Toppersnotes
Unleash the topper in you

ENGINEERS ACADEMY

RPSC - A.En.

Assistant Engineering

CIVIL

Rajasthan Public Service Commission (RPSC)

Volume - 6

Strength of Materials

SIERRA INNOVATIONS

1 INTRODUCTION

THEORY

1.1 MATERIAL CLASSIFICATION

According to behaviour on loading, material can be classified as

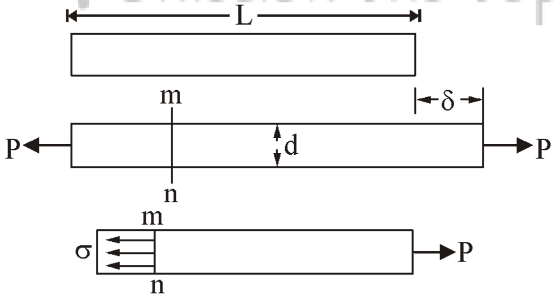
1. **Elastic :** Undergoes deformation when subjected to the external loading and comes back to its original state after removal of load.
2. **Plastic :** Material do not regain its original dimensions and the deformation is permanent.
3. **Rigid :** Does not undergo any deformation when loaded externally.

In statics and dynamics, we deal with forces and motions associated with particles and rigid bodies. In strength of materials, we examine the stresses and strains that occur inside real bodies those deform under loads, here you must understand the difference between rigid body and real body.

1.2 STRESS AND STRAIN

1.2.1 Normal Stress

Consider a prismatic bar loaded by axial forces P at the ends. A prismatic bar is straight structural member having constant cross section throughout its length. The axial force produce a uniform stretching of bar. Here, bar is said to be in tension.



A section taken perpendicular to longitudinal axis of bar is cross-section. Considering free-body diagram, the tensile force P acts on right hand of free body at the other end is force representing the action of removed part of bar upon the part that remains. These forces are continuously distributed over the cross-section. The intensity of force (i.e. force per unit area) is called the stress and is denoted by

Hence under equilibrium, $F = \sigma A$

$\Rightarrow$ $\sigma = \dfrac{F}{A}$

The stress is the force of resistance per unit area offered by a body against the deformation.

When the bar is stretched by force P, as shown in figure, the resulting stresses are tensile stresses and if forces are reversed in direction, causing the bar to be compressed, the stresses are compressive stresses.

As stress acts in direction perpendicular to cut surface, it is referred as normal stress. The normal stresses may be tensile or compressive. The shear stress act parallel to the surface. Conventionally the tensile stresses are taken as positive and compressive stresses are negative.

The unit of stress is N/m^2 also referred as pascal.

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ Kgf/cm}^2 = 0.1 \text{ MPa}$$

$$1 \text{ mPa} = 10^{-3} \text{ N/m}^2$$

It can also be expressed as MPa. i.e., N/mm^2.

1.2.2 Normal Strain

An axially loaded bar undergoes a change in length, becoming longer in tension and shorter when in compression, strain is defined as change in length per unit length.

$$\text{Strain } (\in) = \frac{\delta}{L}$$

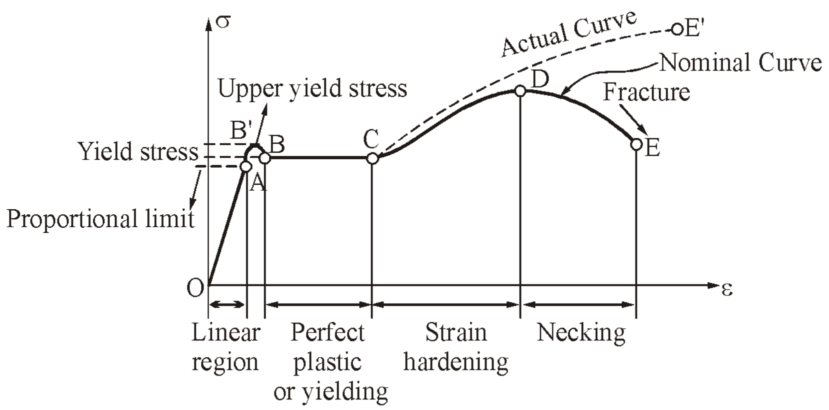
If bar is in tension, the strain is called a tensile strain, representing stretching of material. If the bar is in compression, the strain is compressive strain. The tensile strain is taken as positive and compressive strain is negative. The strain $\in$ is called normal strain because it is associated with normal stresses. As normal strain $\in$ is ratio of two lengths it is a dimensionless quantity i.e. it has no units.

Note : The definition of normal stress and normal strain are based purely on statical and geometrical consideration. It can be used for load of any magnitude and for any material.

1.3 Stress Strain diagram of mild steel in tension

The mechanical properties of material are determined by test performed on small specimen of the material. The most common test is tension test, in which tensile loads are applied on cylindrical specimen. The American Society for Testing and Materials (ASTM) standard tension specimen has diameter of 0.5 in and a gauge length of 2.0 in. The machine used in test is Universal Testing Machine (UTM).

In a static test, the load is applied very slowly and in dynamic test, the rate of loading may be very high. Here, we are analyzing properties based on static test.



The typical stress strain diagram of mild steel is shown in figure. Here, the stress is nominal stress or engineering stress and strain is nominal strain or engineering strain.

$$\text{Nominal stress} = \frac{\text{Load}}{\text{Initial cross section area}} = \frac{P}{A_0}$$

$$\text{True stress} = \frac{\text{Load}}{\text{Actual Area}} = \frac{P}{A_a}$$

$$\text{Nominal strain} = \frac{\Delta L}{L_0}$$

$$\text{True strain} = \frac{\Delta L}{L_a}$$

The nominal stress is obtained by dividing the load P by initial cross sectional area A. The true stress is calculated by using the actual area of the bar.

Similarly, for calculation of strain, if initial gauge length is used nominal strain is obtained. If the actual length is used, true strain is obtained.

1. The diagram begin with straight line from O to A. In this region the stress and strain are directly proportional and behaviour of material is linearly elastic.
2. Beyond point A, linear relationship between stress and strain no longer exists. A is called proportional limit.
3. When load is increased beyond A, the slope of curve become smaller and smaller, unit at point B, the curve becomes horizontal.
4. From B to C, considerable elongation occurs with no increase in tensile force. The phenomenon is known as yielding, region BC is called as yield plateau.
5. In region CD, the material begin to strain hardening, the material undergoes change in its atomic and crystalline structure, resulting in increased resistance of material to further deformation.
6. The load reaches its maximum value and corresponding stress is called ultimate stress.
7. The fracture finally occur at point E as shown in figure. Various properties of material can be deduced from stress-strain diagram, stress corresponding to E is called fracture/rupture stress.

1.4 Some Important Properties of Material

1.4.1 Ductility

The ductility of material by which it can be drawn as wire of small cross-section upon tensile forces.

The percent elongation is defined as

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0} \times 100$$

$$\text{Percent change in area} = \frac{A_f - A_0}{A_0} \times 100$$

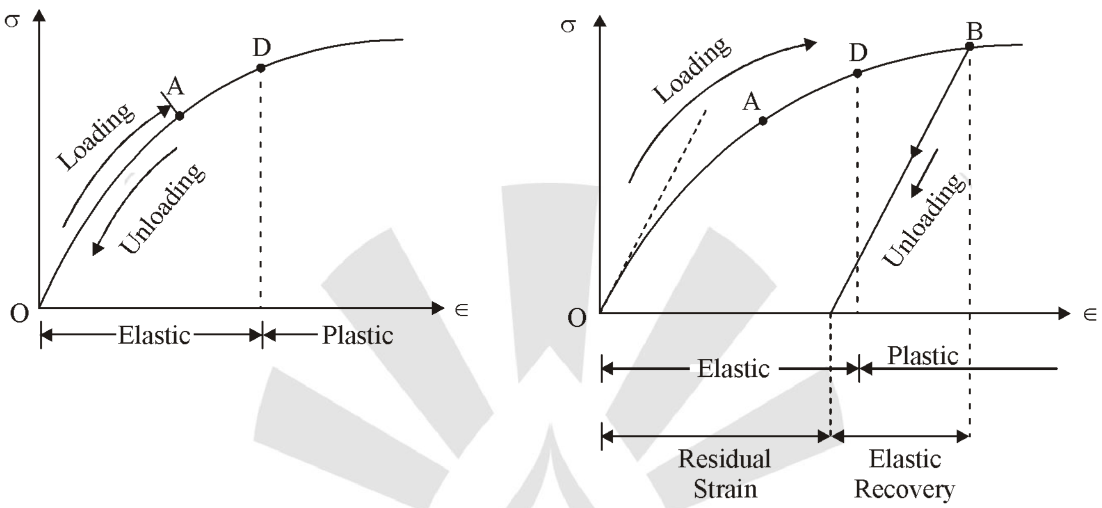
Where, A_0 = Original cross-section area

A_f = Final area at fracture

The material which fails in tension at lower values of strain are classified as brittle materials.

1.4.2 Elasticity and Plasticity

When load is slowly removed, two different situation may occur as shown in figure.



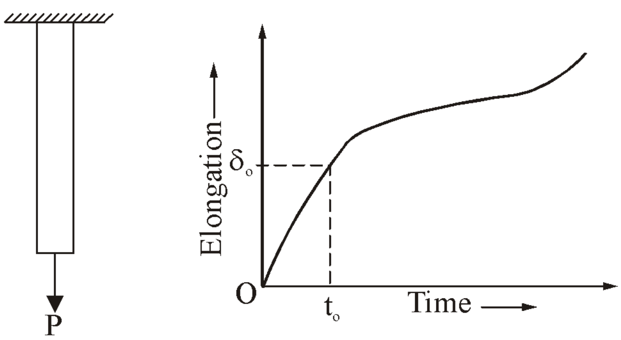
Suppose material is loaded and strain go from O to A. When load is removed, the material follows exactly the same curve back to origin O. This Property of material by which it returns to its original dimension during unloading is called elasticity and material is said to be elastic.

When material is loaded to higher level and when unloading occurs, the material follows line BC (which is parallel to initial slope of OA) on the diagram. When point C is reached, the load has entirely been removed but a residual strain or permanent strain, OC remain in the material. The corresponding residual elongation of bar is called permanent set. Thus, the bar returns partially to original shape, hence the material is said to be partially elastic. Then stress level above which all the strain is not recovered is called elastic limit of material.

The characteristic of material by which it undergoes inelastic strain beyond those at the elastic limit is known as plasticity.

1.4.3 Creep

When a constant load is applied on a material, over a long period of time, a permanent deformation occurs. Which is called creep.



Consider a vertical bar loaded by constant force P. When load P is applied initially, the bar elongates by amount δ_0 during time t_0. Subsequent to time t_0, the load remains constant. However, due to creep, the bar elongates even though load does not change. Creep is more at higher temperature than at ordinary temperature, i.e., creep is proportional to temperature.

1.5 Hooke's law

According to Hooke's law with in the proportional limit, the stress is directly proportional to the strain.

i.e., $\sigma \propto \in$

or $$\frac{\sigma}{\in} = E$$

Where, E = Young's modulus or modulus of elasticity.

When a material behave elastically and also exhibits a linear relationship between stress and strain it is said to be linearly elastic. The linear relationship between stress and strain for bar in simple tension and compression is expressed as:

$$\sigma = E\in$$

Where E is constant of proportionality known as the modulus of elasticity for the material. The modulus of elasticity is slope of stress strain diagram in linearly elastic region. The unit of E is same as the unit of stress.

This equation is known as Hooke's law. The modulus of elasticity E has relatively large values for materials that are stiff. The modulus of elasticity of common materials are

(i) E_{Steel} = 200 GPa

(ii) $E_{Aluminium}$ = 80 GPa

(iii) E_{Wood} = 11 GPa

The modulus of elasticity is often called Young's modulus.

1.5.1 Poisson's Ratio

When prismatic bar is loaded in tension, the axial elongation is accompanied by lateral (direction normal to applied load) contraction. The lateral strain is proportional to axial strain in linear elastic range when material is both homogeneous and isotropic.

The ratio of strain in lateral direction to strain in axial direction is known as Poisson's ratio.

$$\mu = \left| -\left(\frac{\text{Lateral strain}}{\text{Axial strain}}\right) \right|$$

Negative sign shows that both the strain are opposite in nature.

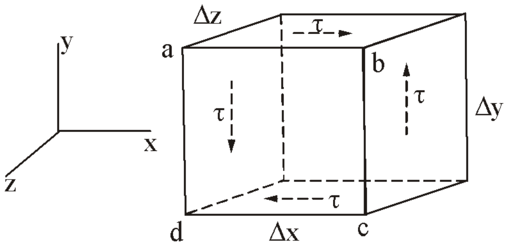
A material is homogeneous if it has the same composition throughout the body and Isotropic material have same properties in all direction.

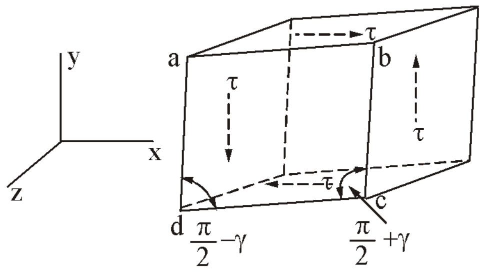
1.5.2 Shear Stress and Strain

Shear Stress act parallel or tangential to the surface. The shear stress is given as

$$\tau = \frac{F}{A}$$

To understand nature of shear stress, let us consider stress element in form of rectangular parallel block having sides of Δx, Δy and Δz.





Considering force equilibrium, total shear force on the top face is $\tau\Delta x\Delta z$ and this force is balanced by equal and opposite force on bottom face. These two forces form a couple having moment about z-axis of magnitude $\tau\Delta x\Delta y\Delta z$. Equilibrium of element requires this moment to be balanced by equal and opposite moment resulting from shear stress acting on side face of element.

This requires the magnitude of shear stress on opposite face of an element to be equal in magnitude and opposite in direction.

Under the action of these stresses, the material is deformed, The angle γ is measure of distortion or change in shape of element and is called shear strain, the unit of shear strains are radians.

A shear stress acting on positive face of element is positive if it acts in positive direction of one of the coordinate axis and negative if it acts in the negative direction of the axis. A shear stress acting on a negative face of an element is positive if it acts in the negative direction of an axis and negative if it acts in the positive direction.

Shear strain in an element is positive when the angle between two positive (or negative) faces is reduced. The strain is negative when angle between two positive (or two negative) faces is increased.

The shear stress-strain diagram can be plotted in same way as in tension-test diagram. Hooke's law in shear is

$$\tau \propto \gamma$$

$$\tau = G\gamma$$

Where, G = Shear modulus of elasticity or modulus of rigidity

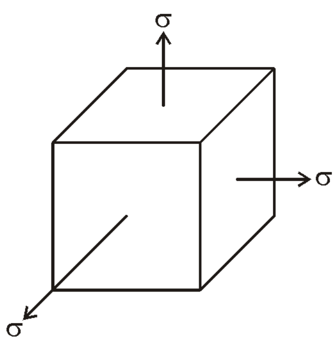
τ = Shear stress

γ = Shear strain

1.5.3 Bulk Modulus

If material is subjected to similar and equal triaxial stresses, then ratio of stress to volumetric strain is called bulk modulus.

In case of hydrostatic stress, $\sigma \propto \in_v$



$$\sigma = K \times \in_v$$

$$K = \frac{\sigma}{\in_v} = \frac{\sigma E}{3\sigma(1-2\mu)}$$

$$E = 3K(1 - 2\mu)$$

For material to be incompressible

Either $\sigma_x + \sigma_y + \sigma_z = 0$

or $\mu = 0.5 \rightarrow$ Not possible

1.5.4 Relation Between Elastic Constants

$$E = 2G(1 + \mu)$$

$$E = 3K(1 - 2\mu)$$

$$E = \frac{9KG}{3K+G}$$

$$\mu = \frac{3K-2G}{6K+2G}$$

Where, E = Modulus of elasticity

K = Bulk modulus

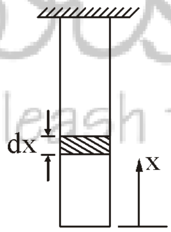
G = Modulus of rigidity

μ = Poisson's ratio

Example 1 : A long wire of specific mass m hang freely under its own weight. Derive formula for tensile stress in the wire.

Solution :

Let the weight per unit volume of the wire be w.



Force on the cross section of the elemental part = w × volume below the section = wAx

$$\therefore \quad \text{Stress} = \frac{wAx}{A} = wx$$

Note : $$w = \frac{mg}{v}$$

Where, m = Total mass of the whole wire

v = Total volume of the whole wire

v = Al = Area × Length