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ENGINEERS ACADEMY

RRB - JE

MECHANICAL

Railway Recruitment Board

Volume - 4

Theory of Machines (TOM)

SIERRA INNOVATIONS

THEORY OF MACHINE

THEORY

1. MACHANISM :

If a number of bodies assembled in such a way that the motion of one causes constrained and predictable motion to others, it is known as 'Mechanism'.

It transmit or modify a motion e.g. slider-crank mechanism, type writer, spring toys.

- **Machine:**

A machine is a machanism or a combination of mechanism which, apart from imparting motion to the part, also transmits and modifies the available mechanical energy into some kind of desired work.

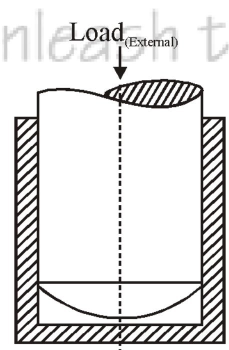
- **Type of Constrained Motion**

Completely constrained motion

When motion between two elements of a pair is in a definite (*single*) direction irrespective of the direction of force applied. It is known as completely constrained motion.

Succesfully constrained motion

When motion between two element of pair is *possible* in *more than one* direction but is made to have motion *only in one* direction by using some *external* means, it is called succesfully constrained motion e.g.



Foot Step Bearing

A pistion in a cylinder of an internal combustion engine is made to have only reciprocating motion due to constrain of the pistion pin (external), cam and follower, shaft in foot step bearing.

Incompletely constrained motion:

When the motion between the elements of a pair is possible in *more than one direction* and depends upon the *direction of force applied,* it is known as incompletely constrained motion e.g. cylindrical shaft in round bearing.

Ridig, Resistant Body

Rigid body:

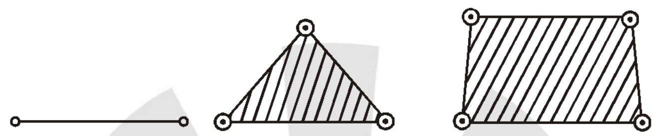
It does not suffer any distortion, under the action of force.

Resistant body:

Those body which are rigid for the purouse they have to serve for e.g. belt drive, where belt is rigid when subjected to tensile forces. Resistant bodies transmit the requried forces with negligivble deformation.

Link

A link id defined as a member of machanism, connecting other member and having motion relative to them.



Binary link Ternary link Quaternary link

Classification of Kinematic Pair

According to nature of contact:

- Lower pair:

A pair of links having *surface or area* contact between the member. e.g. nut turning on a screw, shaft fotating in a bearing, all pair of slider-crank machanism.

- Higher pair

When a pair has point or line contact between the links, it is known as higher pair.

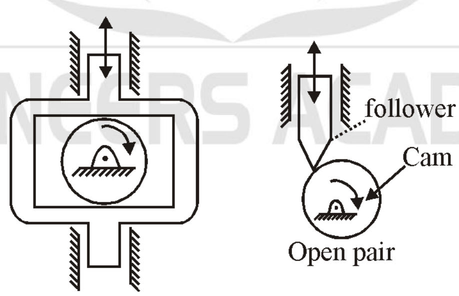
e.g. wheel rolling on a surface, cam and follower pair.

According to Nature of Machanical Constraint

- Closed pair

When the element of pair held *mechanically,* it is known as closed pair.

The contact between the two can be broken by only destruction of *at least one* of the member.



Open pair

Point to remember:

- All lower pairs are closed pair.
- Open (unclosed pair)

When two links of a pair are in contact either due to force of gravity or some spring action.

e.g. cam follower.

According to Nature of Relative Motion

- Sliding pair

  If two links have sliding motion relative to each other, they form sliding pair e.g.

  A rectangular rod in a rectangular hole in a prism.

- Turning pair

  When one link has turning or revolving motion relative to the other, they constitute a turning or revolving pair e.g. curcular shaft revolving in a bearing.

- Rolling pair

  when the link of pair have rolling motion relative to each other, they form a rolling pair, e.g.

  Rolling wheel on flat surface, ball and roller bearing.

- Screw pair (helical)

  It two mating link have turning as well as sliding motion between them form a screw pair e.g. lead screw, nut of lathe.

- Spherical pair

  When one link in the form of a sphere turns inside a fixed link, it is a spherical pair e.g. ball in socket.

Type of Joints

There are three typer of joint.

- Binary joint (B)

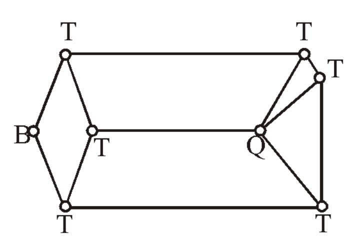
  If two links are joined at the same connection.

- Ternary joint (T)

  if three link are joined at a connection, it is consider equivalent to two binary joint. Since fixing of any one link constitutes two binary joints.

- Quaternary joint (Q)

  If four links are joined at a connection. It is quaternary joint. It is equivalent to three binary joint.



Point to remember:

- If n number of links are connected at a joint, it is equivalent to (n-1) binary joints.

Degree of freedom

The connection of link with another imposes certain constrains on relative motion thus,

Degree of freedom = 6 – number of restraints

Points to remember:

- Number of restraints can never be zero (joint disconncted)
- Number of restraints can never be 6 (joint become rigid.)

Degree of Freedom of Space Machanism (3-D)

$F = 6(L - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5$

Here,

F = Degree of freedom (D.O.F.)

L = Total number of links in machanism

P_1 = Number of pair having one D.O.F.

P_2 = Number of pair having two D.O.F, and so on.

Degree of freedom of plane (2D) machanism (Grueble Criterion)

$F = 3(L - 1) - 2P_1 - P_2$

Here, L = Number of link in a machanism

P_1 = Number of pair shaving one degree of freedom.

P_2 = Number of pair shaving two degree of freedom.

- Kutzback's equation

$F = 3(L - 1) - 2j - h$

Here, L = Number of link

j = Number of binary joint

h = Number of higher joint

- Grubler's Equation

For those machanism which have single degree of freedom and zero higher pair.

$3l - 2j - 4 = 0$

Here,

l = Number of links

j = Number of binary joints

- Degree of Freedom

F = 0 (Frame)

F < 0 (redundant frame), indeterminate streucture

F > 0 (constrained/unconstrained frame)

Point to remember:

- All machanism have minimum 4 number of link.

Kinematic Chain

When all the links are connected in such a way that first link is jointed with the last link, then the structure formed is known as closed chain. The closed chain will be a kinematic chain when the relative motion between the link is either completely constrained or successfully constrained.

- Condition for a kinematic chain

  There are two conditions and both the conditions are equivalent.

- Relation between the number of links and number of pair.

  $l = 2P - 4$

  where, l = Number of link

  P = Number of pair

- Relation between number of binary joints and number of links

  $2J = 3l - 4$

  where, J = Number of binary joint.

  l = Number of link

  (i) If L.H.S. > R.H.S.

  Locked chain or frame or structure

  (ii) If L.H.S. = R.H.S

  Kinematic chain (constrained chain)

  (iii) If L.H.S. < R.H.S

  unconstrained chain

  Redundant chain

  It does not allow any motion of a link relative to other.

  Frame/Structure

  If one of the link of redundant chain is fixed. It is known as structure or a locked system.

- Points to remember:

  No relative motion

  Capable of transmitting force only.

  Power/energy can not be transmitted.

  Degree of freedom of *structure* is zero.

  Degree of freedom of super structure is less than zero.

  Simple Machanism

  All the machanism having 4-links are simple machanism and the mechanism having more than 4 links are compound mechanism.

  There are there different simple machanisms.

- Four bar machanism/Quadric cycle chain.

  It consists of four link and four turnining pair.

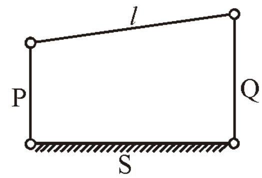
- Slider crank machanism

  It consists four link, three turning pair and one sliding pair.

- Double slider crank machanism.

  It consists four link, two turning pair and two sliding pair.

Four-Bar Machanism



- Grashof's law

$S + l \leq P + Q$

Here,

S = Shortest link length

l = Longest link length

P,Q = Adjacent link length to shortest link.

Case-I:

If $S + l < P + Q$

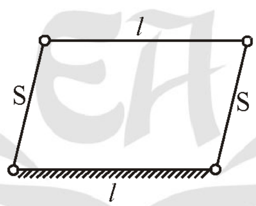
- S is fixed – Double crank mechanism.
- P or Q fixed – Crank rocker mechanism
- l is fixed – Rocker-rocker mechanism

Case-II:

If $S + l = P + Q$

- All link have different length then same as case-1.
- Parallelogram linkage-crank-crank mechanism.

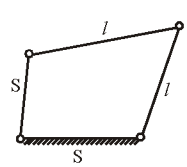
i.e. $S = P$, $l = Q$



S is fixed – Double crank mechanism.

l is fixed– Double crank mechanism.

Deltoid linkage



(a) S is fixed – Crank-crank mechanism

(b) l is fixed – Crank-rocker mechanis.

Case-III:

$s + l > P + Q$

Grashof's law is not satisfied and it will give non grashof's rocker-rocker mechanism.

Inversion of 4-Bar Mechanism

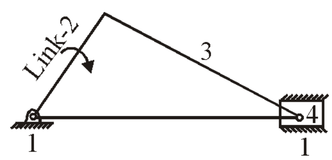
Coupling rod of locomotive	–	Crank-crank mechanism
Beam engine	–	Crank-rocker mechanism
Watt's indicator	–	Rocker-rocker mechanism

Points to remember:

Approximate straight line mechanism are watts indicator, modified Scott-russel mechanism (Grass Hopper mechanism). The tchebicheff streight line mechainsm.

Exact straight line mechanism are Peculiar mechanism, Hart mechanism, Scott-russel mechanism.

Inversion of Slider-Crank Mechanism



First inversion-link 1 is fixed.

Reciprocating engine/compressor.

Second inversion-link 2 is fixed (Crank)

Whitworth quick return mechanism, rotary (redial) engine.

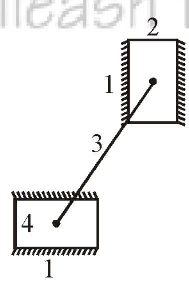
Third inversion-link 3 is fixed (Connecting Rod)

Crank and slotted lever mechanism, oscillating cylinder mechanism.

Fourth inversion-link 4 is fixed (Slider)

Hand pump, bull engine.

Inversion of Double Slider Crank Mechanism



First inversion-link 1 is fixed

Elliptical trammel.

Second inversion-slider 2 is fixed.

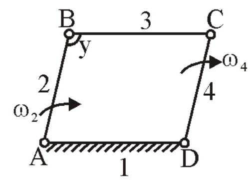
Scotch yoke mechanism.

Third inversion-link 3 is fixed

Oldham coupling.

Mechanical Advantage

$$M.A. = \frac{\text{Output force / torque}}{\text{Input force / torque}}$$



Power input = Power output

$$T_2\omega_2 = T_4\omega_4$$

$$M.A. = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

Points to remember:

If γ is equal to 0° or 180°. ω_4 become zero thus mechanical advantage will be infinity.

Extreme position of linkage is knows as toggle position.

2. Velocity Analysis

Velocity in machines can be dtermined by either analytically or graphically. This chapter deals with graphical analysis.

Velocity Analysis

Let

V_{ao} = Velocity of A relative to O

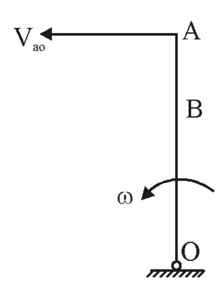
V_{ba} = Velocity of B relative to A

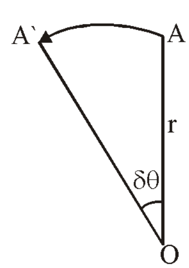
V_{bo} = Velocity of B relative to O

$V_{bo} = V_{ba} + V_{ao}$

Velocity of A relative to O = $\frac{\text{Arc } AA`}{\delta t}$

$$V_{ao} = \frac{r.\delta\theta}{\delta t} = r\frac{d\theta}{dt} = r\omega$$





as $\delta t \to O$, AA` will be perpendicular to OA, Thus the velocity of A is ωr and is perpendicualr to OA.

Points to remember:

The velocity of any point relative to any other point on a fixed link is always zero.

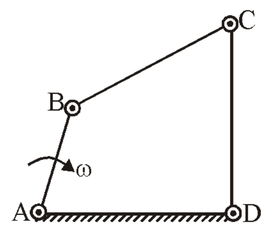
The velocity of an intermediate point on any of links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides link.

The angular velocity of a link about one extremity is the same as the angular velocity about the other.

Velocity of Rubbing

The velocity of rubbing of the two surface will depend upon the angular velocity of a link, relative to the other.

If pin at A



Pin at A joins links AD and AB. AD being fixed the velocity of rubbing will depend only upon angular velocity of AB.

∴ Velocity of rubbing

$$= r_a \omega_{AB}$$

Here, r_a = Radius of pin A

Pin at D

Velocity of rubbing = $r_d \cdot \omega_{cd}$

Pin at B

Both link AB and BC is moving

$$\omega_{ab} = \omega = \text{clockwise}$$

$$\omega_{bc} = \omega = \text{anticlockwise}$$

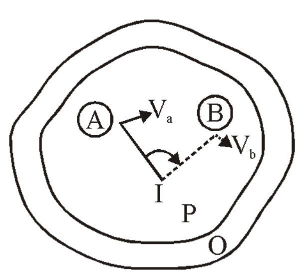
∴ Velocity of rubbing $= r_b (\omega_{ab} + \omega_{bc})$

Pin at C

Velocity of rubbing = $r_c (\omega_{bc} + \omega_{dc})$

Theory of Instantaneous Centre

Instantaneous centre of rotation or vitrual centre.



Let a plane body 'P' having non-linear motion relative to another plane body Q. At any instant, the linear velocities of the point 'A' and 'B' on the body 'P' are 'V_a' and 'V_b' respectively.

If a line is drawn perpendicular to the direction of V_a at 'A', the body can be imagined to rotate about some point on this line. Similarly for point B. If the intersection of the two lines is at 'l', the body 'P' will be rotating about l at the instant.

This point 'l' is known as instantaneous centre of velocity.

Point to remember:

If the direction V_a and V_b are parallel to the l-centre of body lies at infinity.

Centrode:

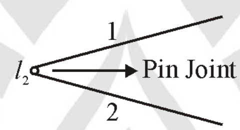
As we know, in general the position of instantaneous centre changes throughout the whole motion. The locus of all these instantaneous centre for a particular link is known as 'Centrode'. It is a *curve*.

Axode:

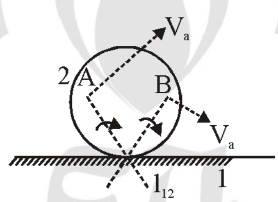
The line passing through instantaneous centre and perpendicular to the plane of motion is known as instantaneous axis. The locus of instantaneous axis for a link during the whole motion is known as 'Axode'. It is a *surface*.

Instantaneous Centre in Different Situation

If two links are attached with a turning pair, the instantaneous centre of such a pair will be at *pin joint*.

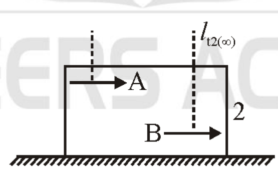


Rolling of a sphere on a plane.



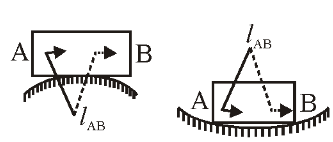
Instantaneous centre will be at point of contant.

Object sliding on a plane surface.



The location of l_{12} will be at infinity but in a direction perpendicular to the *sliding surface*.

Object sliding on a *curved* surface.



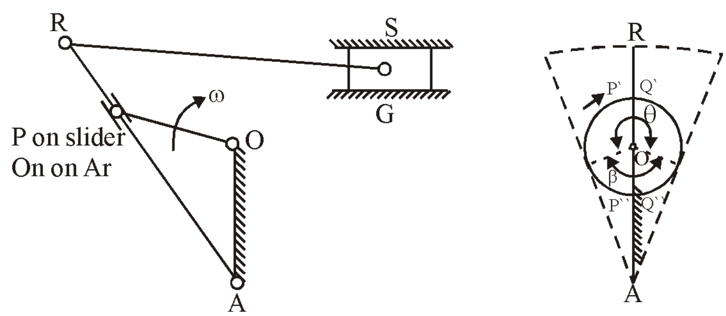
Instantaneous centre will be at the *centre* of curvature of curved surface.

Number of Instantaneous Centre

$$l = \frac{n(n-1)}{2}$$

Here, n = Number of link

Crank and Slotted Lever Mechanism



Let,

r = length of crank (= OP)

l = Length of slotted lever (=AR)

C = Distance between fixed centres (=AO)

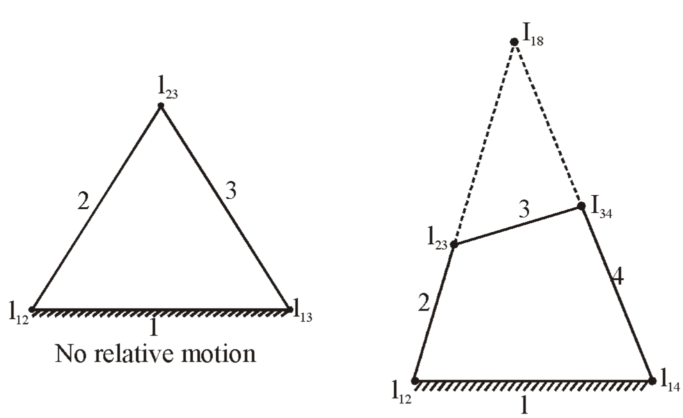
ω = angular velocity of crank

Thus during cutting stroke

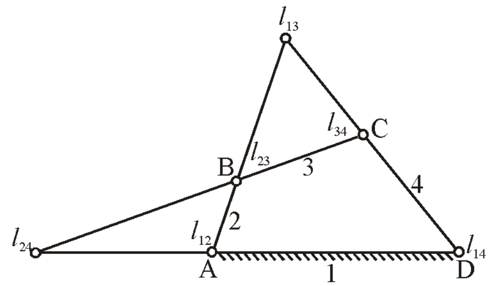
$$\frac{(V_s)_{max}\,(\text{cutting})}{(V_s)_{max}\,(\text{return})} = \frac{c-r}{c+r}; \quad \frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$

Kennedy's Theorem

For the three bodies having the contiuous relative motion there all instaneous centres lies on the same line.



Angular Velocity Ratio Theorem



It is used to find angular velocity of a link if angular velocity of another link is known.

The angular velocity ratio of two links relative to third link is inversely proportional to the distances of their common I-centre from their respective centre of rotation.

$$\frac{\omega_4}{\omega_2} = \frac{I_{24} - I_{12}}{I_{24} - I_{14}}$$

Angular velocity ratio is positive when the common instantanteous center falls outside the other two centers & negative when it falls between them.

3. Acceleration Analysis

The rate of change of velocity with respect to time is known as acceleration and it acts in the direction of the change in velocity.

Tangential and Centripetal Acceleration

The rate of change of velocity in the tangential direction of the motion is known as tangential acceleration.

$$a_t = \frac{dv}{dt}$$

The rate of change of velocity towards the centre of rotation is known as centropetal or radial acceleration.

$$a_c = \frac{v^2}{r}$$

Points to remember:

The tangential component of acceleration occurs due to the angular accelration of link.

the acceleration of intermediate points on the links can be obtained by dividing the acceleration vectors in the same ratio as the points divide the links.

Coriolis Acceleration Component

Coriolis Acceleration Component

Let,

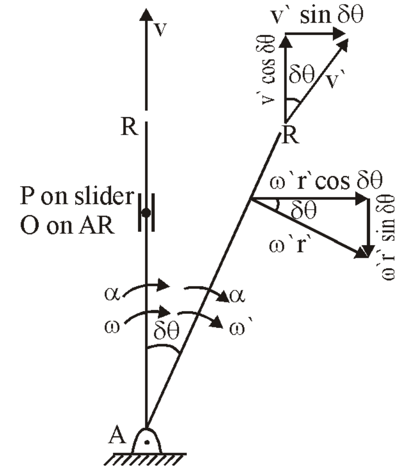
ω = Angular velocity of the link

α = Angular acceleration of the link

v = Linear velocity of the slide of the link

f = Linear acceleration of the slider on the link

r = Radial distance of point P on eht slider.



Acceleration of P along AR =

Acceleration of slider – Centrifugal acceleration

$$= f - \omega^2 r$$

Acceleration of P perpendicular to AR

= 2 wV + Tangential acceleration

$$= 2\,\omega V + r\alpha.$$

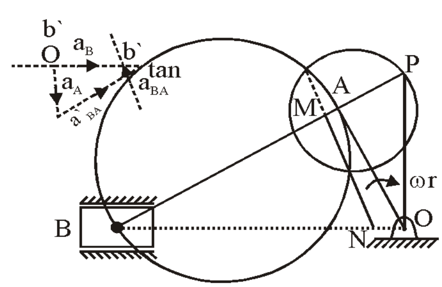
Points to Remember:

The component 2 ωv is known as the Coriolis acceleration component.

The direction of the coriolis acceleration component is obtained by rotating the radial velocity vector 'v' through 90° in the direction of rotation of the link.

Klein's Construction

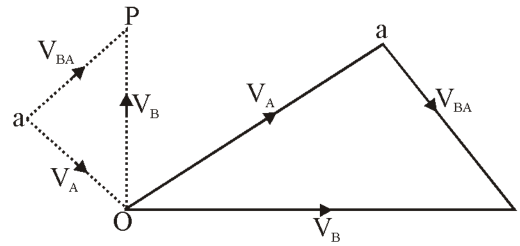
In kleni's construction, the velocity and the acceleration diagrams are made on the configuration diagram itself. The line that represents the crank in the configuration diagram also represents the velocity and acceleration of its moving end in the velocity and acceleration diagram respectively.



$$\frac{a_B}{ON} = \frac{a'B_A}{AM} = \frac{a_A}{OA} = \frac{a_{BA}^{tan}}{NM} = \omega^2$$

OAP – Velocity diagram

OAMN – Acceleration diagram.



$$\frac{V_A}{OA} = \frac{V_B}{OP} = \frac{V_{BA}}{AP} = \omega_{crank}$$

Some Other Important Points

Acceleration images are helpful to find the accelerations of offset points of the links. The acceleration image of link is obtained in the same mannge as a velocity image.

Acceleration of a point on a link relative to a coincident point on a moving link is the sum of absolute acceleration of the coincident point. acceleration of the point relative to coincident point and the Coriilis acceleration.

4. CAM

A cam is mechanical member used to impart desired motion to a follower by direct contatc. The cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating.

It is used in automatic machine, IC engine, machine tools, printing control machanism.

Element of Cam

A driver member known as the cam

A driven member called the follower

A frame which supports the cam and guides the follower.

Point of remember:

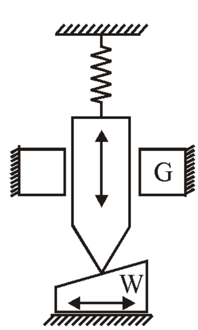
A cam and follower combination belong to the category of higher pair.

Types of Cam

According to Shape

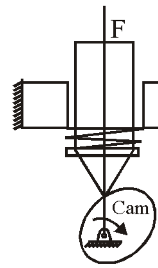
Wedge and flat cams

A wedge cam has a translationsl motion, the follower can either translate or oscillate.



Radial or Disc Cams

A cam in which the follower moves radially from the centre or rotation of the cam is known as a radial or disc cam.

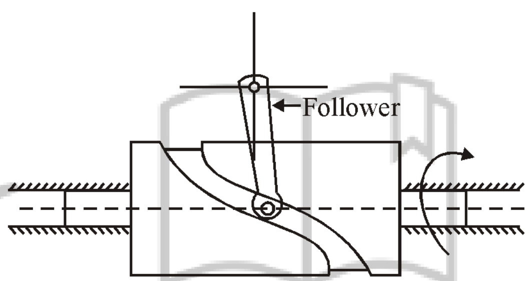


Spiral cams

A spiral cam is a face cam in which a grovve is cut in the form of a spiral. It is used in computer.

Cylindrical cams

In a cylindrical cam, a cylinder which has a circumferential contour cut in surface, rotates about its axis. It is also known as barrel or drum cams.



Conjugate cams

It is a double disc cam and preferred when the requirements are low wear, low noise, better control of the follower, high speed, high dynamic loads etc.

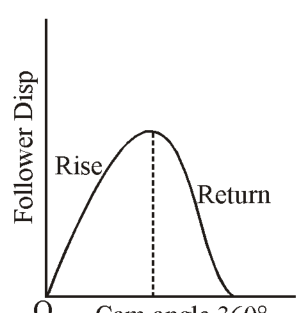
Globoidal cams

It has two types of surface i.e. convex or concave. It is used when moderate speed and the angle of oscillation of the follower is large.

According to follower movement

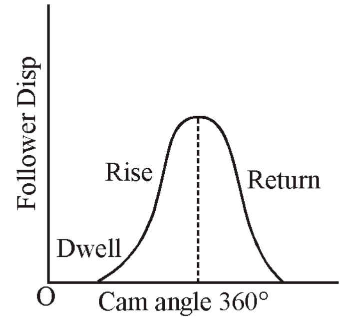
Rise-Return-Rise (RRR)

In this there is alternate rise and return of the follower with no period of dwells. The follower has a linear or an angular displacement.



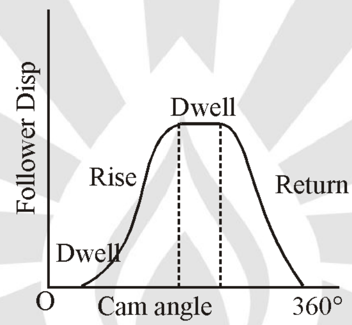
Dwell-Rise-Return Dwell (D-R-R-D-S)

In this cam, there is rise and return of the follower after a dwell.

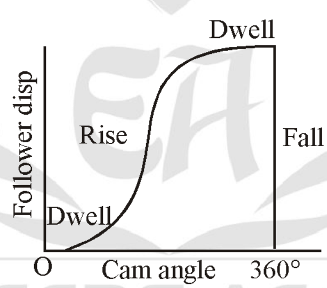


Dwell-Rise-Dwell-Return-Dwell (D-R-D-R-D)

The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell. In case the return of the follower is by a fall, the motion may be known as Dwell-Rise-Dwell (DRD)



D-R-D-R-D



D-R-D

Type of Follower

According to shape

Knife-edge follower:

Its use is limited as it produces a great wear of the surface at the point of contact.

Roller follower:

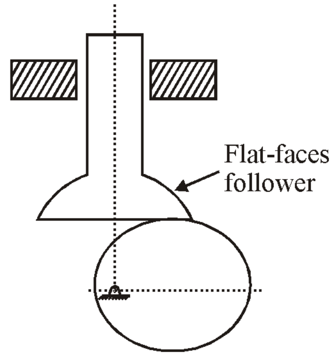
At low speeds, the follower has a pure rolling action, but at high speeds, some sliding also occurs.

Point to remember:

In case of steep rise roller follower is not preferred.

Mushroom follower:

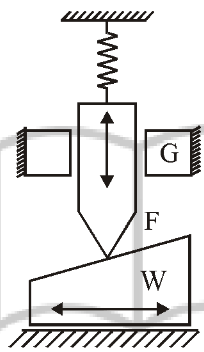
It does not pose the problem of jamming the cam.



According to movement

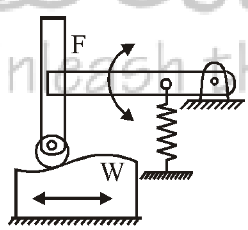
➢ **Reciprocating follower**:

In this type, as the cam rotates the follower reciporcates or translates in the guides.



Oscillating Follower:

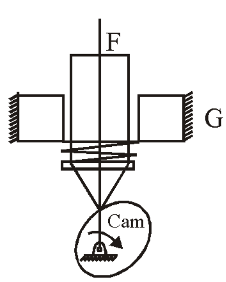
The follower is pivoted at a suitable point on the fram and oscillates as the cam makes the rotary motion.



According to loaction of line of movement

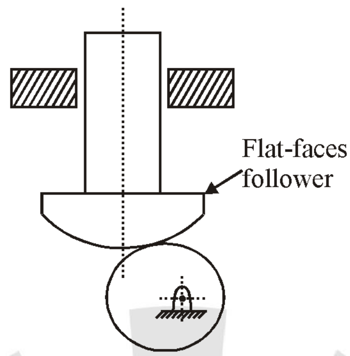
➢ **Radial Follower:**

The follower is known as a radial follower if the line of movement of the follower passes through the centre of rotation of the cam.

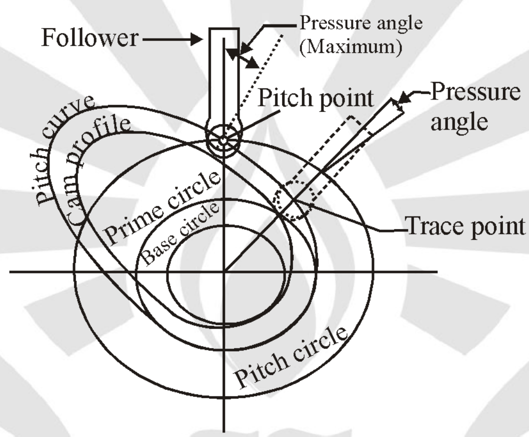


Offset Follower:

If the line of movement of the roller follower is offset from the centre of rotation of the cam, the follower is known as an offset follower.



Terminology of Cam



Base circle

It is the smallest circle trngent to the cam profile drawn from the centre of rotation of a radial cam.

Pressure angle:The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at a point and the direction of the follower motion. It varies in magnitude at all instants of the follower motion. A high value of maximum presure angle in not desired on it might jam the follower in the bearing.

Pitch point: It is the point on pitch curve at which the pressure angle is maximum.

Pitch circle: It is the circle passing through the pitch point and concentric with the base circle.

Angle of Ascent (ϕ_a): It is the angle through which the cam turns during the time the follower rise.

Angle of dwell (δ): Angle of dwell is the angle through which the cam turns while the follower remains stationary at the highest or lowest position.

Angle of Decent (ϕ_d): It is the angle through which the cam turns during the time the follower returns to the initial position.

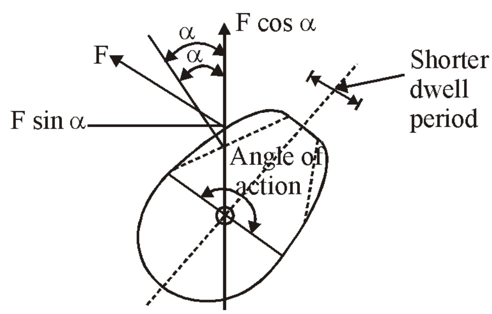
Angle of Action: It is the total angle moved by cam during the time, between the beginning of rise and the end of return of the follower.

Point to remember:

The dynamic effects of acceleration (jarks) usually, limit the speed of the cams.

Force Exerted by Cam

The force exerted by a cam on the follower is always normal to the surface of the cam at the point of contact. The vertical component ($F\cos\alpha$) lifts the follower whereas the horzontal component ($F\sin\alpha$) exerts lateral pressure on the bearing.



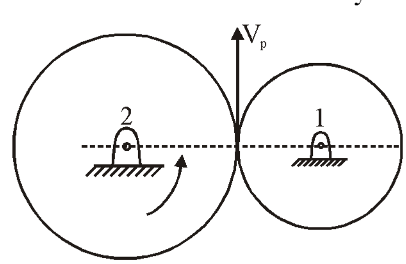
In order to reduce the lateral pressure of $F\sin\alpha$, α has to be decreased which means making the surface more convex and longer. This results in reduced velocity of the follower and more time for the same rise.Minimum value of α cannot be reduced from a certain value.

The increase in the base circle diameter increases the length of the arc of the circle upon which the wedge is to be made. A short wedge for a given rise requires a steep rise or a higher pressure angle, thus increasing the lateral force.

5. Gear

Inrtroduction to Gear

Gears use no intermediate link or connector and transmit the motion by direct contact. The two bodies have either a rolling or a sliding motion along the tangent at the point of contact. No motion is possible along the common normal as that will either break the contact or one body will tend to penetrate into the other.



Point P can be assumed on gear 2 or gear 1.

$$V_p = \omega_2 r_2 = \omega_1 r_2 \qquad \left[\omega = \frac{2\pi N}{60}\right]$$

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Symbols has usual meaning.

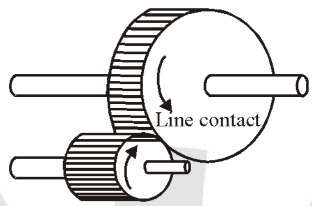
Point to remember:

Gear is a positive drive because no slip occur in its motion.

Classification of Gear

Parallel shaft

Spur Gears : They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load.

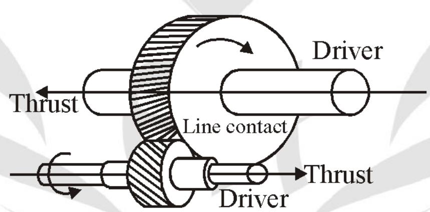


Point to remember:

At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axes of rotation This results in sudden application of the load, high impact stresses and excessive noise at high speeds.

Spur Rack and Pinion : Spur rack is a special case of spur gear wher it is made of infinite diameter so that the pitch surface is plane. The spur rack and pinion conbination converts rotary motion into translatory motion or vice-versa. It is used in lathe in which the rack transmits motion to the saddle.

Helical gears or helical spur gears: In helical gears, the teeth are curved, Two mating gears have the same helix angle; but have teeth of opposite hands.



Points to remember:

At the beginning of engagement, contact occurs only at the point of leading edge of the curved teeth, Thus the load application is gradual which results in low impact stresses & reduction in noise.

The helical gears can be used at higher velocities than the spur gears and have greater load - carrying capacity.

Helical gears have the disadvantage of having end thrust as there is a force component along the gear axis.

Double-helical and Harring bone Gears: A double-helical gear is equivalent to a pair of helical gears secured together, one having a right hadn helix and other a lift hand helix.

Points to remember:

No axial thrust is present.

If the left and the right inclinations of a double-helical gear meet at a common apex and there is no groove in between the gear is known as herringbone gear.

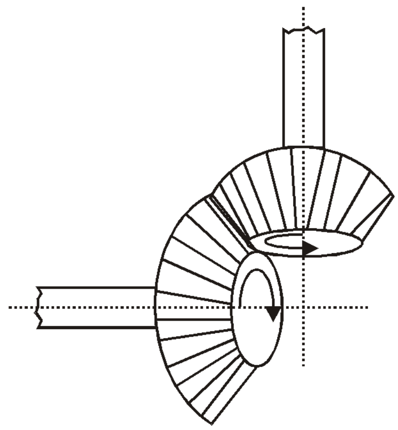
Intersecting Shaft

The motion between two intersecting shafts is equivalent to the rolling of two cones assuming no slipping.

Straight bevel Gears:The teeth are straight, radial to the point of inter section of the shaft axes and vary in cross-section throughout their length.

Points to remember:

Gears of the same size and connecting two shafts at right angle to each other are known as mitre gears.

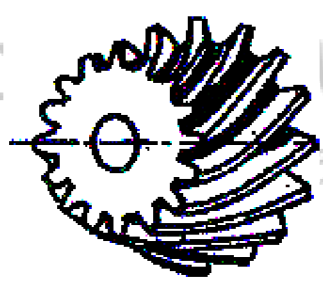


Spiral bevel Gears: When the teeth of a bevel gear are inclined at angle to the face of the bevel, they are known as spiral bevel or helical bevels.

Points to remember:

There is gradual load application and low impact stresses.

These are used for the drive to the differential of an automobile.



Zerol bevel Gears: Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zerol bevel gears.

Skew Shafts

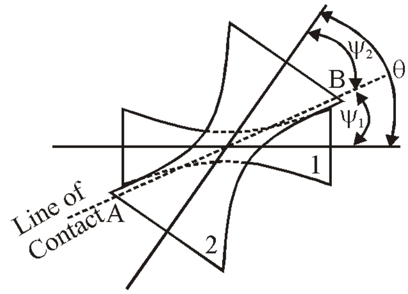
In case of skew (non-parallel, non-intersecting) shafts, a uniform rotary motion is not possible by pure rolling contact.

Points to remember:

If the two hyperboloids rotate on their respective axes, the motion between them, whould be a combination of rolling and sliding action.

Angle between two shafts will be qual to the sum of the angles of generation of two hyperboliods.

$$\theta = \psi_1 + \psi_2$$



Crossed helical gears: The use of crossed-helical gears or spiral gears is limited to light loads. Therse gears are used to drive feed mechanism on machine tools, camshafts and oil pumps in I.C. engine.

Worm Gears: It is a special case of a spiral gear in which the larger wheel usually has a hollow or concave shape.

Points to remember:

the smaller of the two wheels is called the worm which also has large spiral angle.

the sliding velocity of worm gear is higher as compared to other types of gears.

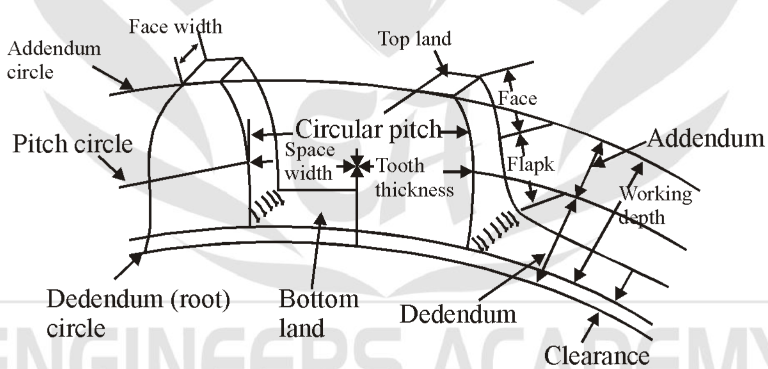
Classification of Gear According to Peripheral Velocity of Gear

Low velocity gear – (0-3) m/s

Medium velocity gear - (3-5) m/s

Large velocity gear – (> 15) m/s

Gear Terminology



Pitch Circle

It is an imaginary circle drawn in such a way that a pure rolling motion on this circle gives the motion which is exactly similar to the gear motion.

Points to remember:

These pitch circle always touch each other for the correct power transmission.

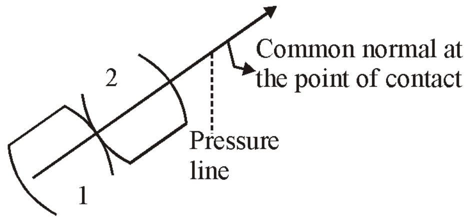
It is not fundamental characteristics of gear.

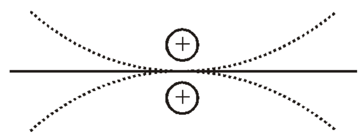
Pitch point

It is a point where the two pitch circle of the mating gear touch each others.

Pressure angle (ϕ)

It is the angle between common normal at a point of contact tangent at a pitch point.





The standard value of pressure angle are 14½, 20°, 25°.

Module (m)

It is defined as the ratio of pitch circle diameter in mm to the number of teeth.

$$m = \frac{D_{(mm)}}{T}$$

Addendum Circle

A circle drawn from top of tooth and concentric to pitch circle is known as addendum circle.

Addendum is Radial distance between pitch circle and addendum circle and it is equal to 1 module.

Point to remember:

Clearance = 0.157 m

Dedendum Circle

A circle drawn from bottom of the teeth and concentric with pitch circle.

Dedendum is Radila distance between pitch circle and dedendum circle and it is equal to 1.157 module.

Circular Pitch (C)

It is a distance along a pitch circle from one point on a tooth to the corresponding point on the next tooth.

$$C = \frac{\pi D}{T}$$

D = Pitch circle-diameter

T = Number of Teeth

Diametral Pitch

It is the ratio of number of teeth to the pitch circle diameter but diameter should be in mm.

$$P_d = \frac{T}{D_{mm}}$$

Relation between circular pitchs (C) diametral pitch (P_d)

Circular pitch × Diametral pitch = π

Tooth Space

It is the thickness of tooth measured along pitch circle.

Tooth Space

The space between the consecutive teeth measured along the pitch circle.

Backlash

It is difference between Tooth space and tooth thickness, which is generally provided to avoid jamming due to thermal expansion.

Face

The portion of tooth profile above the pitch surface.

Flank

The portion of tooth profile below the pitch surface.

Profile.

The curvature containe by face and flank.

Path of contact (POC)

It is the path travelled by point of contact from the starting of engagement to the end of engagement.

POC = Path of approach + Path of Recess.

Arc of Contact (AOC)

It is the path traced by a point on the pitch circle during starting of engagement to the end of engagement.

Gear Ratio (G)

$$G = \frac{T}{t}$$

Here,

T = Number of teeth of gear.

t = Number of teeth o pinion (small gear)

Velocity Ratio (VR)

$$VR = \frac{1}{\text{Gear ratio}}$$

Angle of Action

Angle turned by gear from the beginning of engagement to the end of engagement of a pair of teeth.

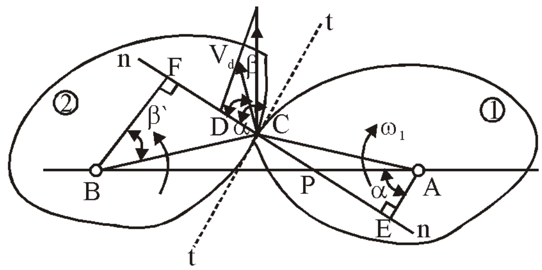
= Angle of approach + Angle of recess

Contact Ratio

$$\frac{\text{Angle of action}}{\text{Pitch angle}} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

Law of Gearing

The law of gearings states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio between two gears.



Let

ω_1 = angular velocity of gear 1 (clockwise)

ω_2 = angular velocity of gear 2 (anticlockwise)

$$\frac{\omega_1}{\omega_2} = \frac{BP}{AP} = \frac{PF}{PE}$$

For constant angular velocity ratio of the two gears the common normal at the point of contact of the two mating teeth must pass through the pitch point.

Velocity of sliding

If the curved surfaces of the two teeth of the gears are to remain in contact one can have a sliding motion relative to the other along the common tangent.

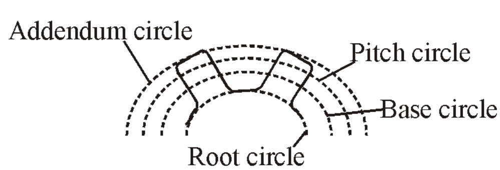
= Sum of angular velocities X distance between the pitch point and point of contact

$$= (\omega_1 + \omega_2)\ PC$$

Type of Prifile

Involute Profile

Involute is a curve generated by point on a tangent which rolls on a circle without slipping. The involute profile on a gear will be generated through a generating circle and this generating circle will be known as base circle. It is a fundamental property of a gear its radius will not change in any condition for a gear.



Points to remember:

A normal on any point of involute profile will be tangent to the base circle.

Tooth profile is always generated from base circle and the profile between root circle and base circle will not be of involute type.

If the centre distance between the two pitch circles varies, the point P is shifted and the speed of the driven gear would very.

For a pair of involute gears velocity ratio is inversely propoetional to the pitch circle diameters as well as base circle diameters.

Path of Contact

$$= \sqrt{R_a^2 - R^2 \cos^2 \phi} + \sqrt{r_a^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$

Here,

r = Pitch circle radius of pinion

R = Pitch circle radius of wheel

r_a = addendum circle raduis of pinion

R_a = Addendum circle radius of wheel

Arc of contact

$$= \frac{\text{Path of contact}}{\cos \phi}$$

Number of pair of teeth in contace

$$= \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

The maximum value of the addendum radius of the wheel to avoid interference.

$$= R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2 \phi}$$

Maximum value of addendum of the wheel

$$= R\left[\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2 \phi} - 1\right]$$

Minimum Number of teeth on the wheel for the given values of the gear ratio the pressure angle and the addendum coefficient (a_w).

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1}$$

Where a_w = Addendum coefficient

Points to remember:

Point of contact will always be at a line tangent to the base circle.

Base circle diameter = pitch circle diameter × cos ϕ.

Cycloidal Profile Teeth

A cycloid is the locus of a point on the circumference of a circle that rools without slipping on the circumference of another circle. in this type, the faces of the teeth are epicyloids and flanks the hypocycloids.

Terminology / Centre Distance of Helical Gear

Helix angle (ψ)

It is the angle at whixch the teeth are inclined to the axis of a gear it is also known as spiral angle.

Normal Circular Pitch (P_n)

Normal circular pitch of simply normal pitch is the shortest distance measured along the normal to the helix between corresponding points on the adjacent teeth. The normal circular pitch of two mating gears must be same.

$$P_n = P \cos \psi$$

Also, we have, $P = \pi m$ as for spur gear

$$P_n = \pi m_n$$

and $$m_n = m \cos \psi$$

Centre distance $= \dfrac{m_n}{2}\left[\dfrac{T_1}{\cos \psi_1} + \dfrac{T_2}{\cos \psi_2}\right]$

Helical and Spiral Gears

In helical and spiral gears, the teeth are inclined to the axis of a gear.

They can be right handed or left handed.

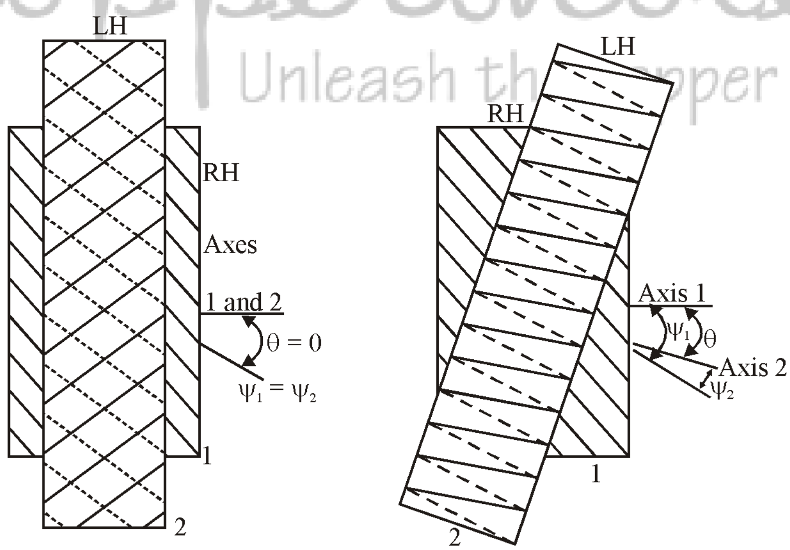
Let, ψ_1 = helix angle for gear 1

ψ_2 = helix angle for gear 2

θ = Angle between shaft

$\theta = \psi_1 + \psi_2$ (for gears of same hand)

$\theta = \psi_1 - \psi_2$ (for gears of opposite hand)



Angle between shafts, $\theta = \psi_1 + \psi_2$ for gears of same hand $\theta = \psi_1 - \psi_2$. for gears of opposite hand.

for $\psi_1 - \psi_2$, $\theta = 0$, a case of helical gears joining parallel shafts.

$$\psi_1 = \psi_2$$

Efficiency of Spiral and Helical Gear

$$\eta = \frac{\cos(\theta+\phi)+\cos(\psi_1-\psi_2-\phi)}{\cos(\theta-\phi)+\cos(\psi_1-\psi_2-\phi)}$$

$$\eta_{max} = \frac{\cos(\theta+\phi)+1}{\cos(\theta-\phi)+1}$$

ϕ = Pressure angle

ψ = Helix angle

θ = Angle between two shaft

Terminology of Worm Gear/Velocity Ratio-Centre Distance/Efficiency

Axial Pitch (p_a)

It is the distance between corresponding points on adjacent teeth measured along the direction of the axis.

Lead (L): The distance by which a helix advances along the axis of the gear for one turn around is known as load.

In a sinle helix, the axial pitch is equal to lead. In a double helix, this is one-half the lead, in triple helix one third of lead, and so on.

Lead Angle (λ): It is the angle at which the teeth are inclined to the normal to the axis of rotation, the lead angle is the complement of the helix angle.

$\psi + \lambda = 90°$

$$\text{Velocity ratio} = \frac{1}{\pi d_2} \quad (d = \text{diameter})$$

Centre distance = $\frac{m_2}{2}[T_1 \cot \lambda_1 + T_2]$

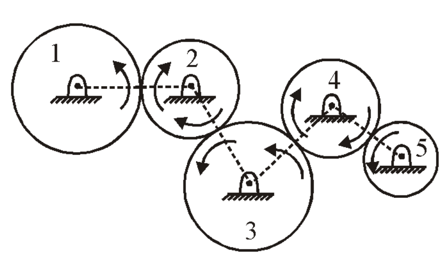
$$\eta = \frac{\tan\lambda_1}{\tan(\lambda_1+\phi)}$$

$$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

6. Gear Train

A Gear train is a combination of gears used to transmit motion from one shaft to another. It is required to obtain large speed reduction within a small space.

Simple Geat Train



A series of gears, capable of receiving and transmitting motion from one gear axes remain fixed relative to the frame and each gear is on separates shaft.

$$\text{Train value} = \frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven gear}}$$

$$= \frac{N_5}{N_1} = \frac{T_1}{T_5}$$

$$\text{Speed ratio} = \frac{1}{\text{train value}}$$

Points to remember:

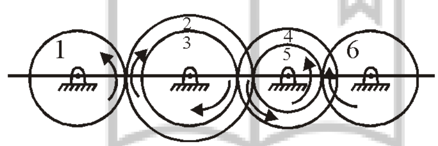
A simple gear train can also have bevel gears.

A pair of mated external gear always move in opposite direction.

All odd numbered gears move in one direction and all even numbered gears in the opposite direction.

Intermediate gears have no effect on the speed ratio and therefore, they are known as idlers.

Compound Gear Train



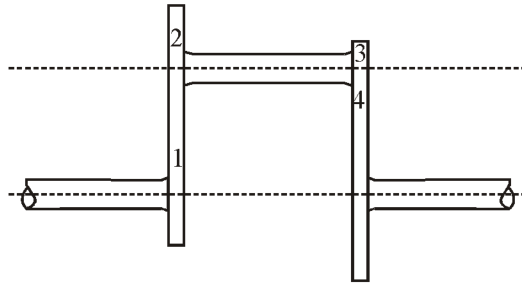
When a series of gears are connected in such a way that two or more gears rotate aobut an axis with the same angular velocity, it is known as compound gear train.

$$\text{Train value} = \frac{\text{Product of number teeth on driving gears}}{\text{Product of number of teeth on driven gears}}$$

$$= \frac{N_6}{N_1} = \frac{T_1 T_3 T_5}{T_2 T_4 T_6}$$

Reverted Gear Train

If the axis of the first and last wheel of a compound gear coincide it is called reverted gear train. Such arrangement is used in clock and in simple lathe where 'back gear' is used to give slow speed to the chuck.



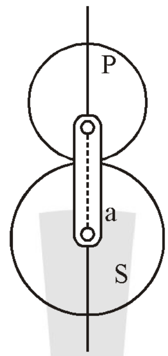
$$\textbf{Train value} = \frac{N_4}{N_1} = \frac{T_1 T_3}{T_2 T_4} = \frac{\text{Product of number of teeth on driving gear}}{\text{Product of number of teeth on driven gears}}$$

If r is the pitch circle radius of a gear.

$$r_1 + r_2 = r_3 + r_4$$

Epicyclic Gear Train

When there exists a relative motion of axes in a gear train. It is calles an epicyclic gear train. Thus in an epicyclic train, the axis of at least one of the gears also move relative to the frame.



If arm 'a' is fixed. Turn 's' through x revolutions in the clockwise direction. Assuming clockwise motion of a wheel as positive and counter clockwise motion as negative.

Revolution made by 'a' = 0

Revolution mnade by 's' = x

Revolution made by 'p' $= -\left(\frac{T_s}{T_p}\right)\times$

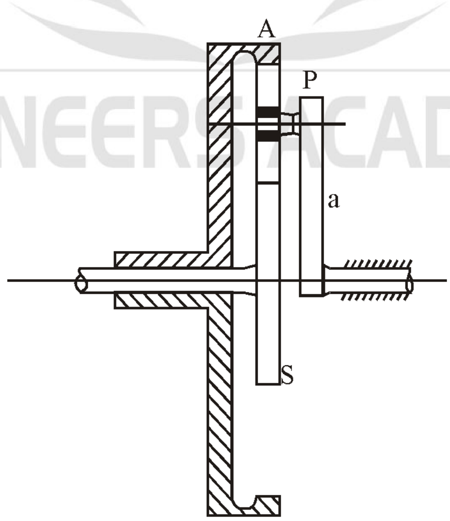
Points to remember:

Lareg speed reductions are possible with epicyclic gears.

In general epicyclic gear trains have two degrees of freedom.

Sun and Planet Gear

➢ When an annular wheel is added to the epicyclic gear train, the combination is usually, referred to as sun and planet gear.



Teh annular wheel gears with the wheel P which can rotate freely on the arm 'a'. The wheel S and P are generally called thw sun and the planet wheels.

If the sun sheel S is fixed, $N_2 = 0$

Speed of the arm, $$\frac{N_a}{N_A} = \frac{1}{(T_S/T_A)+1}$$

If the annular wheel A is fixed, $N_A = 0$

$$\frac{N_a}{N_s} = \frac{T_S/T_A}{1+(T_s/T_A)}$$

Differential Gear

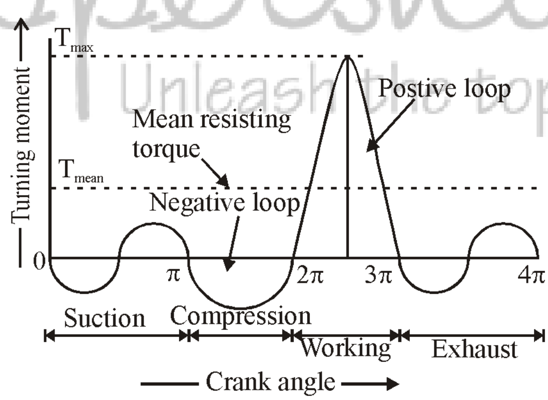
When a vehicles takes a turn, the outer wheels must travel farther than the inner wheels. Since, Both rear wheels are driven by the engine through gearing. Therefore some sort of automatic device is necessary so that the two rear wheels are driven at slightly different speeds. This is accomplished by fitting a differential fear on the power (rear) axle. Differential gear is a device which adds or subtracts angular displacements.

7. Flywheel

A flywheel used in machines serve as a reservoir, which stores energy during the period when supply of energy is more that the requirement, and release it during the period when the requirement of energy is more thatn the supply, Flywheel does not maintain a constant speed, it simply reduce fluctation of speed. It does not control the speed variations caused by the varying load.

Truning Moment Diagram for a Four Stroke Engine

A turning moment diagram for a stroke cycle internal combustion engine is shown in fig. We know that in a four stroke internal engine, there is one working stroke after the crank has turned through two revolutions. i.e., 720° (or 4π radians).



The pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed.

During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained.

During the expansion or working stroke, the fuel burns and the gases ecpand, therefore a large positive loop is obtained in this stroke, the work is done by the gases.

During exhaust stroke, the work is done on the gases, therefore a negative loop is formed.

Work Done Per Cycle

Work done per cycle = $T_{mean} \times \theta$

Where, T_{mean} is mean torque,

θ is angle turned is one cycle

= 2π, in case of two stroke engine

= 4π, in case of 4 stroke engine

Fluctuation of Speed

The difference between the maximum and minimum speed during a cycle is called maximum fluctuation of speed.

The ratio of maximum fluctation of speed to the mean speed is called coefficient of fluctation of speed (C_s).

$$C_s = \frac{N_{max} - N_{min}}{N_{mean}}$$

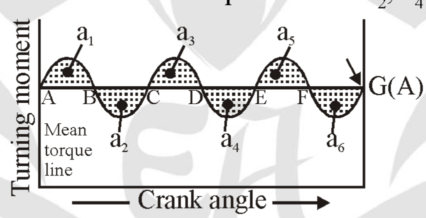
Coefficient of Steadiness

The reciprocal of the coefficient of fluctaution of speed is known as coefficient of steadiness and is denoted by m.

$$m = \frac{1}{C_s} = \frac{N_{mean}}{N_{max} - N_{min}}$$

Maximum fluctuation of Energy

The turning moment diagram for Multi-cylinder engine is shown in diagram. The horizontal line AG represents the mean line. Let a_1, a_3, a_5 be area above mean torque line and a_2, a_4 and a_6 be area below mean torque line.



Let the energy is the flywheel at A = E.

∴ Maximum fluctuation of energy

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

We can also write

$$\Delta E = E_{max} - E_{min} = \frac{1}{2} I (\omega^2_{max} - \omega^2_{min})$$

$$\Delta E = I\omega^2_{mean} C_s$$

Here,

I = Maxx moment of inertia of the flywheel about its axis of rotation

ω_{max} = Maximum angular spedd during cycle.

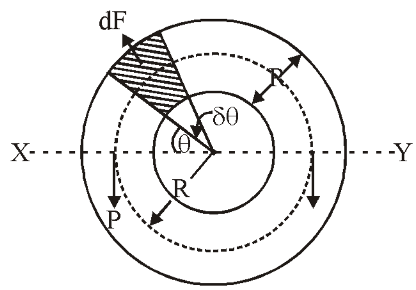
ω_{min} = Minimum angular speed during cycle.

$$\omega_{mean} = \frac{\omega_{max} + \omega_{min}}{2} \text{ during cycle}$$

Coefficient of Fluctuation of Energy (C_E)

$$C_E = \frac{\text{Mamimum flucturation of energy}}{\text{Work done per cycle}}$$

Dimensions of The Flywheel RIM



Let D = Mean diameter of rim in meters.

R = Mean radius of rim in meters.

A = Cross-sectional area of rim in m^2.

ρ = Density of rim material in kg/m^3

N = Speed of the flywheel in r.p.m.

ω = Angular velocity of the flywheel in rad/s.

v = Linear velocity at the mean radius in m/s

$= \omega . R$

σ = Tensile stress of hoop stress in N/m^2 due to the centrifugal force.

∴ Total vertical upward force tending of burst the rim across the diameter XY.

$$= 2\rho.A.R^2.\ \omega^2$$

This vertical upward force will produce tensile stress of hoop stress and it is resisted by 2P,

$$2P = 2\sigma.A$$

$$2.\rho.A.R^2.\ \omega^2 = 2\sigma.\ A$$

or $$\sigma = \rho.\ R^2.\ w^2 = \rho.\ v^2$$

∴ $$v = \sqrt{\frac{\sigma}{\rho}}$$

Mass of the Rim

$$m = \text{Volume} \times \text{density} = \pi.D.A.\rho$$

∴ $$A = \frac{m}{\pi.D.\rho}$$

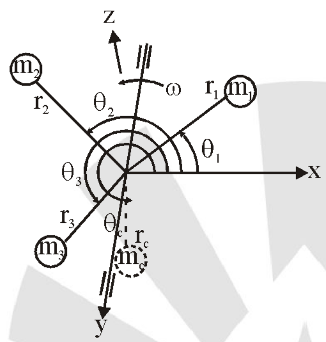
8. Balancing

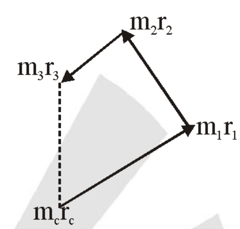
Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible, eliminate completely.

The most common approach to balancing is by redistributing the mass which may be accomplished by addition or removal of mass from various machine memebers.

Static Balancing

A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.





The rotor is said to be statically balanced. If vector sum of centrifugal force (F) is zero.

$$F = m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^3$$

Here, ω = constant angular velocity

m_1, m_2 and m_3 = rotating masses.

r_1 r_2 and r_3 = radil of masses.

If f is not zero, then introduce a counter weight of mass 'm_c' at radius r_c to balance the rotor so that.

$$\Sigma mr + m_c r_c = 0$$

and
$$\tan \theta_c = \frac{\Sigma mr \sin\theta}{\Sigma mr \cos\theta}$$

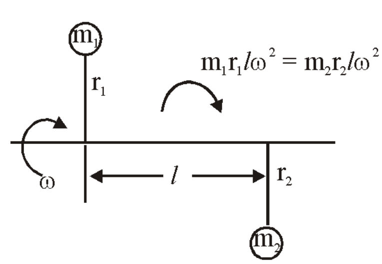
and
$$m_c r_c = \sqrt{(\Sigma mr \cos\theta)^2 + (\Sigma mr \sin\theta)^2}$$

Dynamic Balancing

A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.

$$\text{Force} = mr$$

$$\text{Couple} = mrl$$



Balancing of Raciprocating Mass.

Acceleration (α) and force (F) of the reciprocating mass of a slider crank mechansim.

$$\alpha = \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$\text{Force (F)} = mr\omega^2 \cos\theta + mr\omega^2 \frac{\cos 2\theta}{n}$$

$mrw^2 \cos\theta$ – primary accelerating force

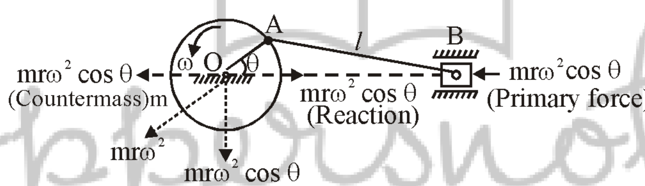
$\frac{mr\omega^2}{n} \cos 2\theta$ – Secondary accelerating force.

$$n = \frac{l}{r} = \frac{\text{Length of connecting rod}}{\text{Radius of crank}}$$

Points to remember:

- Maximum value of primary force = $mr\omega^2$.
- Maximum value of secondary force = $\frac{mr\omega^2}{n}$
- Secondary force is small as compared to primary force for slow speed engines.

Primary Balancing



If c is the fraction of the reciprocaitng mass then.

Primary force balanced by the mass = c $mr\omega^2 \cos\theta$

Primary force unbalanced by the mass = $(1 - c)\, mr\omega^2 \cos\theta$

Vertical component of centrifugal force which remains unbalanced

= $cmr\,\omega^2 \sin\theta$.

Resultant unbalanced force

$$= \sqrt{[(1-c)mr\omega^2 \cos\theta]^2 + [cmr\omega^2 \sin\theta]^2}$$

Points to remember:

- The resultant unbalanced force is minimum when c = 1/2.
- 2/3 of the reciprocating mass is balanced.
- The unbalanced force is zero at the ends of the stroke when $\theta = 0°$ or $180°$ & maximum at the middle when $\theta = 90°$. The magnitude of the unbalanced force remains the same i.e. equal to $mr\omega^2$.

Secondary Balancing

$$\text{Secondary force} = mr\omega^2 = \frac{\cos 2\theta}{n}$$

$$= mr(2w)^2 \frac{\cos 2\theta}{4n}$$

Points to remember:

- Its frequency is twice that of the primary force and the magnitude 1/n times the magnitude of the primary force.
- For complete balancing of reciprocating mass. Primary force, primary couple, secondary force secondary couple all must be balanced.

Effect of Partial Balancing in Locomotive

Hammer Blow

It is the maximum vertical unbalanced force caused by the mass provided to balance the reciprocating masses. Its value is

$$= mrw^2$$

Points to remember:

- It varies as square of the speed.
- At high speeds, force of the hammer blow could exceed the static load on the whels and the wheels can ber lifted off the rail when the direction of the hammer blow will be vertically upwards.

Variation of Tractive Force

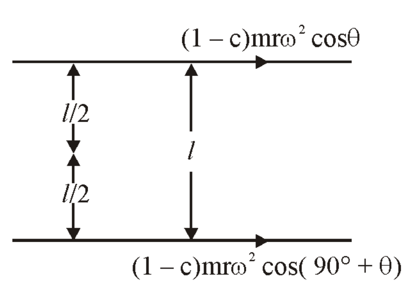
A variation in the tractive force (effort) of an engine is caused by the unbalanced portion of the primary force which acts along the line of stroke of a locomotive engine.

Total unbalanced primary force or the variation in the tractive force = $(1 - c)\ mrw^2\ (\cos\theta - \sin\theta)$. this is maximum when $\theta = 135°$ or $315°$.

Maximum variation in tractive force.

$$= \pm \sqrt{2}\,(1 - c)\ mr\omega^2$$

Swaying Couple



Unbalanced primary forces along the lines of stroke the separated by a distance l apart and thus constitute a couple. This tends to make the leading wheels sway from side to side.

Swaying douple (about engine centre line.)

$$= (1 - c)\, mr\omega^2 (\cos\theta + \sin\theta)\, \frac{1}{2}$$

This is maximum when θ = 45° or 225°

Maximum swaying couple $\pm \frac{1}{\sqrt{2}}\ (1 - c)\, mrw^2 l$

9. Governors

The function of a Governor is to maintain the speed of an engine within specified limits whenever there is a variation of load. The operation of a flywheel is continuous whereas that of a governor is more or less intermittent.

Type of Governors

Centrifugal governors

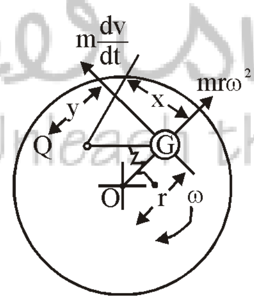
Its action depends on the change of speed and centrifugal effect produced by the masses, known as governor balls, which rotate at a distance from the axis of rotation.

The valve is operated by the actual change of engine speed in the case of centriffugal governors.

Inertia Governors

The positions of the balls are affected by the forces set up by an angular acceleration or deceleration of the spindle, in addition to centrifugal forces on the balls.

It is by the rate of change of speed in case of inertia governors.



Let, r = Radial distance OG

v = Tengential velocity of G = ωr

ω = Angular velocity of disc

Centrifugal force of the rotating mass.

F = (radially outwards) $mr\omega^2$

If the engine shaft is accelerated due to increase in speed, the ball mass does not get accelerate at the same amount on account of its inertia, the inertia force being equal to

$$F_1 = ma = m\frac{dv}{dt}$$

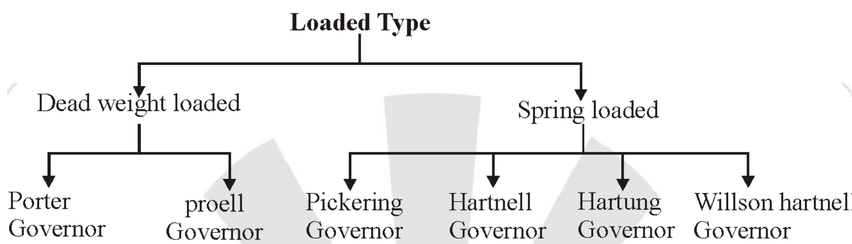
Points to Remember:

- Response of inertia governors is faster than that of centrifugal governors.
- In inertia governor, both forces i.e., centrifugal force and inertia force are in action.

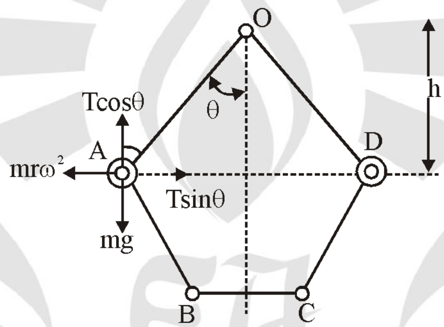
Type of Centrifugal Governor

Pendulum type — Watt Governor (simplest)

Loaded type



Watt Governor



Let,

m = Mass of each ball

h = Height of the governor

w = Weight of each ball

ω = Angular velocity of the balls, arms and the sleeve

T = Tension in the arm

r = Radial distance of ball-centre from spindle-axis

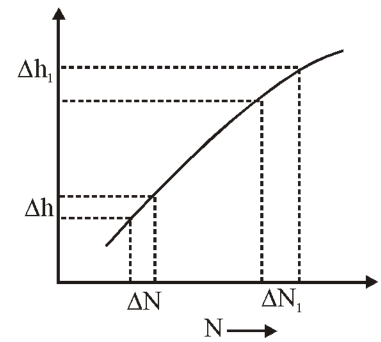
Assumption

Link is massless.

Sleeve is frictionless.

$$h = \frac{895}{N^2} \text{ (metre)}$$

Variation of height 'Δh' with speed



Points to remember:

Height 'h' is independent of mass of ball.

At higher speed, sensitivity will decrease.

Porter Governor

If the sleeve of a Watt governor is loaded with a heavy mass, it becomes a Porter governor.

Let,

M = Mass of the sleeve

m = Mass of each ball

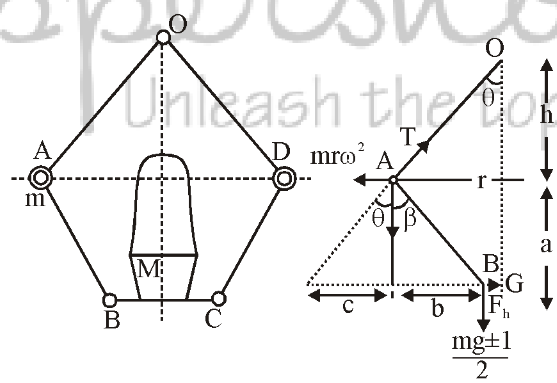
f = Force of friction at the sleeve

h = Height of the governor

r = Distance of the centre of each ball from axis of rotation.

θ = Angle between arm and spindle axis.

β = Angle between link and spindle axis.



$$\omega^2 = \frac{1}{mn}\left(\frac{2mg + (Mg \pm f)(1+k)}{2}\right)$$

or

$$N^2 = \frac{895}{h}\left(\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right)\left[k = \frac{\tan\beta}{\tan\theta}\right]$$

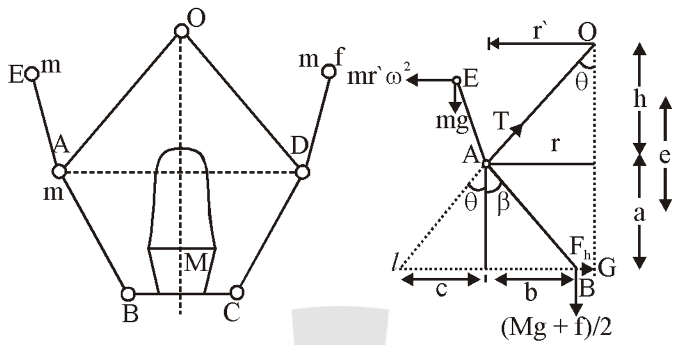
If

$k = 1, f = 0$

$$N^2 = \frac{895}{h}\left(\frac{m+M}{m}\right)$$

Procell Governor

A Porter Governor is known as a Proell. Governor if the two balls (Masses) aer fixed on the upward extensions of the lower links which are in the form of bent links BAE and CDF shown in the below figure.



After taking r` = r

$$h = \frac{1}{m\omega^2} \cdot \frac{a}{e}\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

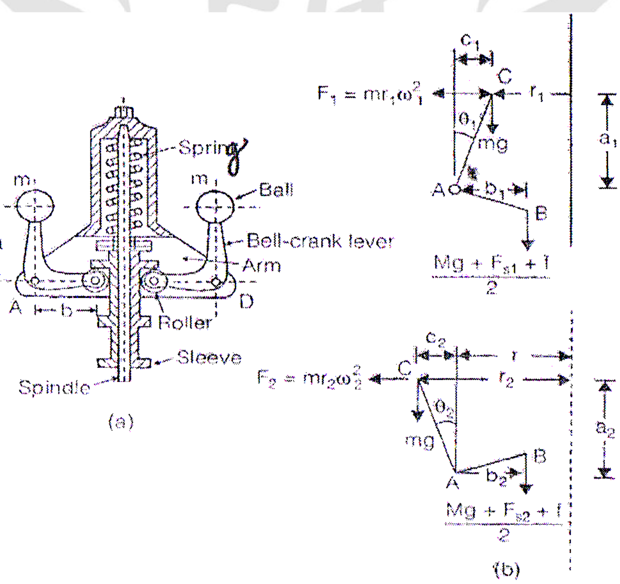
$$N^2 = \frac{895}{h}, \frac{a}{e}\left(\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right)$$

If $k = 1, f = 0$

$$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{m+M}{m}\right)$$

Hartnell Governor

In this governor, ball are controlled by a spring.



Let, Centrifugal force (F) = $m r \omega^2$

F_s = Spring force

Taking moments about the fuicrum A.

$$F_1 a_1 = \frac{1}{2}(Mg + F_{s1} + f)\, b_1 + mgc_1$$

$$F_2 a_2 = \frac{1}{2}(Mg + F_{s2} + f)\, b_2 + mgc_2$$

Neglect obliquity of the arm in that case,

$$a_1 = a_2 = a,\ b_1 = b_2 = b,\ c_1 = c_2 = 0$$

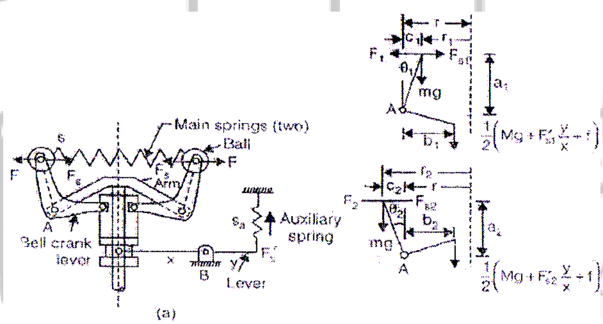
$$F_1 a_1 = \frac{1}{2}(Mg + F_{s1} + f)\, b$$

$$F_2\, a_2 = \frac{1}{2}(Mg + F_{s2} + f)\, b$$

Let, K = Stiffness of the spring

$$K = 2\left(\frac{a}{b}\right)^2 \left(\frac{F_2 - F_1}{r_2 - r_1}\right)$$

Wilson-Hartnell Governor (Radial-Spring Governor)



Let, s = Stiffness of each of the main springs

S_a = Stiffness of the auxiliary spring

$F`_s$ = Force applied by the auxiliary spring

Assuming that the sleeve moves up, take moments about the fulcurm A in two positions.

$$F_1 a_1 - F_{s1} a_1 = \frac{1}{2}\left(Mg + F`_{s1}\frac{y}{x} + f\right) b_1 + mgc_1$$

$$F_2 a_2 - F_{s2} a_2 = \frac{1}{2}\left(Mg + F`_{s2}\frac{y}{x} + f\right) b_2 + mgc_2$$

If obilquity effects are neglected.

$$a_1 = a_2 = a,\ b_1 = b_2 = b \text{ and } c_1 = c_2 = 0$$

$$(F_1 - F_{s1})\,a = \frac{1}{2}\left(Mg + F`_{s1}\frac{y}{x} + f\right)b$$

$$(F_2 - F_{s2})a = \frac{1}{2}\left(Mg + F`_{s2}\frac{y}{x} + f\right)b$$

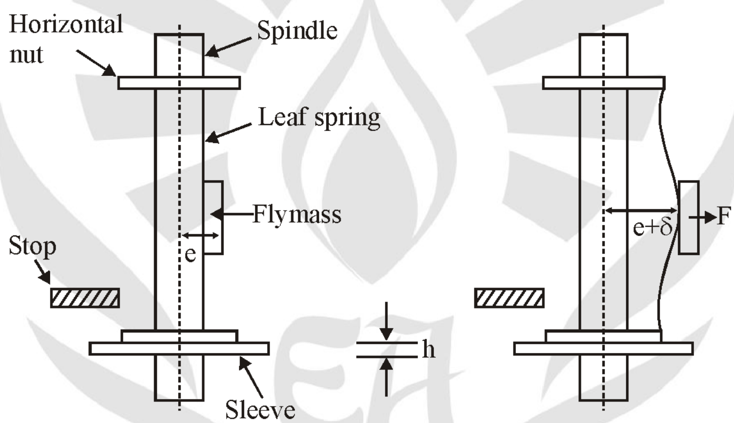
The main spring consists of two springs. Therefore, the force exerted is given by,

$$F_{s2} - F_{s1} = 2 \times \text{Force exerted by each spring}$$
$$= 2 \times \text{Stiffness of each spring} \times \text{Elongation of each spring}$$
$$= 2 \times s \times 2 \times (r_2 - r_1)$$
$$= 4s\,(r_2 - r_1)$$

and also

$$\frac{F_2 - F_1}{r_2 - r_1} = 4s + \frac{S_a}{2}\left(\frac{b}{a}\frac{y}{x}\right)^2$$

Pickering Governor



Let m = Mass fixed to each spring

e = Distance between spindle axis and centre of mass when the governor is at rest.

ω = Angular speed of the sleeve

δ = Deflection of the centre of the leaf spring for spindle speed ω

E = Modulus of elasticity of the spring material

l = Moment of inertia of the corss -section of the spring aobut neutral axis = $\frac{bt^3}{12}$ b and t being the width and the thickness of the leaf spring.

Centrifugal force, $F = m(e + \delta)\,\omega^2$

Assume leaf spring is a beam of uniform cross section fixed at both ends and carrying a load at the centre.

δ

$$= \frac{Fl^3}{192EI} = \frac{m(e+\delta)\omega^2 l^3}{192EI}$$

Point to Remember:

- Pickenning governor is used in gramophone

Sensitiveness of Governor

A governor is said to be sensitive when it readily responds to a small change of speed.

$$\text{Sensitiveness} = \frac{\text{mean speed}}{\text{range of speed}}$$

$$= \frac{N}{N_2 - N_1} = \frac{N_1 + N_2}{2(N_2 - N_1)}$$

When N = Mean speed

N_1 = Minimum speed corresponding to full load conditions

N_2 = Maximum speed corresponding to no-load conditions

Hunting

Sensitiveness of a governor is a desirable quality. However, if a governor is too sensitive, it may fluctuate continuously. This phenomenon of fluctuation is pronounced as hunting.

Isochronism

A governor with sensitivity equal to infinity is treated as isochronous governor. For all position of sleeves, governor has same equilibrium speed.

Stability

A governor is said to be stable if it brings the speed of the engine to the required value and there is not much hunting. The ball masses, occupy a definite position for each speed of the engine within the working range. The stability and the sensitivity are two opposite characteristics.

Effort of Governor

The effort of the governor is the mean force acting on the sleeve to raise or lower it for a given change of speed. At constant speed, the governor is in equilibrium and the resultant force acting on the sleeve is zero. However, when the speed of the governor increases or decreases, a force is exerted on the sleeve which tends to move it. When the sleeve occupies a new steady position, the resultant force acting on it again becomes zero.

Power of Governor

The power of a governor is the work done at the sleeve for a given percentage change of speed.

Power = Effort of governor × displacement.

Controlling Force

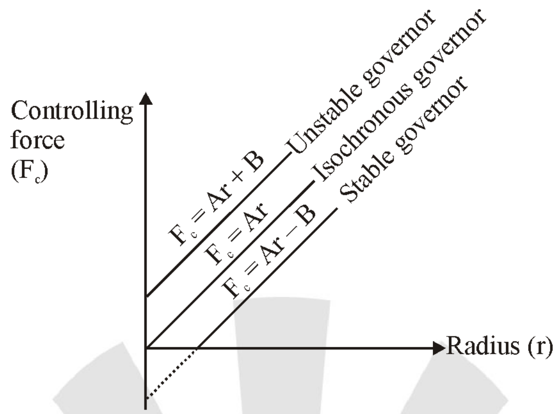
Controlling force is equal and opposite to the centrifugal force and acts readily inward. It is supplied by

Gravity of mass of ball in case of watt governor.

Gravity of mass of ball and dead weight of sleeve in case of porter and proell governor.

Gravity of ball masses and spring force in harthell and hartung governs.

Controlling force curve for spring loaded governor.



Where, A, B, > 0

Points to remember:

- Controlling force curve is parabolic curve in case of dead weight governor.
- Controlling force curve is straight line is case of spring loaded governor.

10. Vibration

Vibration is a periodic motion of small magnitude. But for sake of simplicity we can assume it as simple harmonic motion of small amplitude.

Some important Term of Vibration

Period

It is the time taken by a motion to repeat itself, and is measured in seconds.

Cycle

It is the motion completed during one time period.

Frequency

Frequency is the number of cycles of motion completed in one second. It is expressed in hertz (Hz) and is equal to one cycle per second.

Resonance

When the frequency of the external force is the same as that of the natural frequency of the system, a state of resonance is said to have been reached. Resonance results in large amplitudes of vibrations and this may be dangerous.

Type of Vibration

Free Vibrations

Elastic vibrations in which there are no friction and external forces after the initial release of the body, are known as free or natural vibrations.

Forced Vibrations

When a repeated force continuously acts on a system, The vibrations are said to be forced vibration. The frequency of the vibrations is that of the applied force and is independent of there own natural frequency of vibrations.

Damped Vibrations

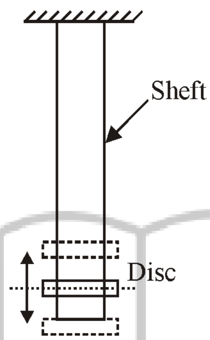
When the energy of a vibrating system is gradually dissipated by friction and other resistance the vibrations are said to be damped vibration.

Un-damped Vibrations (Hypothetical)

When there is no friction or resistance present in system to contract vibration then body execute un-damped vibration.

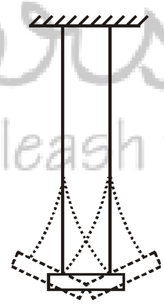
Longitudinal Vibrations

If the shaft is elongated and shortened so that the same moves up and down resulting in tensile and compressive stresses in the shaft, the vibrations are said to be longitudinal.



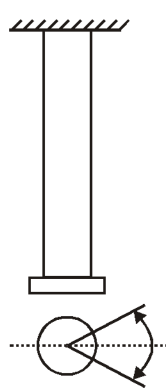
Transverse Vibrations

When the shaft is bent alternately and tensile and compressive stresses due to bending result, the vibrations are said to be transverse vibration.

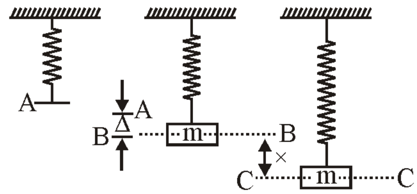


Torsional Vibrations

When the shaft is twisted and untwisted alternately and torsional shear stresses are induced. The vibrations are known as torsional vibrations.



Free Longitudinal Vibrations



∴ Equation of equilibrium

$$m\ddot{x} + sx = 0$$

Here,

s = Stiffness of the spring

m = Mass

x = Displacement

Natural circular frequency (ω_n)

$$\omega_n = \sqrt{\frac{s}{m}}$$

Natural linear frequency (f_n)

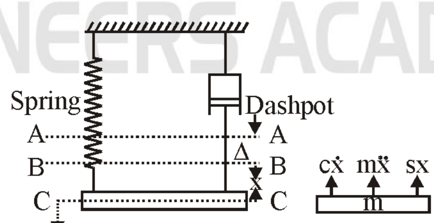
$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

Here, mass of spring is neglected,

If we consider mass of spring is m_1 then

$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m + \frac{m_1}{3}}}$$

Damped Longitudinal Vibration



Let s = Stiffness of the spring

c = Damping coefficient (damping force per unit velocity)

ω_n = Frequency of natural undamped vibrations

x = Displacement of mass from mean position at time t.

$v = \dot{x}$ = velocity of the mass at time t

$f = \ddot{x}$ = acceleration of the mass at time t

Equation of equilibrium,

$$m\ddot{x} + c\dot{x} + sx = 0$$

Damping factor

$$\zeta = \frac{c}{c_c} = \frac{\text{Actual dampring coefficient}}{\text{Critical damping coefficient}}$$

$$\zeta = \sqrt{\frac{(c/2m)^2}{s/m}} = \frac{c}{2\sqrt{sm}}$$

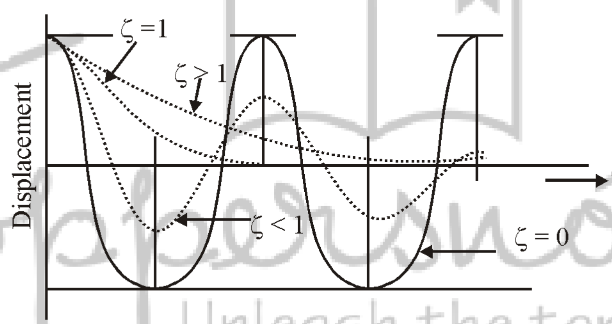
Here, $C_c = \sqrt{sm}$

Thus when

$\zeta = 1$, the damping is critical

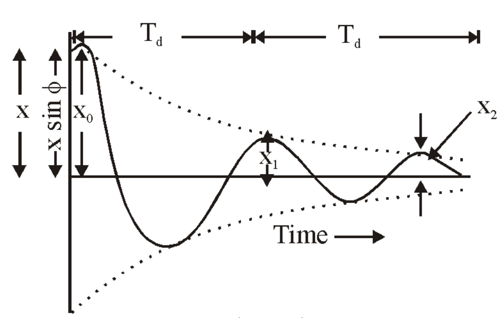
$\zeta > 1$, the system is over-damped

$\zeta < 1$, the system is under-damped



Logarithmic Decrement (δ)

The ratio of two successive oscillations is constant in an underdamped system. Natural logarithm of this ratio is called logarithmic decrement.

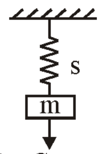


$$\delta = \ln\left(\frac{X_n}{X_{n+1}}\right) = \ln e^{(\zeta\omega_n T_d)} = \zeta\omega_n t_d$$

$$= \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

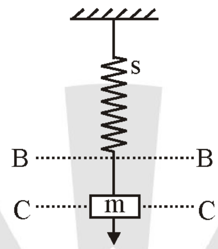
Forced Vibration

Step-input force



$$\ddot{x} + \frac{s.x}{m} = 0$$

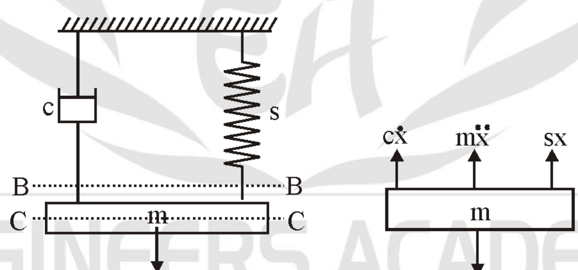
Harmonic force



$$m\ddot{x}+sx = F_0 \sin \omega t$$

$$x = x\sin(\omega_n t + \phi) + \frac{\frac{F_o}{s}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\sin \omega t$$

Forced-Damped Vibrations



$$m\ddot{x}+c\dot{x}+sx = F_o \sin \omega t$$

Damped Free Response

$$x = X e^{-\zeta\omega nt.} \sin(\omega d \,.\, t + \phi)$$

Steady State Response

$$X = \underbrace{\frac{Xe^{-\zeta\omega nt}.\sin(\omega_d.t+\phi_1)}{\text{Dapmed free response}}} + \underbrace{\frac{F_o \sin(\omega t - \phi)}{\sqrt{(s-m\omega)^2+(c\omega)^2}}}_{\text{Steady state response}}$$

Magnification Factor

The ratio of the amplitude of the steady-stae response to the static deflection under the action of force F_o is known as the magnification factor (MF).

$$MF = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

Transmissiblity

It is defined as the ratio of the force transmitted (to the foundation) to the force applied. It is a measure of the effectiveness of the vibration isolating material.

$$\in = \frac{\sqrt{1+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left[\frac{\omega}{\omega_n}\right]^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

At resonance, $\omega = \omega_n$

$$\in = \frac{\sqrt{1+(2\zeta)^2}}{2\zeta}$$

No dampoint ($\zeta = 0$)

$$\in = \frac{1}{\pm\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]}$$

Case 1: $\frac{\omega}{\omega_n} < \sqrt{2} \Rightarrow \in > 1$

Case 1: $\frac{\omega}{\omega_n} = \sqrt{2} \Rightarrow \in = 1$

Case 1: $\frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow \in < 1$

TRANSVERSE VIBRATION

Natural Linear Frequency

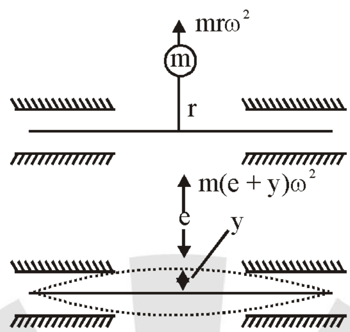
$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

Δ = Static deflection

Whirling of shaft

y = Displacement of shaft from axis

e = The distance by which centre of mass of shaft is displaced from shaft axis.



$$y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

Case 1: At resonance, $\omega = \omega_n$

$$\omega_c = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\Delta}}$$, the diflection y is infinitely large

Case 2: If

$$\omega > \omega_n$$

$$y < 0$$

Shaft will deflect in opposite direction.

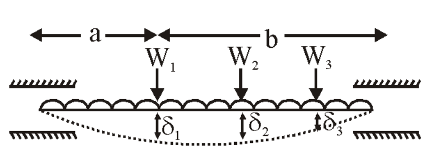
Case 3: If $y = -e$ when $\omega >> \omega_n$

Shaft will deflect so that the axin of rotation passer through the c.g. of the man.

Dunkerley's Method

Let W_1, w_2, W_3, ... be the concentrated loads on the shaft due to masses m_1, m_2, m_3.... and δ_1, δ_2, δ_3 ......the static deflections of this shaft under each load when that load acts alone on the shaft. Let the shaft carry a uniformly distributed mass of m per unit length over its whole span and the static delflection at mid-sapn due to the load of this mass be δ_s. Also , let

f_n = Frequency of transverse vibration of the whole system.



$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + ... + \frac{\delta_{shaft}}{1.27}}} \text{HZ}$$

If mass of shaft is negligible

$$\delta_{shaft} = 0$$

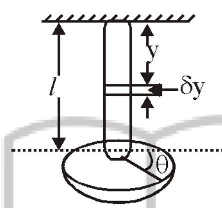
$$\Rightarrow \quad f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + ...}}$$

$$\therefore \quad \delta_1 = \frac{W_1 a^2 b^2}{3EI\,l}$$

$$\delta_{shaft} = \frac{5Wl^4}{384EI}$$

TORSIONAL VIBRATION

Free Torsional Vibration (Single Rotor)



Let

θ = Angular displacement of the disc from its equilibrium position at any instatn.

$q = \left(\frac{GJ}{l}\right)$ = torsional stiffness of the shaft

Here,

G = Modulus of rigidly of the shaft material

J = Polar moment of inertia of the shaft cross-section

$$I\ddot{\theta} + q\theta = 0$$

$$\omega_n = \sqrt{\frac{q}{I}}$$

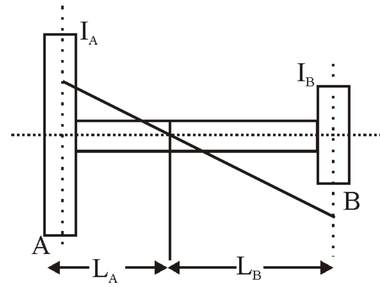
$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$$

Inertia effect of mass of shaft

$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{1 + \frac{I_1}{3}}}$$

Here, I_1 = Moment of inertia of shaft

Free Torsional Vibrations (Two Rotor System)



Let

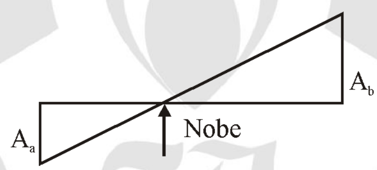
L_a and L_b = lengths of two portions of the shaft

l_a and l_b = Moment of inertia of rotors A and B respectively

q_a and q_b = Torsional stiffness of lengths l_a and l_b of the shaft respectively.

f_{na} and f_{nb} = natural frequencies of torsional vibrations of rotors A and B respectively.

$$\frac{L_A}{L_B} = \frac{l_B}{l_A} \text{ or } L_A l_A = L_B l_B$$

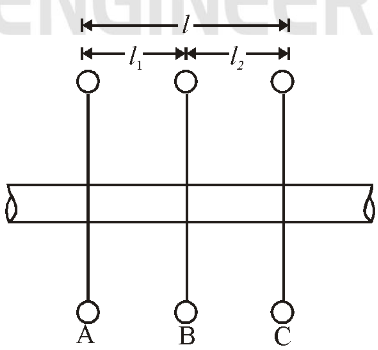


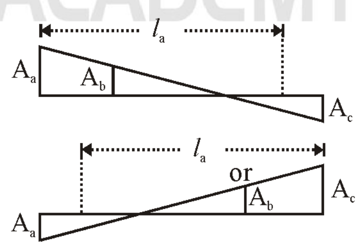
Point of remember:

At node displacement will be zero.

Single Node system

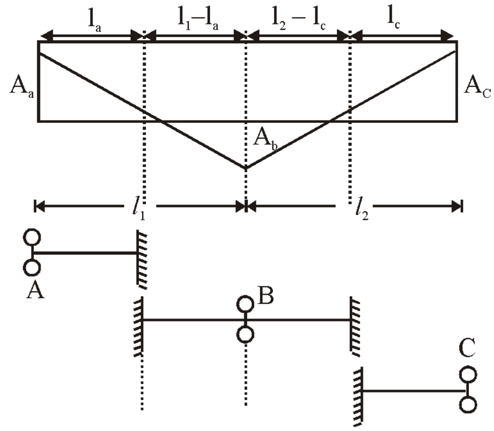
Single node case observed when whether ‘A’ and ‘B’ rotate is same direction and ‘C’ in opposite direction or ‘B’ and ‘C’ in same direction and ‘A’ in opposite direction.





Double Node System

Double node case occur when ‘A’ and ‘C’ are in same direction and ‘B’ is in opposite direction.



$$f_{na} = \frac{1}{2\pi}\sqrt{\frac{k_{\theta A}}{I_A}}$$

$$f_{nb} = \frac{1}{2\pi}\sqrt{\frac{GI_p}{I_b}\left(\frac{1}{l_1 - l_a} + \frac{1}{l_2 - l_c}\right)}$$

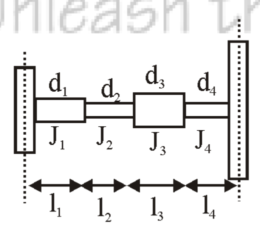
$$f_{nc} = \frac{1}{2\pi}\sqrt{\frac{k_{\theta c}}{I_c}}$$

$\therefore$
$$f_{na} = f_{nb} = f_{nc}$$

$$I_a l_a = I_c l_c$$

$$\frac{1}{I_a l_a} = \frac{1}{I_b}\left(\frac{1}{l_1 - l_a} + \frac{1}{l_2 - l_c}\right)$$

Torsionally Equivalent Shaft



$$l = l_1\left(\frac{d}{d_1}\right)^4 + l_2\left(\frac{d}{d_2}\right)^4 + l_3\left(\frac{d}{d_3}\right)^4 + l_4\left(\frac{d}{d_4}\right)^4$$

d & l are the diameter & the length respectively of the torsionally equivalent shaft.

Diameter d is usually chosen an one of the existing diameter of the stepper shaft.

❑❑❑