

RRRB - JE

←————→

MECHANICAL

Railway Recruitment Board

Volume - 6

Industrial Engineering



FORECASTING

THEORY

1.1 | FORECASTING

Forecasting means estimation of type, quantity and quality of future works e.g. sales etc. It is a calculated economic analysis.

1.1.1 Basic Elements of Forecasting

- Trends
- Cycles
- Seasonal Variations
- Irregular Variations

1.2 | SALES FORECASTING TECHNIQUES

- Historic Estimation
- Sales Force Estimation
- Trend Line (or Time-series Analysis) Technique
- Market Survey
- Delphi Method
- Judge Mental Techniques
- Prior Knowledge
- Forecasting by Past Average
- Forecasting from Last Period's Sales
- Forecasting by Moving Average
- Forecasting by Weighted Moving Average
- Forecasting by Exponential Smoothing
- Correlation Analysis
- Linear Regression Analysis.

1.2.1 Average Method

Forecast sales for next period = Average sales for previous period

$$F_{t+1} = \frac{S_t + S_{t-1} + \dots + S_1}{t}$$

Where,

F_{t+1} = Forecast sales for next period

S_t, S_{t-1}, \dots, S_1 are sales of t number of periods.

Example:

Period No.	1	2	3	4	5	6
Sales	7	5	9	8	5	8

Forecast sales for Period No

$$F_{6+1} = \frac{7+5+9+8+5+8}{6} = 7$$

1.2.2 Forecast by Moving Average

In this method the forecast is neither influenced by very old data nor does it solely reflect the figures of the previous period.

Example:

Year	Period	Sales	Four-period average forecasting
1987	1	50	
	2	60	
	3	50	
	4	40	
1988	1	50	Forecast for 1988 period $1 = \frac{50+60+50+40}{4} = 50$
	2	55	Forecast for 1988 period $2 = \frac{60+50+40+50}{4} = 50$

1.2.3 Weighted Moving Average

A weighted moving Average allows any weights to be placed on each element, providing of course, that the sum of all weights equals one.

Example:

Period	Sales
Month-1	100
Month-2	90
Month-3	105
Month-4	95
Month-5	110

Forecast (weights 40%, 30%, 20%, 10% of most recent month)

Forecast for month-5 would be:

$$F_5 = 0.4 \times 95 + 0.3 \times 105 + 0.2 \times 90 + 0.1 \times 100 = 97.5$$

Forecast for month-6 would be:

$$F_6 = 0.4 \times 110 + 0.3 \times 95 + 0.2 \times 105 + 0.1 \times 90 = 102.5$$

1.2.4 Exponential Smoothing

New forecast = α (latest sales figure) + (1 - α) (old forecast) [VIMP]

$$F_{t+1} = \alpha d_t + (1 - \alpha) F_t$$

Where,

α is known as the smoothing constant.

The size of α should be chosen in the light of the stability or variability of actual sales, and is normally from 0.1 to 0.3.

The smoothing constant, α , that gives the equivalent of an N-period moving average can be calculated

as follows, $\alpha = \frac{2}{N+1}$

For e.g. if we wish to adopt an exponential smoothing technique equivalent to a nine period moving

average then, $\alpha = \frac{2}{9+1} = 0.2$

Basically, exponential smoothing is an average method and is useful for forecasting one period ahead. In this approach, the most recent past period demand is weighted most heavily. In a continuing manner the weights assigned to successively past period demands decrease according to exponential law.

1.2.5 Generalized Equation

$$F_t = \alpha (1 - \alpha)^0 d_{t-1} + \alpha (1 - \alpha)^1 d_{t-2} + \alpha (1 - \alpha)^2 d_{t-3} + \dots + \alpha (1 - \alpha)^{k-1} d_{t-k} + (1 - \alpha)^k F_{t-k}$$

[Where k is the number of past periods]

It can be seen from above equation that the weights associated with each demand of equation are not equal but rather the successively older demand weights decrease by factor (1 - α). In other words, the successive terms $\alpha (1 - \alpha)^0$, $\alpha (1 - \alpha)^1$, $\alpha (1 - \alpha)^2$, $\alpha (1 - \alpha)^3$ decreases exponentially.

This means that the more recent demands are more heavily weighted than the remote demands.

Exponential smoothing method of Demand Forecasting:

- Demand for the most recent data is given more weightage.
- This method requires only the current demand and forecast demand.
- This method assigns weight to all the previous data.

Regression Analysis:

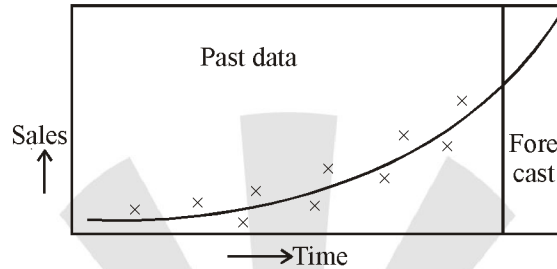
Regression analysis is also known as method of curve fitting. On this method the data on the past sales is plotted against time, and the best curve called the 'Trend line' or 'Regression line' or 'Trend curve'. The forecast is obtained by extrapolating this trend line or curve.

For linear regression

$$y = a + bx$$

$$a = \frac{\sum y - b \sum x}{n}$$

$$b = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$



$$\text{Standard error} = \sqrt{\frac{\sum (x - y_1)^2}{(n - 2)}}$$

Example: What are moving average and exponential smoothing models for forecasting? A dealership for Honda city cars sells a particular model of the car in various months of the year. Using the moving average method, find the exponential smoothing forecast for the month of October 2010. Take exponential smoothing constant as 0.2:

Jan.	2010	80 cars
Feb.	2010	65 cars
March	2010	90 cars
April	2010	70 cars
May	2010	80 cars
June	2010	100 cars
July	2010	85 cars
Aug.	2010	65 cars
Sept.	2010	75 cars

Solution: (i) **Moving average model for forecasting:** Refer theory part of this book.
 (ii) **Exponential smoothing model for forecasting:** Refer theory part of this book

Months	Sells cars	Forecast demand ($n = 3$)
Jan.	80	
Feb.	65	
March	90	
April	70	$(80 + 65 + 90)/3 = 78.33$
May	80	$(65 + 90 + 70)/3 = 75$

June	100	$(90 + 70 + 80)/3 = 80$
July	85	$(70 + 80 + 100)/3 = 83.33$
Aug.	60	$(80 + 100 + 85)/3 = 88.33$
Sep.	75	$(100 + 85 + 60)/3 = 81.67$

Forecast of oct. by exponential smoothing method

$$F_{\text{oct}} = F_{\text{sep}} + \alpha (D_{\text{sep}} - F_{\text{sep}})$$

$$\alpha = 0.2 \quad F_{\text{sep}} = 81.67 \quad D_{\text{sep}} = 75$$

$$F_{\text{oct}} = 81.67 + 0.2 (75 - 81.67)$$

$$F_{\text{Oct}} = 80.33 \simeq 81$$

Forecast for the month of October using moving average

$$F_{\text{oct}} = \frac{D_{\text{July}} + D_{\text{Aug}} + D_{\text{Sep}}}{3}$$

$$= \frac{75 + 60 + 80}{3} = 71.67$$

Example: Explain the need for sales forecasting. How are forecasting methods classified?

The past data about the load on a machine centre is as given below:

Month	Load, Machine-Hours
1	585
2	611
3	656
4	748
5	863
6	914
7	964

- (i) If a five month moving average is used to forecast the next month's demand, compute the forecast of the load on the centre in the 8th month.
- (ii) Compute a weighted three month moving average for the 8th month, where the weights are 0.5 for the latest month, 0.3 and 0.2 for the other months, respectively.

Solution: Most organizations are not in a position to wait until orders are received before they begin to determine what production facilities, process, equipment, manpower, or materials are required and in what quantities. Most successful organizations anticipate the future and for their products and translate that information into factor inputs required to satisfy expected demand. Forecasting provides a blue print for managerial planning. Forecasting is the estimation of the future on the basis of the past.

In many organizations, sales forecasts are used to establish production levels, facilitate scheduling, set inventory levels, determine man power loading, make purchasing decisions, establish sales conditions (pricing and advertising) and aid financial planning (cash budgeting and capital budgeting).

A good forecast should have the following attributes. It should be accurate, simple, easy, economical, quick and upto date. Following are the basic steps involved in a systematic demand forecast.

- (i) State objectives
- (ii) Select method
- (iii) Identify variables
- (iv) Arrange data
- (v) Develop relationship
- (vi) Prepare forecast and interpret
- (vii) Forecast in specific units.

(i) Forecast for 8th month on the basis of five month moving average

$$= (964 + 914 + 863 + 748 + 656)/5 = 829$$

(ii) Forecast for 8th month on the basis of weighted average

$$= 0.5 \times 964 + 0.3 \times 914 + 0.2 \times 863 = 928.8$$

Example: (i) List common time-series forecasting models. Explain simple exponential smoothing method of forecasting demand. What are its limitations?

(ii) The monthly forecast and demand values of a firm are given below:

<i>Month</i>	<i>Forecast units</i>	<i>Demand units</i>
Jan	100	97
Feb	100	93
Mar	100	110
Apr	100	98
May	102	130
Jun	104	133
Jul	106	129
Aug	108	138
Sep	110	136
Oct	112	124
Nov	114	139
Dec	116	125

Calculate Tracking Signal for each month. Comment on the forecast model.

Solution: (i) Component of time series models

- (1) Trend (T)
- (2) Cyclic variation (C)
- (3) Seasonal variation (S)
- (4) Random variation (R)

Exponential Smoothing

This is similar to the weighted average method. The recent data is given more weightage and the weightages for the earlier periods are successfully being reduced. Let x_t is the actual (historical) data of demand during the period t . Let \pm is the weightage given for the period t and F_t is the forecast for the time t then forecast for the time $(t + 1)$ will be given as

$$(ii) \text{ Tracking signal} = \frac{\text{Cumulative deviation}}{MAD} = \frac{\sum(x_t - F_t)}{MAD}$$

Where,

$$MAD = \text{Mean Absolute deviation} = \frac{\text{Sum of absolute deviations}}{\text{Total number of datas}} = \frac{\sum(x_t - F_t)}{n}$$

Month	Forecast	Demand	($x_t - F_t$)	MAD		T.S. = $\frac{\sum(x_t - f_t)}{MAD}$
January	100	97	-3	3	-3	-1
February	100	93	-7	5	-10	-2
March	100	110	10	6.67	0	0
April	100	98	-2	5.5	-2	-0.3636
May	102	130	28	10	26	2.6
June	104	133	29	13.167	55	4.177
July	106	129	23	14.571	78	5.353
August	108	138	30	16.5	108	6.545
September	110	136	26	17.55	134	7.635
October	112	124	12	17	146	8.588
November	114	139	25	17.727	171	9.646
December	116	125	9	17	180	10.588

$$\text{Mean square error (MSE)} = \frac{\sum |x_t - F_t|^2}{n} = \frac{4742}{12} = 395.167$$

$$\text{Upper limit} = 3 \times \sqrt{MSE} = 3 \times \sqrt{395.167} = 59.636$$

Since upper limit of T.S < 59.636 hence model should not be revised.

Example: Demand for a certain item has been as shown below:

The forecast for April was 100 units with a smoothing constant of 0.20 and using first order exponential smoothing what is the July forecast? What do you think about a 0.20 smoothing constant?

Time	Actual Demand
April	200
May	50
June	150

Solution: Using exponential smoothing average:

$$\begin{aligned}F_{\text{may}} &= \alpha \times D_{\text{April}} + (1 - \alpha) F_{\text{April}} \\ &= 0.2 \times 200 + (1 - 0.2) \times 100 = 120\end{aligned}$$

$$\begin{aligned}F_{\text{June}} &= \alpha \times D_{\text{May}} + (1 - \alpha)F_{\text{may}} \\ &= 0.2 \times 50 + (1 - .2) \times 120 = 106\end{aligned}$$

$$\begin{aligned}F_{\text{july}} &= \alpha \times D_{\text{june}} + (1 - \alpha) \times F_{\text{june}} \\ &= 0.2 \times 150 + 0.8 \times 106 = 114.8 = 115\end{aligned}$$

Example: In a time series forecasting model, the demand for five time periods was 10, 13, 15 18 and 22. A linear regression fit results in as equation $F = 6.9 + 2.9 t$ where F is the forecast for period t . The sum of absolute deviation for the five data is?

Solution: Sum of absolute deviation

$$\begin{aligned}&= |D_1 - F_1| + |D_2 - F_2| + |D_3 - F_3| + |D_4 - F_4| + |D_5 - F_5| \\ &= |10 - 6.9 - 2.9 \times 1| + |13 - 6.9 - 2.9 \times 2| + |15 - 6.9 - 2.9 \times 3| \\ &\quad + |18 - 6.9 - 2.9 - 2.9 \times 4| + |22 - 6.9 - 2.9 \times 5| \\ &= 0.2 + 0.3 + 0.6 + 0.5 + 0.6 = 2.2\end{aligned}$$

