the magnetic meridian plane = Bcosθ

  $$\frac{B_B}{B_H} = \tan\theta$$



MOTION OF CHARGED PARTICLE IN ELECTRIC AND MAGNETIC FIELD

Trajectory of a Charged Particle Moving in Uniform Electric Field

- Let a positively charged particle having charge +q and mass m enter at origin O with velocity v along X-direction in the region where electric field E is along the Y-direction (see Fig. 21,2).
- Force acting on the charge +q due to electric field E is given by

  $$\vec{F} = q\vec{E}$$

  Acceleration of the charged particle is

  $$\vec{a} = \frac{\vec{F}}{m} \text{ or } \vec{a} = \frac{q\vec{E}}{m} \quad \text{...(i)}$$

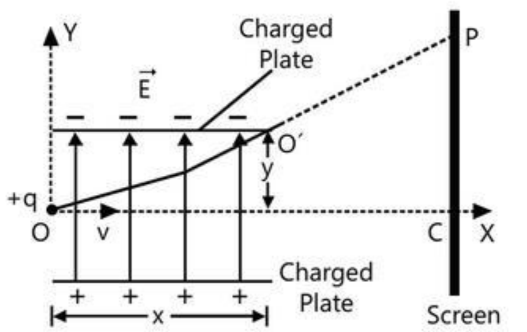


Figure: Charged particle moving in electric field

- The charged particle will accelerate in the direction of E, deviating from its straight line path.
- During its motion in the region of electric field, along X-axis we have

  $u_x = v$

  $a_x = 0$

  $x = vt$

  $$t = \frac{x}{v} \qquad ...(ii)$$

  along Y- axis we have

  $u_y = 0$

  $a_y = qE / m$

  $$y = \frac{1}{2}a_y t^2$$

  $$y = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

  Using Eq, (ii), we get

  $$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2 \text{ or } y = \frac{qEx^2}{2mv^2} = Kx^2 \qquad ...(iii)$$

  where

  $$K = \frac{qE}{2mv^2}i$$

  k is a constant. Thus The charged particle follows a parabolic path.

Trajectory of a Charged Particle Moving in Uniform Magnetic Field

(a) Magnetic force acting on a charged particle travelling parallel (θ = 00) or antiparallel (θ = 180^0) to B, will be zero. Thus the trajectory of the particle is a straight line.

(b) If the particle's velocity v is perpendicular to B, i.e. θ =90^0, then the magnetic force is F = qvB, and the direction of this force is always perpendicular to v. The charged particle goes in a circular path. (see Fig. 21.3).

(c) If the charged particle's velocity v makes an angle θ with B, the particle moves in a helical route. The perpendicular component vsinθ drives the charged particle in a circular route, whereas the parallel or antiparallel component vcosθ remains unaltered because there is no magnetic force along the direction of B. As a result, the charged particle follows a helical route

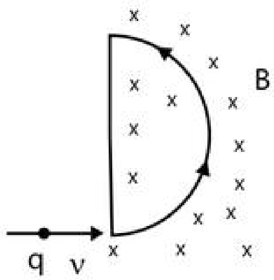


Figure: Charged particle moving in uniform magnetic field in electric field

(d) The magnetic force on the component of velocity perpendicular to the magnetic field gives the charged particle with the centripetal force to follow a circular trajectory of radius r.

$$qv_\perp B = \frac{mv_\perp^2}{r}$$

$$\text{or } r = \frac{mv_\perp}{qB}$$

Angular velocity,

$$\omega = \frac{v_\perp}{r} = \frac{qB}{m}$$

Frequency

$$f = \frac{qB}{2\pi m}$$

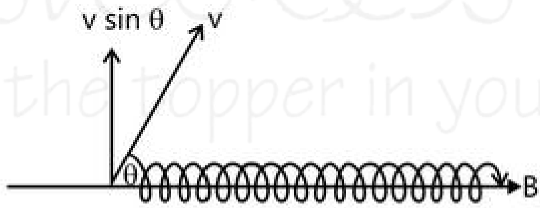


Figure: Charged particle moving in helical path in uniform magnetic field

$$T = \frac{2\pi m}{qB}$$

Time period T is independent of v.

DISCOVERY OF ELECTRON

The simplified version of Thomson's experiment is depicted in Fig. 21.5. By connecting a battery across their terminals, an electric field E is created in the region between the deflecting plates. The magnetic field B is directed into the figure's plane from the region between the deflecting plates.

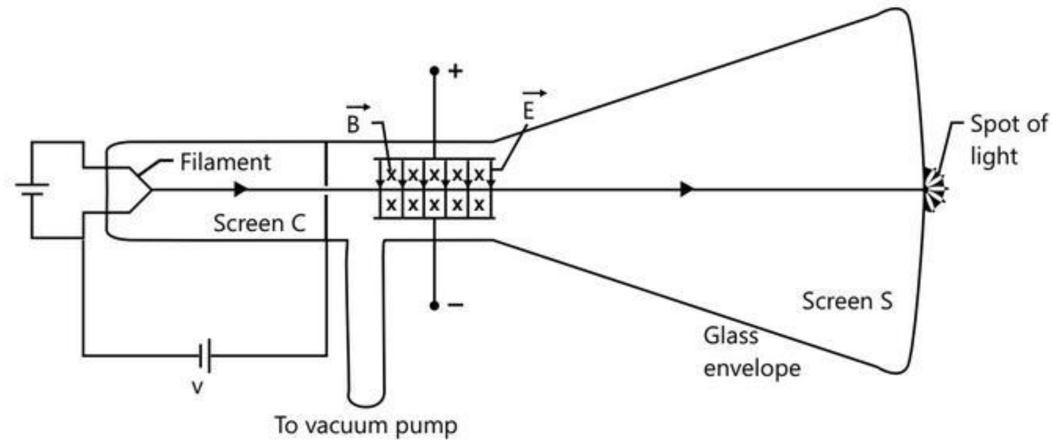


Figure: Thomson's experimental set up

- Charged particles (electrons) are accelerated by an applied potential difference V produced by a hot filament at the rear of the evacuated cathode ray tube.
- They produce a narrow beam after passing through a slit in screen C. They then pass through the area between the deflecting plates on their way to the centre of fluorescent screen S, where they generate a bright spot.
- Crossed-fields E and B in the region between the deflecting plates can deflect them away from the screen's centre. The deflection of charged particles can be adjusted by varying the size and direction of the fields, E and B.
- When both fields E and B are turned off, the charged particle beam arrives at the screen undeflected.
- When field E is activated and the charged particle beam is deflected.
- Keeping the field E unaffected, the B field is also enabled. The magnitude of B is changed until the deflection of the charged particles is zero. The magnetic force balances the electric force on the charged particles in this circumstance.

$$q\vec{E} = -q\vec{v}\times\vec{B}$$

$$\vec{E} = -\vec{v}\times\vec{B}$$

- The ratio of magnitudes of E and B in this situation gives the speed of the charged particles.

$$v = \frac{E}{B}$$

- When only the E field is activated, the displacement of the charged particles in the y-direction, when they reach the end of the plates,

$$y = \frac{|q|EL^2}{2mv^2}$$

where v is the particle's speed along the x-axis, m is its mass, q is its charge, and L is the length of the plates. The direction of deflection of charged particles indicates that the particles are negatively charged.

- Substituting the value of v in terms of E and B we get,

$$y = \frac{|q|B^2L^2}{2mE}$$

$$\frac{m}{|q|} = \frac{B^2L^2}{2yE}$$

- Thus in this way the mass to charge ratio of electrons was discovered.

HALL EFFECT

- The Hall Effect is defined as the generation of a voltage difference (the Hall voltage) across a current-carrying wire in a magnetic field perpendicular to the current. The hall voltage is generated in the opposite direction of the electric current in the conductor. Edwin Hall found it in 1879. The Hall Effect helps us to determine whether the

- The charge carried in a conductor is either positively or negatively charged, as well as the number of charge carries per unit volume of the conductor.
- External magnetic field B, points into the plane of a copper strip of width d, carrying a current I as shown in Fig. 21.6.
- The magnetic force F_m will act on each electron travelling towards the strip's right edge. As electrons build on the right edge, positive charges accumulate on the left edge, creating an electric field E within the strip that is directed from left to right.
- This field applies an electric force Fe on each electron near the strip's left edge. The electric field E causes the hall potential difference V over the width of the strip to be V = Ed.
- When the electric and magnetic forces balance each other,

  $eE = ev_dB$ or $E = v_db$

  The drift speed v_d is given as

  $$v_d = \frac{J}{ne} = \frac{I}{neA}$$

  So we obtain $n = \frac{BI}{V\ell e}$ where $\ell\left(\frac{\text{Cross} - \text{section Area}}{\text{Width}}\right)$ is the thickness of the strip.

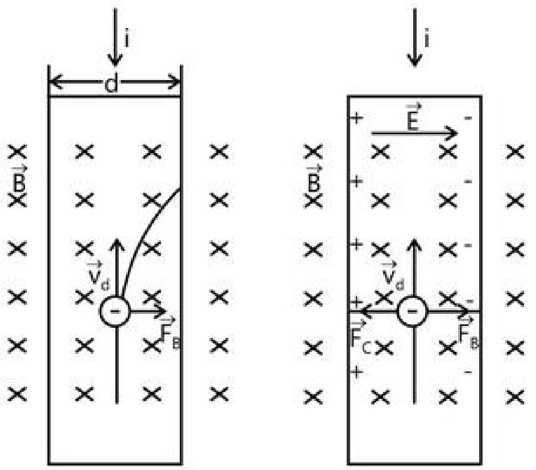


Figure: Hall Effect in conductor

MAGNETIC FORCE ON A CURRENT CARRYING WIRE

- If the number of free electrons per unit volume in a conductor is n, then the total charge of free electrons in an infinitesimal volume dV in the conductor will be

  dq = ne dV
- If the magnetic field at the elementary volume's position is B and the drift velocity of free electrons is v_d, then the magnetic force on the elementary volume is

  $$d\vec{F} = ne\left[\vec{v}_d \times \vec{B}\right]dV$$
- Now we know that the current density is given as

  $$\vec{j} = ne\vec{v}_d$$

  $$d\vec{F} = \left[\vec{j} \times \vec{B}\right]dV$$

  We can write, after introducing the vector in the direction of current, $\vec{j}dV = \vec{j}\Delta Sd\ell = Id\vec{\ell}$.

  Here ΔS is the area of cross-section and the length of the elementary volume dV.

  So

  $$d\vec{F} = I\left[d\vec{\ell} \times \vec{B}\right]$$

  The total magnetic force on the conductor is

  $$\vec{F} = I\int\left[d\vec{\ell} \times \vec{B}\right]$$
- If the field B is constant throughout the length of the wire and perpendicular to it, we can write for a thin straight wire of length L

  F = I L B

  In vector form we can write,

  $$\vec{F} = I\vec{L} \times \vec{B}$$

  where L is a length vector with magnitude L that runs down the wire segment in the direction of the (conventional) current.
- The following are some key points about the force on a current carrying conductor in a magnetic field:

  (a) In a uniform magnetic field, the force, dF = IBdl sinθ is independent of the present element's position vector. As a result, this force is non-central. (A central force varies with position vector.) r,F = f (r)

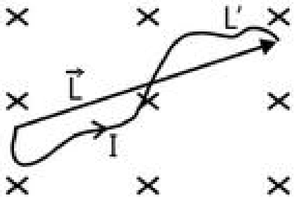


Figure: Current carrying conductor in uniform magnetic field

(b) The force dF is always perpendicular to the plane in which B and dl are located. Vectors B and dl are not always perpendicular to one other.

(c) As explained above, the total magnetic force on the conductor is

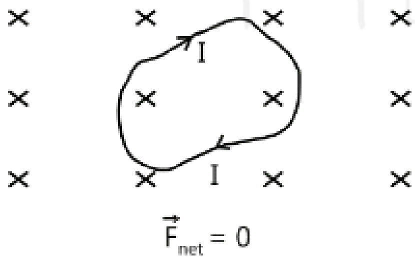
$\vec{F} = I\int\left[d\vec{\ell}\times\vec{B}\right]$

For uniform magnetic field, B can be taken out from the integral.

$\vec{F} = I\int\left[d\vec{\ell}\right]\times\vec{B}$

- According to the law of vector addition $\int d\vec{\ell}$ is equal to the length vector L from initial to final point of the conductor as shown in Fig. 21.7. For a conductor of any arbitrary shape the magnitude of vector L is different from the actual length L' of the conductor.

$\therefore\ \vec{F} = I\vec{L}\times\vec{B}$



(d) For a current carrying closed loop of any arbitrary shape placed in a uniform magnetic field

$\vec{F} = I\left[\oint d\vec{\ell}\right]\times\vec{B} = 0$

When we add all of the elementary vectors dl around the closed loop, the vector sum is zero because the final point is the same as the initial position.

$\therefore\ \oint d\vec{\ell} = 0$

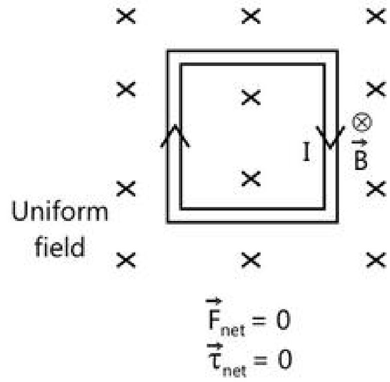


Figure: Area vector of closed loop is in direction of uniform magnetic field

- As a result, the net magnetic force on a current loop in a homogeneous magnetic field is always zero. However, different sections of the loop experience different net forces, even if the vector sum of all these forces is zero.
- As a result, the loop may undergo some microscopic contraction or expansion, putting it under tension.
- Although the resultant of magnetic forces operating on the loop is zero, the resultant torque may not be zero.
- As a result, with a homogenous magnetic field, the torque on a loop is not necessarily zero.

(e) When a current-carrying closed loop is placed in a non-uniform magnetic field, it will experience non-zero net force and torque.

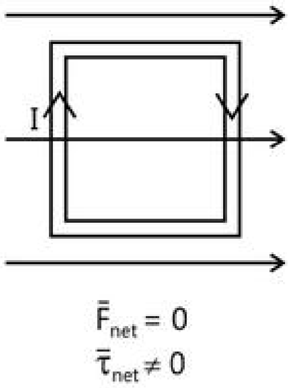


Figure: Area vector of closed loop is perpendicular to uniform magnetic field magnetic field

- In a non-uniform field, even a conductor of arbitrary shape that does not form a loop will feel torque. If the conductor is free to move, it will move in both directions at the same time.

  (f) When a current carrying conductor or closed loop moves or spins in a magnetic field, the kinetic energy obtained is attributable to the work done by magnetic forces rather than the energy given by the electric source that keeps current flowing in the conductor/loop.

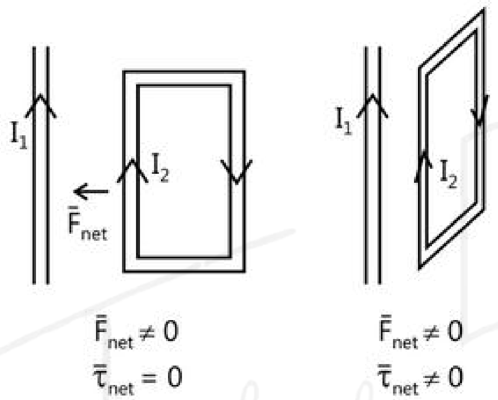


Figure: Closed loop in non-uniform magnetic field

- Magnetic forces operating on a current carrying conductor produce no network. Despite what appears to be the case,

  $$W = \int \vec{F}.d\vec{r} = \int \left[I \int \left(d\vec{\ell} \times \vec{B} \right) \right].d\vec{r} = \Delta K$$

  but actually the kinetic energy is supplied by the electric source.

Fleming's Left Hand Rule

- If the thumb and the first two fingers of the left hand are stretched perpendicular to each other, and the first finger points in the direction of the magnetic field and the second middle finger points in the direction of the current in the conductor, then the thumb's direction gives the direction of force on the conductor.

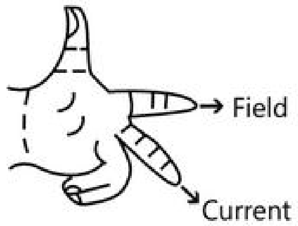
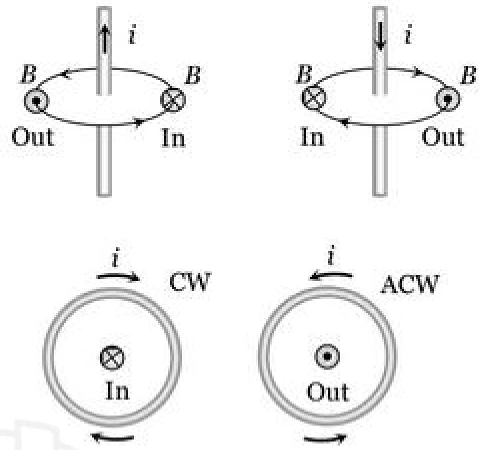


Figure: Fleming's Left hand Rule



TORQUE ON A CURRENT LOOP

- Consider a square loop PQRS with side l and area A = l^2 (Figure). Let us introduce a unit vector n normal to the plane of the loop, the direction of which is connected to the current direction in the loop by the right-hand screw rule. The loop's area can be expressed in vector form as.

  $\vec{A} = \ell^2 \hat{n}$

- If the current I in the loop is anti-clockwise, the vector n will be oriented towards the reader along the perpendicular to the plane of the paper, as shown in Fig. 21.13. Assume the loop is placed in a uniform magnetic field B directed perpendicular to the plane of the paper, i.e. along the vector n. In this case, the magnitude of magnetic force on each branch of the loop is, i.e.

  $I\ell B$, i.e. $|\vec{F}_1| = |\vec{F}_2| = |\vec{F}_3| = |\vec{F}_4| = I\ell B$

- The direction of force on each branch can be found by Fleming's left hand rule.
- We can easily see that $F_1 = -F_3$ and F_1 and F_3 have same line of action. Similarly $F_2 = -F_4$ and F_2 and $-F_4$ have same line of action. So, the net force as well as the net torque on the loop PQRS is zero.

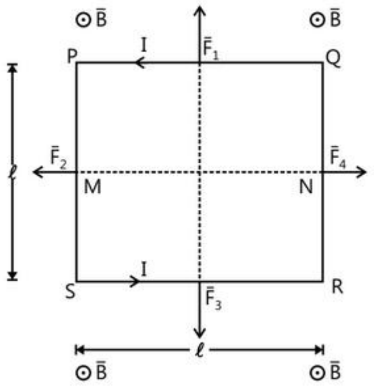


Figure: Zero torque on closed loop in uniform magnetic field

- Now suppose the loop is rotated with an angle θ about the line MN as shown in Fig. 21.14). So the angle between vector n and B will be θ.
- In this situation each of the sides Q'R' and S'P' makes an angle (90-θ) with the magnetic field B so that

  $\left|\vec{F}_2\right| = \left|\vec{F}_4\right| = I\ell B\cos\theta$

  and again we have F_2 = -F_4 and F_2 and F_4 have same line of action. The side PQ shifts to P'Q' and RS shifts to R'S' such that PQ II P'Q' and RS II R'S' so that

  $\left|\vec{F}_1\right| = \left|\vec{F}_3\right| = I\ell B$

  and again we have F1 = -F3, but the lines of action of F1 and F3 are displaced from each other by a distance of lsinθ. This forms a force couple, and the torque due to it will have magnitude

  $\tau = \left(I\,\ell\,B\right)\ell\sin\theta = I\ell^2 B\sin\theta = IAB\sin\theta$

- This torque is directed along the line MN.

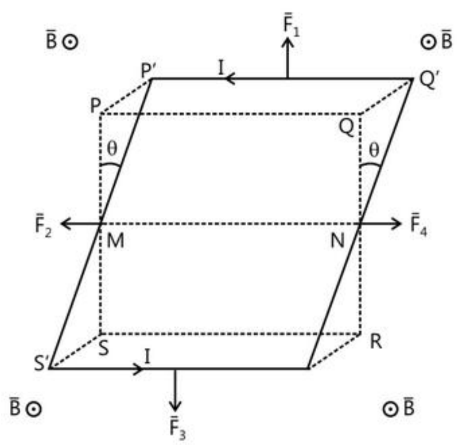


Figure: 14 Non-zero torque on closed loop in uniform magnetic field

In vector form we can write

$\vec{\tau} = I\vec{A} \times \vec{B}$

Defining magnetic dipole moment of the loop as,

$\vec{M} = I\vec{A} = IA\hat{n}$

we can write torque as

$\vec{\tau} = \vec{M} \times \vec{B}$

If the number of turns in the loop is N then we have,

$\vec{M} = NI\vec{A} = NI\,A\hat{n}$

Note that although this formula has been derived for a square loop, it comes out to be true for any shape of the loop.

MAGNETIC DIPOLE MOMENT

- Every current carrying loop behave like a magnetic dipole. It has two poles, north (N) and south (S) similar to a bar magnet. (see Fig. 21.15) Magnetic field lines are closed paths directed from the North Pole to the South Pole in the region outside the magnetic dipole and from South Pole to North Pole inside the magnetic dipole.

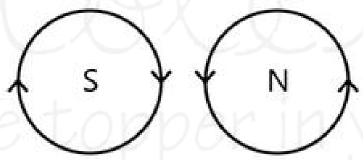


Figure: North and South Pole of current coil

- The magnetic dipole moment of each loop is defined as, M = NIA. where N denotes the number of turns in the loop, I is the current in the loop, and A denotes the area of the loop's cross-section. M can be directed using any of the following methods:
  (a) We travel inside the magnetic dipole in the direction of M, from South Pole to North Pole. The sense of current can be used to identify the North and South Poles of a current loop. The South Pole is the side where the current appears to flow clockwise, while the North Pole is the opposite side where it appears to flow anticlockwise.

(b) Vector M is perpendicular to the plane of the loop. The right hand screw rule connects the direction of M to the present direction in the loop. Curl the fingers of the right hand around the perimeter of the loop in the direction of current, as shown in Fig.21.16. The thumb is then stretched perpendicular to the plane of the loop and directed towards M.

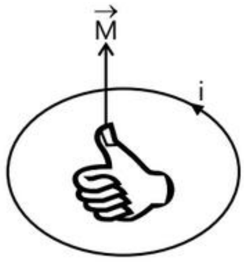


Figure: Right hand screw rule

- The potential energy U of a magnetic dipole placed in a uniform magnetic field is
$U = -MB\cos\theta$
$U = -\vec{M}.\vec{B}$
- For a bar magnet we define the magnetic dipole moment as
$\vec{M} = m\vec{\ell}$
Here m is the pole strength of the bar magnet and vector L is directed from South Pole to North Pole.
The unit of magnetic dipole moment is. A -m^{-2}

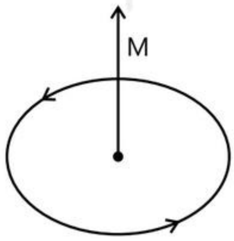


Figure: Direction of magnetic moment

- The magnetic field at a long way on the magnetic axis of a bar magnet with magnetic dipole moment M equals
$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right)$
- The magnetic field at a large distance x on the perpendicular bisector of a bar magnet having magnetic dipole moment M is
$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right)$

BIOT-SAVART LAW

- The magnetic field for a long distance on a bar magnet's magnetic axis with magnetic dipole moment M $d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\cdot\frac{Id\vec{\ell}\times\vec{r}}{r^3}$

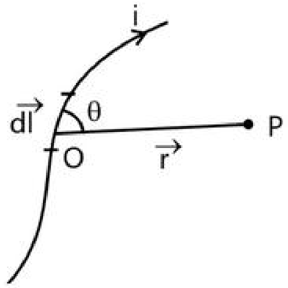


Figure: Magnetic field due to current element $d\ell$

- The position vector r here is the vector from the centre of the element of length dl to the point of observation P. The direction of dl is parallel to the direction of current I passing through it θ If is the angle formed by r with respect to the conductor's length dl, the amplitude of magnetic induction is given by
$\left|d\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{Id\ell(r\sin\theta)}{r^3}$
$\left|d\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{Id\ell(\sin\theta)}{r^2}$
Here μ_0 is the permeability of free space and
$\frac{\mu_0}{4\pi} = 10^{-7}$ Tesla-meter/ampere
- The direction of dB is perpendicular to the plane containing current element dl and radius vector r which joins dl to P.
- The total magnetic induction due to the conductor is given by.
$\vec{B} = \int d\vec{B}$
- The magnetic intensity H at any point in the magnetic field is related to the magnetic induction as
$H = \frac{B}{\mu}$ or $B = \mu H$
where μ is permeability of the medium. The unit of magnetic intensity H is A-m^{-1}

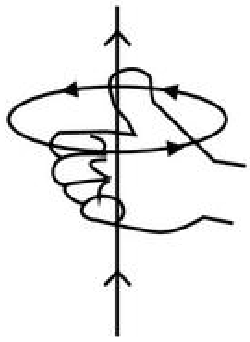


Figure: Right hand thumb rule

Maxwell's Cork Screw Rule:
When a right-handed cork screw is turned so that its tip moves in the direction of current flow through the conductor, the direction of rotation of the screw's head gives the direction of magnetic field lines around the conductor.

Right Hand Rule: If we hold the conductor in the right hand with the thumb stretched in the direction of current, the direction in which the fingers curl indicates the magnetic field direction.

Application of Biot-Savart Law

Biot-Savart law is used to find the magnetic field due to current carrying conductors.

Magnetic Induction Due to Infinitely Long Straight Current Carrying Conductor

- Assume I flows through a long straight current carrying conductor. We want to find the magnetic field at point P, which is perpendicular to the conductor. As illustrated in Fig. 21.23.
- Biot-Savart law gives the magnitude of the field dB at P due to an infinitesimal element of length dl as:

$$|d\vec{B}| = dB = \frac{\mu_0 I d\ell \sin(90+\alpha)}{4\pi x^2}$$

where X is the distance between the current element and point P. The field dB is directed into the plane of the figure and perpendicular to it.

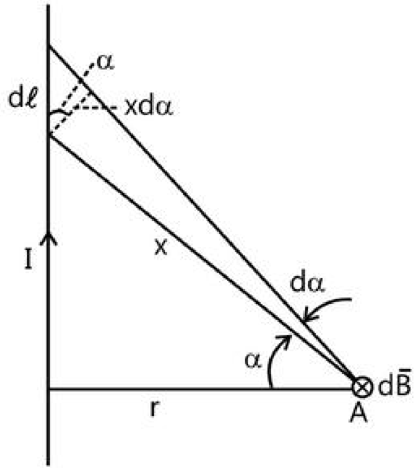


Figure: Magnetic field due to infinitely long straight wire

Now from Fig. 21.23. it is clear that, dlcosα and x = r / cosα so we can write,

$$dB = \frac{\mu_0 I}{4\pi}\frac{\cos\alpha\, d\alpha}{r} \qquad ...(i)$$

- Because the conductor is infinitely long, as the angle varies from -π/2 to π/ 2, the infinitesimal element covers the entire length of the conductor, and the field dB is directed into the plane of the figure for all infinitesimal elements that comprise the conductor.
- Thus, we may combine the magnitudes of dB due to all the infinitesimal constituents to get the entire field magnitude as,

$$B = \frac{\mu_0 I}{4\pi r}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\alpha\, d\alpha = \frac{\mu_0 I}{2\pi r}$$

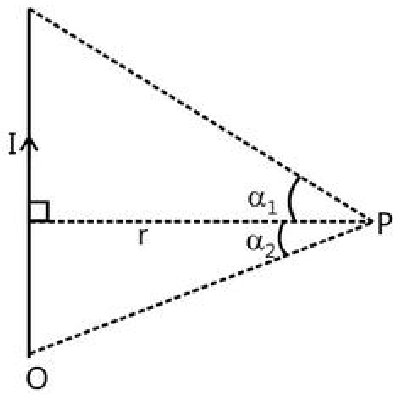


Figure: Magnetic field due to finite straight wire

A Straight Conductor of Finite Length

- If a conductor of finite length subtends an angle α_1 on one side α_2 and on the other side of perpendicular from point P as shown in Fig. 21.24 then we can write,

$$B = \frac{\mu_0 I}{4\pi r}\int_{-\alpha_2}^{\alpha_1} \cos\alpha\, d\alpha = \frac{\mu_0 I}{4\pi r}\left|\sin\alpha\right|_{-\alpha_2}^{\alpha_1}$$

$$= \frac{\mu_0 I}{4\pi r}\left[\sin\alpha_2 + \sin\alpha_2\right] \qquad \text{...(ii)}$$

At the End of a Straight Conductor of Infinite Length

In this case, the angle varies from 0 to π/2, and we can write

$$B = \frac{\mu_0 I}{4\pi r}\int_{0}^{\frac{\pi}{2}} \cos\alpha\, d\alpha = \frac{\mu_0 I}{4\pi r}$$

At The End of a Straight Conductor of Finite Length

In this case, (see Fig. 21.25) the angle varies from 0 to α, and we can write

$$B = \frac{\mu_0 I}{4\pi r}\int_{0}^{\alpha} \cos\alpha\, d\alpha = \frac{\mu_0 I \sin\alpha}{4\pi r}$$

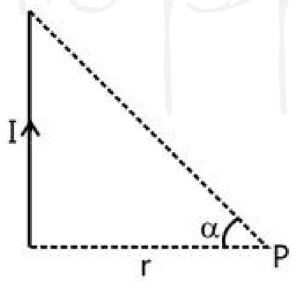


Figure: Magnetic field at end of straight wire of finite length

At a Point Along the Length of the Straight Conductor Near Its End

In this case (see Fig. 21.26) $\alpha_1 = \pi/2$ and , $\alpha_2 = -\pi/2$ and thus equation (ii)gives B = 0 . Actually in this case the value of does not vary at all i.e. it is constant (at all points of the wire we have α = π /2), thus dα = 0and thus equation (i) gives .dB = 0

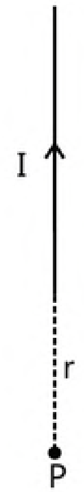


Figure: Magnetic field along length of straight wire

Magnetic Field on the Axis of a Current Carrying Circular Arc

If a current I flows in a circular arc of radius R lying in the Y-Z plane with centre at origin O and subtending an angle ϕ at O, then Biot-Savart Law gives the magnetic field dB at a point P on the -axis with coordinates (X,0,0) due to a small elementary arc of length dl= Rdθ at a distance r from P as:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$$

where r is a vector from midpoint of dl to P.

As shown in Fig. 21.28 the coordinates of dl are (0, Rcosθ, Rsinθ), where θ is the angle between the radius of the arc through dl and the X-axis.

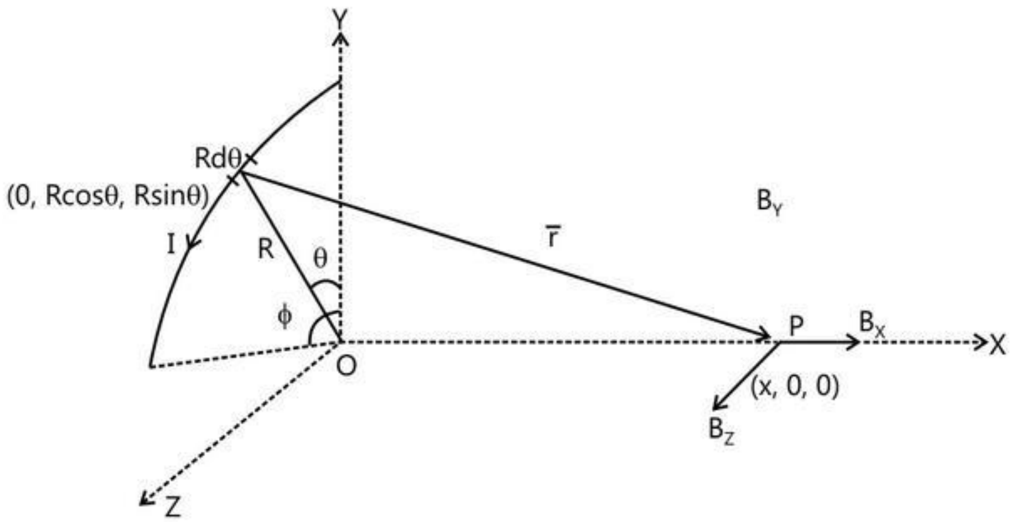


Figure: Magnetic field at a point on the axis of current carrying arc

So we can write

$\vec{r} = x\hat{i} - R\cos\theta\hat{j} - R\sin\theta\hat{k}$ ...(ii)

Magnitude

$r = \sqrt{x^2 + R^2}$ ...(iii)

Let us express dl in Cartesian coordinates system as shown in Fig. 21.29.

$d\vec{\ell} = -R\sin\theta\, d\theta\,\hat{j} + R\cos\theta\, d\theta\,\hat{k}$ ...(iv)

Put (ii), (iii) and (iv) in (i) to get

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(-R\sin\theta\, d\theta\,\hat{j} + R\cos\theta\, d\theta\hat{k}\right)\times\left(x\hat{i} - R\cos\theta\,\hat{j} - R\sin\theta\hat{k}\right)}{\left(\sqrt{x^2+R^2}\right)^3}$$

$$\Rightarrow\ d\vec{B} = \frac{\mu_0 I}{4\pi\left(x^2+R^2\right)^{3/2}}\left(R^2 d\theta\,\hat{i} + xR\cos\theta\, d\theta\,\hat{j} + xR\sin\theta\, d\theta\,\hat{k}\right)$$

Resultant magnetic field at is

$$\vec{B} = \frac{\mu_0 I}{4\pi\left(x^2+R^2\right)^{3/2}}\left(R^2\int_0^{\phi} d\theta\,\hat{i} + xR\int_0^{\phi}\cos\theta\, d\theta\hat{j} + xR\int_0^{\phi}\sin\theta\, d\theta\hat{k}\right)$$

$$\Rightarrow\ \vec{B} = \frac{\mu_0 I}{4\pi\left(x^2+R^2\right)^{3/2}}\left[R^2\phi\hat{i} + xR\sin\phi\hat{j} + xR\left(1-\cos\phi\right)\hat{k}\right]$$

Thus B can be resolved into components parallel to the x,y and z the axes.

$$B_x = \frac{\mu_0 I R^2\phi}{4\pi\left(x^2+R^2\right)^{3/2}}$$

$$B_y = \frac{\mu_0 I R x\sin\phi}{4\pi\left(x^2+R^2\right)^{3/2}}$$

$$B_z = \frac{\mu_0 I R x\left(1-\cos\phi\right)}{4\pi\left(x^2+R^2\right)^{3/2}}$$

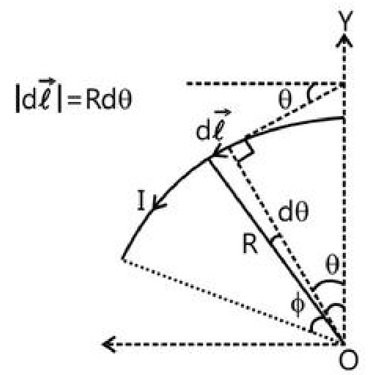


Figure: Vector is in the YZ plane

The field at center of the arc: At center x = 0, so

$$B_x = \frac{\mu_0 I \phi}{4\pi R}$$

$$B_y = 0$$

$$B_z = 0$$

Thus at the center the field is normal to the plane of the arc.

For a semicircular loop, the angle subtended at the center is,

$\phi = \pi$, so $B = \frac{\mu_0 I}{4r}$

Magnetic Field on the Axis of a Current Carrying Circular Loop

The field on the axis of a current carrying circular loop (see Fig. 21.30) can be determined by putting the value of angle subtended at the centre as into the expression for a current carrying circular arc derived in the previous article.

$$\therefore \quad \vec{B} = \frac{\mu_0 I}{4\pi\left(x^2 + R^2\right)^{3/2}}\left[R^2(2\pi)\hat{i} + xR\sin 2\pi\hat{j} + xR(1-\cos 2\pi)\hat{k}\right]$$

$$\therefore \quad \vec{B} = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}\hat{i}$$

Thus field B is directed along the axis of the circular loop.

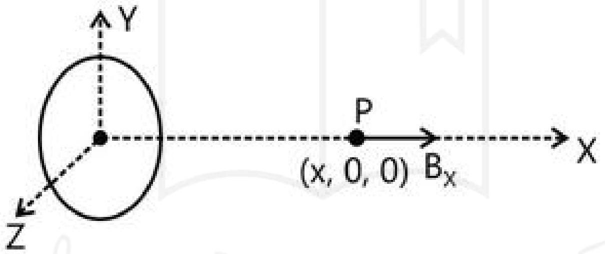


Figure: Magnetic field at a point on the axis of circular loop

For a coil having N circular turns,

$$B = \frac{\mu_0 N I R^2}{2\left(R^2 + x^2\right)^{3/2}}$$

The field at center of the coil:

At center x = 0, so

$$B_0 = \frac{\mu_0 N I R^2}{2R^3}$$

$$B_0 = \frac{\mu_0 N I}{2R}$$

The following is the direction of B at the centre of a circular current carrying arc or closed circular loop:

If we curl the fingers of the right hand in the direction of the current in the arc/loop, the stretched thumb points in the direction of the field at the centre.

If the point P is at a very large distance from the coil, then $X^2 >>> R^2$

$$B = \frac{\mu_0 N I R^2}{2x^3}$$

If A is area of one turn of the coil,

$$A = \pi R^2 \quad B = \frac{\mu_0 N I A}{2\pi x^3}$$

Different forms of Biot-Savarts law

Vector form	**Biot-Savarts law in terms of current density**	**Biot-savarts law in terms of charge and it's velocity**
Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{i\left(d\vec{l}\times\hat{r}\right)}{r^2} = \frac{\mu_0}{4\pi}\cdot\frac{i\left(d\vec{l}\times\vec{r}\right)}{r^3}$	In terms of current density $d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{J}\times\vec{r}}{r^3}dV$ where	In terms of charge and it's velocity, $d\vec{B} = \frac{\mu_0}{4\pi}q\frac{\left(\vec{v}\times\vec{r}\right)}{r^3}$

⇒ Direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$. This is given by right hand screw rule.	$j=\frac{i}{A}=\frac{idl}{Adl}=\frac{idl}{dV}$ = current density at any point of the element, dV = volume of element	$\because i d\vec{l}=\frac{q}{dt}d\vec{l}=q\frac{d\vec{l}}{dt}=q\vec{v}$

FORCE BETWEEN PARALLEL CURRENTS

- Consider two long wires kept parallel to one other and separated by a tiny distance relative to their lengths.
- Suppose currents I_1 and I_2 flow through the wires in the same direction (see Fig. 21.33). Consider a small element dl of the wire carrying current I_2 The magnetic field at due to the wire carrying current I_1 is

  $\vec{B}=\frac{\mu_0 I_1}{2\pi d}\left(-\hat{k}\right)$

  (B is normal to and directed into the plane of the figure)

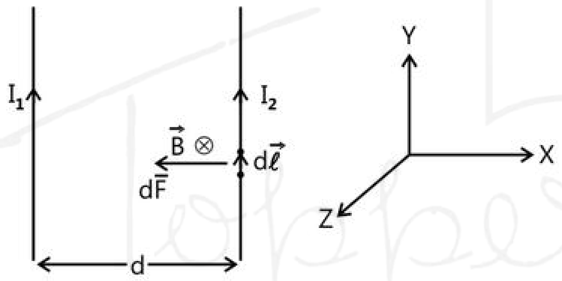


Figure: Force between parallel currents

The magnetic force on this element is

$d\vec{F}=I_2 d\vec{\ell}\times\vec{B}=I_2 d\ell\left(\hat{j}\right)\times B\left(-\hat{k}\right)$

$d\vec{F}=I_2 d\ell B\left(-\hat{i}\right)=\frac{\mu_0 I_1 I_2}{2\pi d}d\ell\left(-\hat{i}\right)$

(directed towards the wire carrying current I_1)

- Thus the wire carrying current I_2 is attracted towards the wire carrying current I_1. By Newton's third law the force acting on wire carrying current I_1 will also be attractive. Thus the two wires are attracted towards each other.
- The force per unit length on each of the wires due to the other wire will be,

  $\frac{dF}{d\ell}=\frac{\mu_0 I_1 I_2}{2\pi d}$

Parallel currents attract each other, and antiparallel currents repel each other.

Note: Memorizing various formula of magnetic field due to ring and wire carrying current would easily help in calculating magnetic field due to complicated wire systems. Also, be careful about the direction of field in every problem you solve.

AMPERE'S LAW

- This law is also known as the 'Theorem on Circulation of Vector B'.
- This law states that the line integral or circulation of magnetic field vector B around a closed channel is equal to μ_0 times the algebraic sum of the currents enclosed by the closed path.

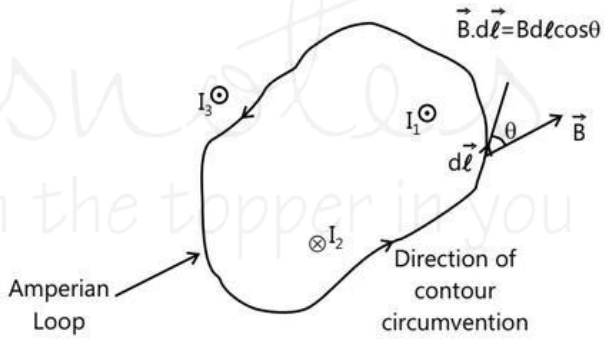


Figure: Current enclosed by amperian loop

$\oint\vec{B}\cdot d\vec{\ell}=\mu_0 I_{enc}$

- The closed path is also called Amperian loop.
- I_{enc} is the algebraic sum of all currents travelling through the closed path's region. Current is assumed to be positive if it flows in the direction associated with the evasion of the closed path by the right-hand screw rule. Curling the fingers of the right hand around the closed passage in the direction of circumvention yields the positive direction of current. Negative current flows in the opposite direction.