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ENGINEERS ACADEMY

# RPSC - A.En.

## Assistant Engineering

## ELECTRICAL

**Rajasthan Public Service Commission (RPSC)**

## Volume - 4

## Power Systems

SIERRA INNOVATIONS

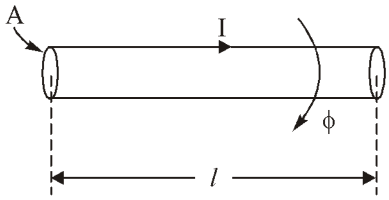
# 1

# MODELLING OF TRANSMISSION LINE

## 1.1 PARAMETERS PERFORMANCE

A particular conductor of cross-sectional area 'A' and length '$l$' having resistance R.

$$R = \rho \frac{l}{A}$$




$$\Rightarrow \qquad R = \frac{l}{\sigma A}$$

$\rho \rightarrow$ Resistivity

$\sigma \rightarrow$ Conductivity

Whenever a current is passed through a conductor it produces a flux '$\phi$'.

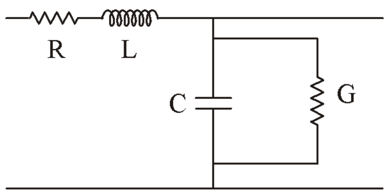
Where, $\phi \propto I$

$\Rightarrow \qquad \phi = LI$

So there exists an inductance also

$$L = \frac{\phi}{I}; \; L \rightarrow \text{Inductance}$$

There is some capacitance exists between two conductors where air behaves as insulator (dielectric). Practically ideal dielectric can't exist in nature, so there must be dielectric loss and losses are represented by resistance or conductance

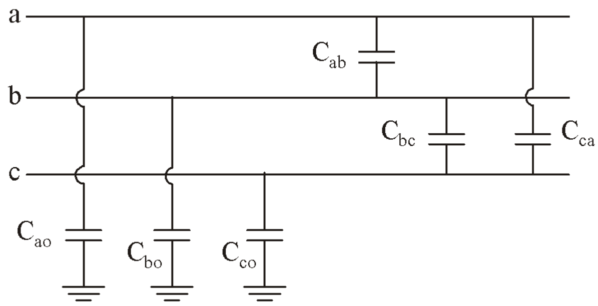



$G \rightarrow$ Shunt conductance/leakage

So there are four parameters R, L, C, G in power line. In power line transmitted power is represented in "MW" and dielectric loss will be in "Watt". So as compared to power transferred dielectric loss is negligible i.e. leakage is neglected.

All the transmission line contains resistance and inductance and in between the conductors there is capacitance and leakage present through out the transmission line. So that R, L, C, G are called as "distributed parameters".

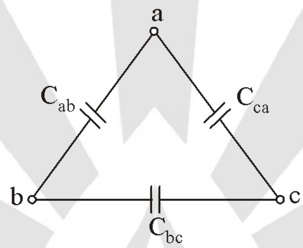
**The capacitances in 3ϕ line**




$C_{ao}$, $C_{bo}$, $C_{co}$ ⇒ Capacitance between line and ground

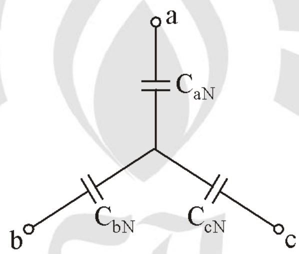
$C_{ab}$, $C_{bc}$, $C_{ca}$ ⇒ Capacitance between two lines.

Here capacitance between lines are in delta form and they can be represented as




Δ-Y conversion:

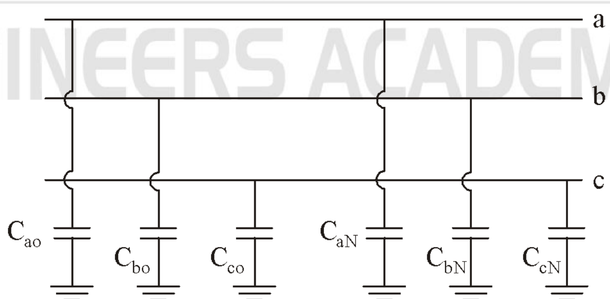
≡




$C_{aN}$ = Capacitance between phase ‘a’ and Neutral.

$C_{bN}$ = Capacitance between phase ‘b’ and Neutral.

$C_{cN}$ = Capacitance between phase ‘c’ and Neutral.

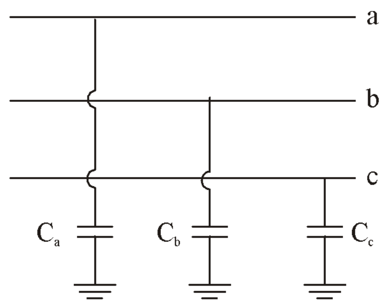



Total capacitance between phase ‘a’ and ground

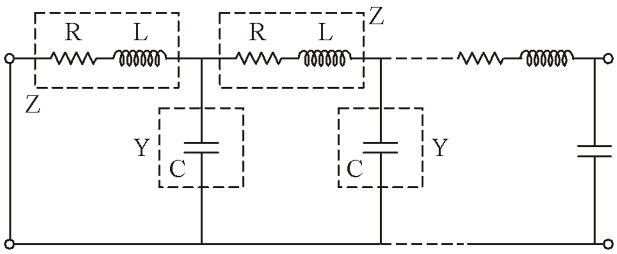
$$C_a = C_{aN} + C_{ao}$$

$$C_b = C_{bN} + C_{bo}$$

$$C_c = C_{cN} + C_{co}$$




## 1.2 Distributed Parameter Model




L ⇒ Length of the line

Per unit length parameters:

Resistance = R

Inductance = L

Capacitance = C

Total series impedance: $Z = (R + j\omega L) \times l$

Total shunt impedance: $Z_{sh} = \dfrac{1}{j\omega Cl} = \dfrac{1}{j(2\pi f)Cl} = \dfrac{1}{j(2\pi C)fl}$

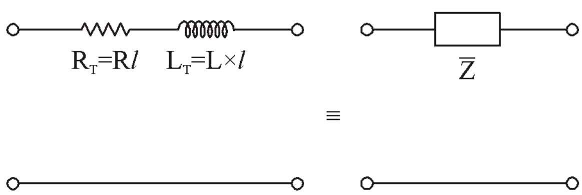
Total shunt admittance: $Y = j(2\pi C)fl$

If "$fl$" ($f \times l$) is very small say $fl < 4000$ Hz km

⇒ Shunt impedance $Z_{sh} \to$ very large

⇒ Shunt admittance $Y \to$ very small

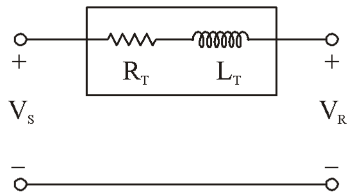
i.e. effect of shunt capacitance neglected and total series impedance can be represented as concentrated hence the simplified circuit will be




Where $\bar{Z}$ is the total series impedance

It is said to be short line model

Short line model:




$$(f \times l) < 4000 \text{ Hz km}$$

$$f \rightarrow \text{Frequency of line}$$

$$l \rightarrow \text{Length of line}$$

For power line

$$f = 50 \text{ Hz}$$

$$\because \quad f \times l < 4000 \text{ Hz km}$$

$$\therefore \quad l < \frac{4000}{50} \text{km}$$

$$l < 80 \text{ km}$$

$$Z_{sh} \rightarrow \text{Neglected}$$

### *1.2.1 Medium Line Model*

If (f × *l*) is not small say

$$4000 < f \times l < 12000 \text{ Hz km}$$

$$Z_{sh} = \frac{1}{j(2\pi C)(fl)}$$

$$f \times l \rightarrow \text{Increases}$$

$$Z_{sh} \rightarrow \text{Decreases}$$

$\therefore$ $Z_{sh}$ is small but it will not be neglected.

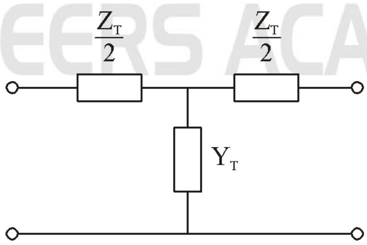
Series impedance $Z = \left(R + j2\pi(f\,L)\right) \times l$

i.e. series impedance will increases.

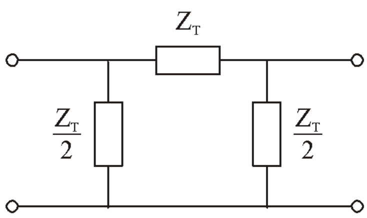
Shunt admittance is not very large so leakage current can't be neglected but it can be assumed to be concentrated either at the centre of line (i.e. T-model) or equally at the corners of the line (i.e. π-model).

### *1.2.2 Nominal T-Model*






**Nominal π-Model:**



These two are the medium line models.

For power line frequency $f = 50\text{ Hz}$

$\therefore$ $80\text{ km} < l < 240\text{ km}$

### *1.2.3 Long Line Model*

(Exact distributed if f$l$ is large i.e. parameter model)

$$(f \times l) > 12000\text{ Hz km}$$

For power line frequency

$$f = 50\text{ Hz}$$

$$l > 240\text{ km}$$

If $(f \times l)$ is large $Z_{sh} = \dfrac{1}{j(2\pi C)(fl)}$

$$Z_{sh} \rightarrow \text{Decreases}$$

Series impedance $Z = (R + j\omega L) \times l$

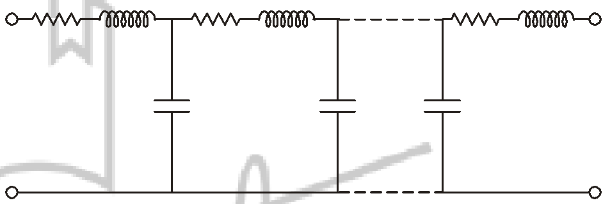
**Note:**

1. Series impedance

   $Z_{(\text{Long line})} > Z_{(\text{Medium line})} > Z_{(\text{Short line})}$

2. Shunt impedance

   $Z_{sh(\text{Long line})} > Z_{sh(\text{Medium line})} > Z_{sh(\text{Short line})}$



Exact distributed parameter model i.e. long line

For communication line if f = 5 MHz (In general communication line is used at high frequencies)

Let $f \times l_1 = 4000\text{ Hz km}$

$\Rightarrow$ $l_1 = \dfrac{4000}{5 \times 10^6}\text{ km}$

$\therefore$ $l_1 = 80\text{ cm}$

Similarly $f \times l_2 = 12000\text{ Hz km}$

$\Rightarrow$ $l_2 = \dfrac{12000}{5 \times 10^6}\text{ km}$

$\therefore$ $l_2 = 240\text{ cm}$

If $f = 5\text{ MHz}$

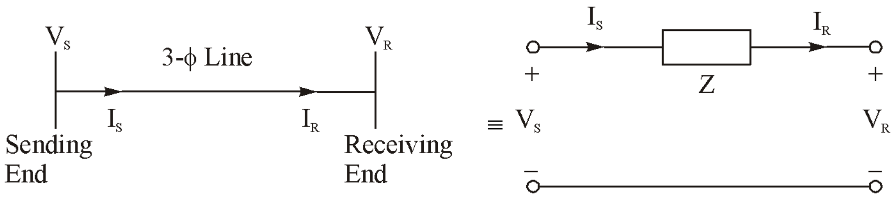
For short line model $l < 80\text{ cm}$

For medium line model $80\text{ cm} < l < 240\text{ cm}$

For long line model $l > 240\text{ cm}$

So for communication line always long line model is used practically

### 1.2.4 Short Line




Total series impedance

$$\bar{Z} = R + jX_L = R + j\omega L$$

$$V_S \rightarrow \text{Sending end voltage}$$

$$V_R \rightarrow \text{Receiving end voltage}$$

$$I_S \rightarrow \text{Sending end current}$$

$$I_R \rightarrow \text{Receiving end current}$$

According to the diagram

$$V_S = V_R + Z I_R \quad \text{...(i)}$$

and

$$I_S = I_R \quad \text{...(ii)}$$

Transmission Parameter

$$V_S = AV_R + BI_R \quad \text{...(iii)}$$

$$I_S = CV_R + DI_R \quad \text{...(iv)}$$

Equation (i) and (ii) also written as

$$V_S = 1 \cdot V_R + Z \cdot I_R$$

$$I_S = 0 \cdot V_R + 1 \cdot I_R$$

When compare with equation (iii) and (iv)

$$A = D = 1$$

$$B = Z$$

$$C = 0$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$



In case of no load

$$I_R = 0$$

$$\therefore \quad V_S = V_R$$

i.e.

$$V_{R(\text{no load})} = V_S$$

Voltage regulation

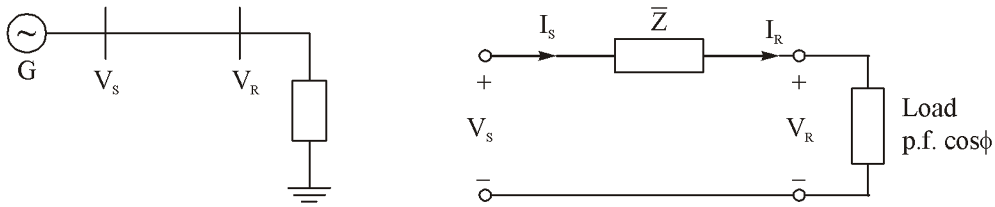
$$V.R. = \frac{V_{R(nl)} - V_{R(fl)}}{V_{R(fl)}} \times 100 = \frac{V_S - V_R}{V_R} \times 100$$

We know that

$$\bar{V}_S = \bar{V}_R + Z\,\bar{I}_R$$

$$\bar{V}_S = \bar{V}_R + \bar{I}_R \times R + j\,\bar{I}_R \times X \quad \text{...(A)}$$

When load is connected



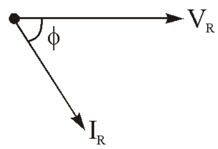

(i) (ii)

At lagging power factor $\cos\phi$, receiving end current $I_R$ lags $V_R$ by angle $\phi$.

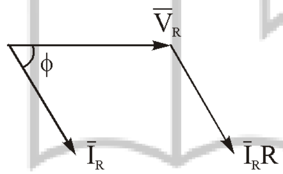
Phasor diagram $\quad \bar{V}_S = \bar{V}_R + \bar{I}_R R + j\,\bar{I}_R X$

**Step-(i) :** Draw $V_R$ along X-axis i.e. reference phase

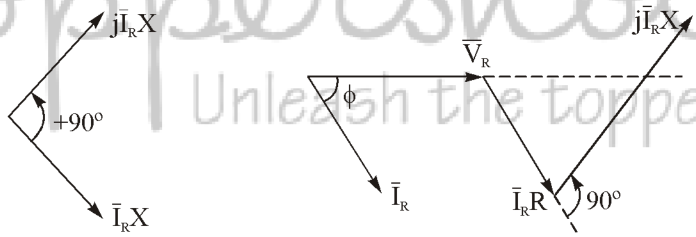

**Step-(ii) :** $I_R$ lags $V_R$ by angle.

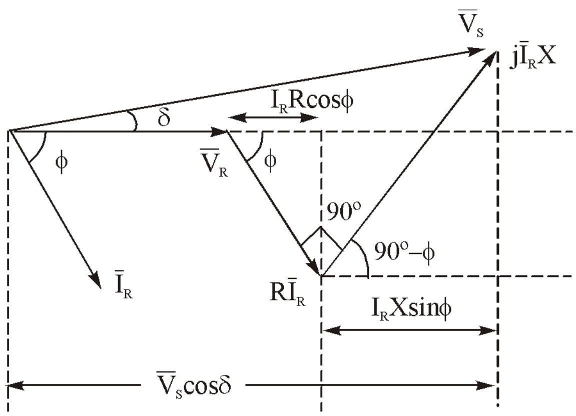



**Step (iii) :** Add $\bar{I}_R\, R$ with $\bar{V}_R$




**Step (iv) :** Add $j\,\bar{I}_R\, X_L$ with $\bar{I}_R R$




As per diagram $\quad |\bar{V}_S|\cos\delta = |\bar{V}_R| + |\bar{I}_R| R\cos\phi + |\bar{I}_R| X\sin\phi$

Due to transient stability criterion the value of 'δ' is small, so that $\cos\delta \approx 1$

$$\therefore \quad |\bar{V}_S| = |\bar{V}_R| + |\bar{I}_R|(R\cos\phi + X\sin\phi)$$

$$\Rightarrow \quad |V_S| - |\bar{V}_R| = |\bar{I}_R|(R\cos\phi + X\sin\phi)$$

$$\Rightarrow \quad \frac{|\bar{V}_S| - |\bar{V}_R|}{|\bar{V}_R|} = \frac{|\bar{I}_R|}{|\bar{V}_R|}(R\cos\phi + X\sin\phi)$$

i.e. at lagging p.f. cos ϕ

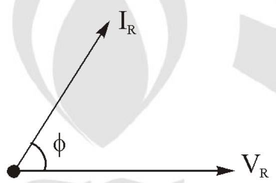
$$\text{Voltage regulation} = \frac{|\bar{I}_R|}{|\bar{V}_R|}(R\cos\phi + X\sin\phi)$$

At leading power factor cos ϕ, i.e. $I_R$ leads the voltage $V_R$ by an angle ϕ.

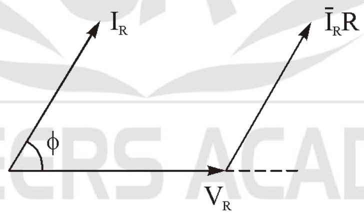
**Phasor Diagram:**

**Step (i) :** Draw $V_R$ along X-axis

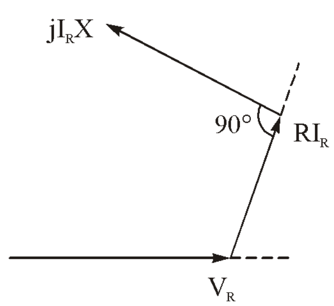

**Step (ii) :** $I_R$ leads $V_R$ by an angle ϕ.



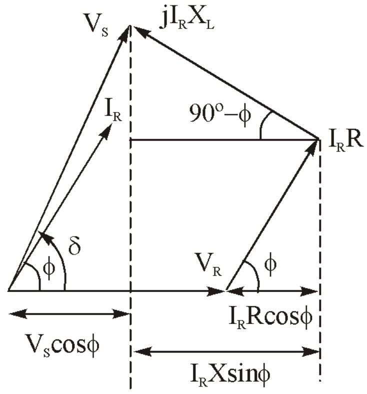

**Step (iii) :** Add $RI_R$ with $V_R$




**Step (iv) :** Add j $I_R X$ with $RI_R$



**Step (v) :** Final phasor diagram




As per diagram $$|\bar{V}_S|\cos\delta = |\bar{V}_R| + |\bar{I}_R| R\cos\phi - |\bar{I}_R| X_L \cos\phi$$

Due to transient stability criterion the value of δ is small. So that cos ϕ = 1.

$$\therefore \quad |\bar{V}_S| = |\bar{V}_R| + |\bar{I}_R|(R\cos\phi - X\sin\phi)$$

$$\Rightarrow \quad |\bar{V}_S| - |\bar{V}_R| = |\bar{I}_R|(R\cos\phi - X\sin\phi)$$

$$\Rightarrow \quad \frac{|\bar{V}_S| - |\bar{V}_R|}{|\bar{V}_R|} = \frac{|\bar{I}_R|}{|\bar{V}_R|}(R\cos\phi - X\sin\phi)$$

$$\therefore \quad \text{Voltage Regulation} = \frac{|\bar{I}_R|}{|\bar{V}_R|}(R\cos\phi - X\sin\phi)$$

For maximum voltage regulation

(It is worst regulation)

$$\frac{dVR}{d\phi} = 0$$

At lagging power factor cosϕ

$$\Rightarrow \quad \frac{|\bar{I}_R|}{|\bar{V}_R|}\frac{d}{d\phi}[R\cos\phi + X\sin\phi] = 0$$

$$\Rightarrow \quad -R\sin\phi + X\cos\phi = 0$$

$$\Rightarrow \quad X\cos\phi = R\sin\phi$$

$$\Rightarrow \quad \tan\phi = \frac{X}{R}$$

$$\therefore \quad \cos\phi = \frac{R}{Z};\ \sin\phi = \frac{X}{Z} \qquad \left[\text{Where } Z = \sqrt{R^2 + X^2}\right]$$

$$(\text{V.R.})_{max} = \frac{|\bar{I}_R|}{|\bar{V}_R|}[R\cos\phi - X\sin\phi] = \frac{|\bar{I}_R|}{|\bar{V}_R|}\left[R\cdot\frac{R}{Z} + X\cdot\frac{X}{Z}\right]$$

$$= \frac{|\bar{I}_R|}{|\bar{V}_R|}\left[\frac{R^2 + X^2}{Z}\right]$$

$$\therefore \quad (\text{V.R.})_{max} = \frac{Z|\bar{I}_R|}{|\bar{V}_R|}$$

At lagging power factor $\tan\phi = \dfrac{X}{R}$

At leading power factor $\cos\phi$ $\quad \text{V.R.} = \dfrac{|\bar{I}_R|}{|\bar{V}_R|}[R\cos\phi - X\sin\phi]$

For zero voltage regulation

$$\frac{|\bar{I}_R|}{|\bar{V}_R|}[R\cos\phi - X\sin\phi] = 0$$

$$\Rightarrow \quad \tan\phi = \frac{R}{X}$$

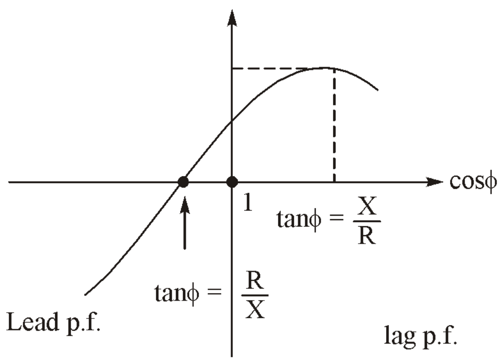
In case of unity power factor

i.e. $\cos\phi = 1$

i.e. $\sin\phi = 0$

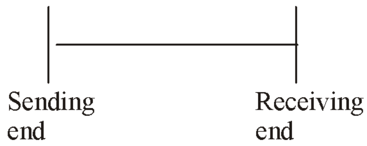
$$\text{Voltage regulation} = \frac{|\bar{I}_R|R}{|\bar{V}_R|}$$

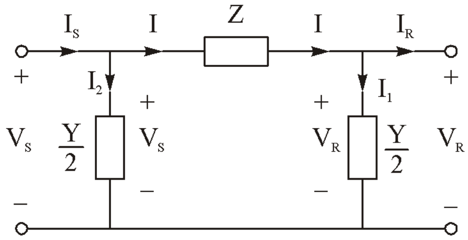
Voltage regulation and power factor curve



### *1.2.5 Medium Line Model*

Nominal $\pi$-model:




According to the diagram

$$I_1 = \frac{Y}{Z}Z \text{ and } I = I_R + I_1$$

$$V_S = V_R + IZ$$

$$\Rightarrow \qquad V_S = V_R + Z\left[I_R + \frac{Y}{2}V_R\right]$$

$$\therefore \qquad V_S = \left(1+\frac{YZ}{2}\right)V_R + ZI_R$$

$$I_2 = \frac{Y}{2}V_S \text{ and } I_S = I + I_2$$

$$I_S = I + \frac{Y}{2}V_S$$

$$\Rightarrow \qquad I_S = \left(I_R + \frac{Y}{Z}V_R\right) + \frac{Y}{2}V_S$$

$$\Rightarrow \qquad I_S = \left(I_R + \frac{Y}{2}V_R\right) + \frac{Y}{2}\left[\left(1+\frac{YZ}{2}\right)V_R + ZI_R\right]$$

$$\Rightarrow \qquad I_S = Y\left[1+\frac{YZ}{4}\right]V_R + \left[1+\frac{YZ}{2}\right]I_R$$

$$A = D = 1+\frac{YZ}{2}$$

$$B = Z$$

$$C = Y\left[1+\frac{YZ}{4}\right]$$

Similarly in case of T-model

$$A = D = 1+\frac{YZ}{2}$$

$$B = Z\left[1+\frac{YZ}{4}\right]$$

$$C = Y$$

**Example 1 :** A 3-ϕ 100 kV long line has constants per km per conductors as follows: Resistance = 0.5 Ω, Inductance = 2 mH and capacitance to neutral is 0.015 μF. Calculate the voltage required at generating end in order that a load of 10 MVA at 0.8 power factor (lag) may be supplied at 120 kV.

***Solution :***

Length = 100 km (given) ; f = 50 Hz

Total resistance $R = 0.5 \times 100 = 50\ \Omega$

Total inductance $L = 2 \times 10^{-3} \times 100 = 0.2\ \text{H}$

Total capacitance $C = 0.015 \times 10^{-6} \times 100 = 1.5\mu\text{F}$

Total series impedance

$$Z = R + j\omega L = 50 + j \times 2\pi \times 50 \times 0.2 \quad (\omega = 2\pi f)$$

$$\therefore \quad Z = 50 = j62.83$$

$$\bar{Z} = 80.29\angle 51.5^\circ$$

$$\bar{Y} = j\omega C = j \times 2\pi \times 50 \times 1.5 \times 10^{-6} = j4.71 \times 10^{-4}\ \mho$$

$$A = D = 1 + \frac{YZ}{2}$$

$$= 1 + \frac{4.71 \times 10^{-4}\angle 90^\circ \times 80.29\angle 51.5^\circ}{2} = 0.985\angle 0.68^\circ$$

$$B = Z = 80.29\angle 51.5^\circ$$

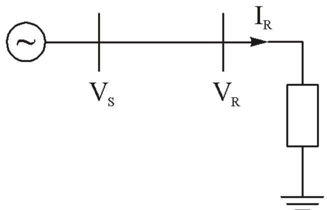
$$C = Y\left[1 + \frac{YZ}{4}\right] = 4.71 \times 10^{-4}\angle 90^\circ \left[1 + \frac{4.71 \times 10^{-4}\underline{|\ 90^\circ} \times 80.29\underline{|\ 51.5^\circ}}{4}\right]$$

$$= 4.68 \times 10^{-4}\underline{|\ 90.32^\circ}$$

Voltage at receiving end $V_{R(L-L)} = 120\ \text{kV}$

Per phase voltage $V_{R(ph)} = \frac{120}{\sqrt{3}} = 69.3\text{kV}$

$$\therefore \quad \bar{V}_R = 39.3\angle 0^\circ \times 10^3\ \text{V}$$




Load $S = 10\ \text{MVA}$

p.f. $\cos\phi = 0.8$ lagging $\Rightarrow \phi = 36.9^\circ$

Load $S = 3\times V_{(ph)R} \times I_{(ph)R}$

$\Rightarrow$ $I_R = \dfrac{10\times10^6}{3\times69.3\times10^3} = 48.1\text{ A}$

As $I_R$ lags $\bar{V}_R$ by $\phi = 36.9°$

$I_R = 48.1\angle -36.9° \text{ A}$

Voltage at sending end $\bar{V}_S = A\bar{V}_R + B\bar{I}_R$

$\Rightarrow$ $\bar{V}_S = 0.985\angle0.68°\times69.3\times10^3\angle0° + 80.29\angle51.5°\times48.1\angle-36.9°$

$\Rightarrow$ $\bar{V}_S = 72\angle1.4° \text{ kV}$

$V_{S(ph)} = 72 \text{ kV}$

$\therefore$ $V_{S(L-L)} = \sqrt{3}\times V_{S(ph)} = \sqrt{3}\times 72$

$= 124.7 \text{ kV}$

To determine:

(i) Sending end power

(ii) Transmission efficiency

(iii) Voltage regulation

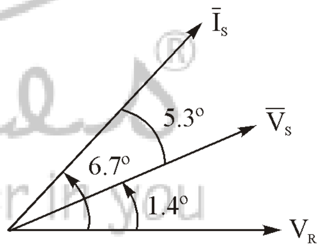
$\bar{I}_S = C\bar{V}_R + D\bar{I}_R$

$\bar{I}_S = 4.68\times10^{-4}\angle90.32\times69.3\times10^3\angle0°$

$+0.985\angle0.7\times48.1\angle-36.9°$

$\bar{I}_S = 38.254\angle6.7°$

$\bar{V}_S = AV_R + B\bar{I}_R$




$\bar{I}_S$ leads $\bar{V}_S$ by 5.3°

$\therefore$ $\phi_s = 5.3°$

$\therefore$ Sending end power factor $= \cos\phi$

$= \cos 5.3°$

$= 0.99$ (leading)

(i) Sending end power $P_s = 3\times V_{s(ph)}\times I_{s(ph)}\times\cos\phi_s = 3\times72\times10^3\times38.23\times0.99$

$= 8.175$ mW

(ii) Receiving end power $= S_R\cos\phi = 10 \times 0.8 = 8$ MW

(iii) Transmission efficiency $\eta = \frac{P_R}{P_S} = \frac{8}{8.17} = 97.9\%$

(iv) At no load $I_R = 0$

Voltage $V_S = AV_R$

$\therefore$ $$V_R = \frac{|V_S|\angle\delta}{|A|\angle\alpha}$$

or $$V_{R(NL)} = \frac{72\angle 1.4^o}{0.985\angle 0.68^o} = 73.46\angle 0.72^o$$

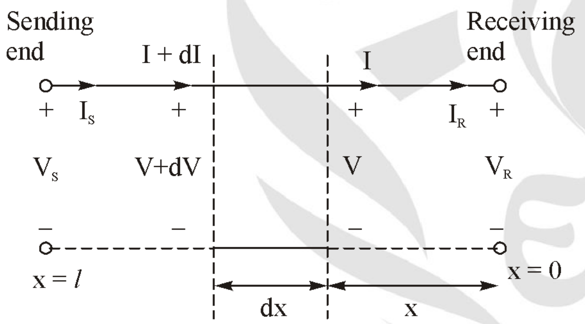
$$V.R. = \frac{|V_R|_{NL} - |V_R|_{FL}}{|V_R|_{FL}} \times 100 = \frac{73.46 - 69.3}{69.3} \times 100$$

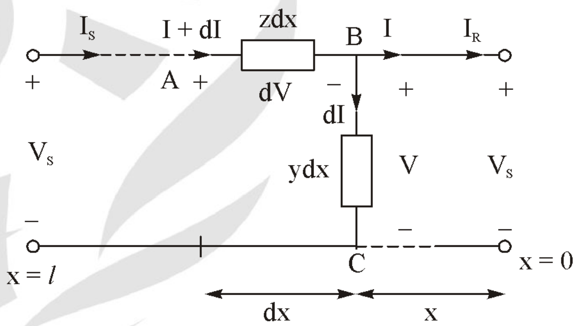
$$= 6\%$$

### *1.2.6 Long Line: (Distributed Parameter Model)*

Series impedance per unit length $\bar{z} = r + j\omega L$

Shunt admittance per unit length $\bar{y} = g + j\omega C$




Let at a distance ‘x’ from receiving end

voltage = V and current = I

Voltage between point A and B = dV

$$dV = (I + dI)z\,dx$$

$$dV = zI\,dx + z\,dI \cdot dx \quad \text{...(i)}$$

As $z\,dI\,dx << z\,I\,dx$

$$dV = z\,I\,dx \quad \text{...(ii)}$$

$$dI = Vy\,dx$$

$\Rightarrow$ $$\frac{dI}{dx} = Vy \quad \text{...(A)}$$

$$\frac{dI}{dx} = zI$$

Differentiating w.r.t. x

$$\Rightarrow \quad \frac{d}{dx}\left[\frac{dV}{dx}\right] = z\left[\frac{dI}{dx}\right]$$

Now from equation (A)

$$\Rightarrow \quad \frac{d^2V}{dx^2} = yzV$$

$$\Rightarrow \quad \frac{d^2V}{dx^2} - yzV = 0$$

Let $\quad yz = \gamma^2$ (Constant)

$$\Rightarrow \quad \frac{d^2V}{dx^2} - \gamma^2 V = 0$$

After solving the equation $\quad V = C_1 e^{\gamma x} + C_2 e^{-\gamma x}$ ...(B)

Where $C_1$, $C_2$ are constants.

Now, $\quad I = \frac{1}{z} \cdot \frac{dV}{dx}$

$$\Rightarrow \quad I = \frac{1}{z} \cdot \frac{d}{dx}\left[C_e e^{\gamma x} + C_2 e^{-\gamma x}\right]$$

$$\Rightarrow \quad I = \frac{1}{z} \cdot \gamma\left[C_1 e^{\gamma x} - C_2 e^{-\gamma x}\right]$$

Let $\quad \frac{\gamma}{z} = \frac{\sqrt{yz}}{z} = \sqrt{\frac{y}{z}} = \frac{1}{Z_0}$

$$\therefore \quad \sqrt{\frac{y}{z}} = \frac{1}{Z_0}$$

$$\Rightarrow \quad I = \frac{1}{Z_0}\left[C_1 e^{\gamma x} - C_2 e^{-\gamma x}\right]$$

At receiving end $\quad x = 0$

$$V = V_R \text{ and } I = I_R$$

$\therefore \quad V_R = C_1 + C_2$ ...(iii)

$I_R = \frac{1}{Z_0}[C_1 - C_2]$ ...(iv)

From equation (iii) and (iv) $\quad C_1 + C_2 = V_R$

$$C_1 - C_2 = Z_0 I_R$$

$$C_1 = \frac{1}{2}[V_R + Z_0 I_R]$$

And $$C_2 = \frac{1}{2}[V_R - Z_0 I_R]$$

Put the value of $C_1$ and $C_2$ in equation (B)

$$\bar{V} = C_1 e^{\gamma x} + C_2 e^{-\gamma x}$$

$$\Rightarrow \quad \bar{V} = \frac{1}{2}[V_R + Z_0 I_R]e^{\gamma x} + \frac{1}{2}[V_R - Z_0 I_R]e^{-\gamma x}$$

$$\Rightarrow \quad \bar{V} = \bar{V}_1 + \bar{V}_2$$

$$\bar{V}_1 = \frac{1}{2}(V_R + Z_0 I_R)e^{\gamma x}$$

and $$V_2 = \frac{1}{2}(V_R - Z_0 I_R)e^{\gamma x}$$

As $$\bar{I} = \frac{1}{2}\left(I_R + \frac{V_R}{Z_0}\right)e^{\gamma x} + \frac{1}{2}\left(I_R + \frac{V_R}{Z_0}\right)e^{-\gamma x}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

Propagation constant $$\gamma = \sqrt{yz}$$

$$\gamma = \alpha + j\beta$$

$$Z_0 = \sqrt{\frac{z}{y}} = R_0 + jX_0$$

Where $Z_0$ = characteristic impedance

Or Surge impedance of line

$$Z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{z_T + l}{y_T + l}} = \sqrt{\frac{Z_T}{Y_T}}$$

i.e. characteristic impedance doesn't depend on length of the line.

$$\because \quad \bar{V}_1 = \left(\frac{V_R + Z_0 I_R}{2}\right)e^{\gamma x} = \left(\frac{V_R + Z_0 I_R}{2}\right)e^{\alpha x} \cdot e^{j\beta x}$$

Where $\gamma = \alpha + j\beta$ and $\alpha$ = Attenuation constant

and $\beta$ = Phase constant

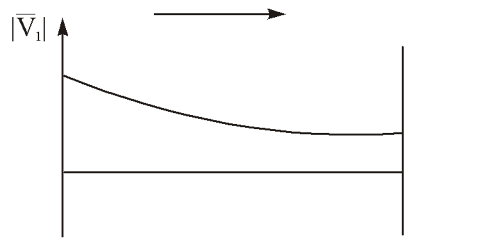
$$|\bar{V}_1| = \left|\frac{V_R + Z_0 I_R}{2}\right|e^{\alpha x}$$

and $$\angle \bar{V}_1 = \beta x$$

If $x \to$ increases

$\Rightarrow$ $|\bar{V}_1| \to$ increases

**Graphical Analysis :** Direction of propagation of wave



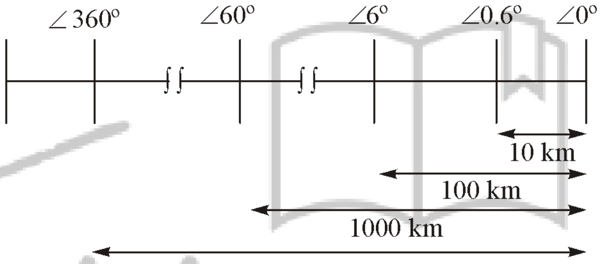

Forward travelling wave i.e. from sending to receiving

If it is considered like a wave. Waves which magnitude and phase changes at each and every point. When wave propagates i.e. wave is moving in a medium, there is a loss of energy. Due to energy loss there is decrement in magnitude. Here magnitude end so the direction of propagation is from sending end to receiving end. (Here voltage is in sinusoidal form).

**Wave Length :** ($\lambda$) → The distance corresponding to a phase change of $2\pi$ or 360º, i.e.

After how much distance of travel, wave will be in same phase

**Example 2**






Where $\beta = 0.06º/km$

***Solution :*** i.e. per km change in phase is 0.06º.

∴ For $2\pi$ or 360º change the total distance covered is 6000 km.

i.e. $\lambda = 6000$ km

i.e. $\lambda \times \beta = 2\pi$ (where β-rad/km)

Wavelength

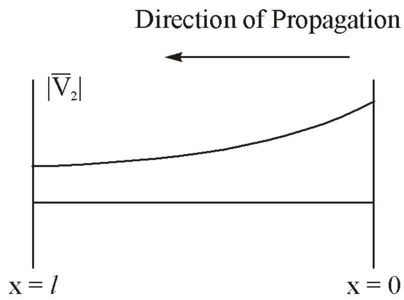
$$\Rightarrow \quad \lambda = \frac{2\pi}{\beta}$$

Similarly for backward travelling wave:

$$\bar{V}_2 = \left(\frac{V_R - Z_0 I_R}{2}\right) e^{-\gamma x}$$

$$\bar{V}_2 = \left(\frac{V_R - Z_0 I_R}{2}\right) e^{-\alpha x} \times e^{-j\beta x}$$

$$|\bar{V}_2| = \left|\frac{V_R - Z_0 I_R}{2}\right| \times e^{-\alpha x}$$




If $x \to$ increases

$\Rightarrow$ $|\bar{V}_2| \to$ decreases

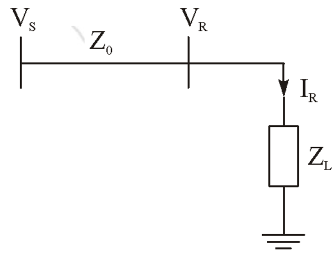
and $\angle \bar{V}_2 = -\beta x$

$V_S =$ Sending end voltage

$V_R =$ Receiving end voltage

$Z_0 =$ Characteristic impedance of line

$Z_L =$ Load impedance




if $Z_0 = Z_L$ then

$Z_L$ = surge impedance loading (SIL)

and $V_R = Z_L I_R = Z_0 I_R$

V and I can also be written as $V = V_R\left[\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right] + Z_0 I_R\left[\frac{e^{\gamma x}-e^{-\gamma x}}{2}\right]$

and $I = \frac{V_R}{Z_0}\left[\frac{e^{\gamma x}-e^{\gamma x}}{2}\right] + I_R\left[\frac{e^{\gamma x}+e^{-\gamma x}}{2}\right]$

i.e. $V = V_R \cosh\gamma x + Z_0 I_R \sinh\gamma x$

$$I = \frac{V_R}{Z_0}\sinh\gamma x + I_R \cosh\gamma x$$

At sending end $x = l$

$\therefore$ $V_S = V_R \cosh\gamma l + Z_0 I_R \sinh\gamma l$

and $I_S = \frac{V_R}{V_0}\sinh\gamma l + I_r \cosh\gamma l$



Now compare with ABCD parameter

$$A = D = \cosh\gamma l$$

$$B = Z_0 \sinh\gamma l$$

$$C = \frac{1}{Z_0}\sinh\gamma l$$

$$\gamma = \alpha + j\beta = \sqrt{yz}$$

Where, $y = g + j\omega C$ and $z = r + j\omega L$

For lossless line: $r = 0$ and $g = 0$

$\therefore$ $$\gamma = \sqrt{(j\omega C)(j\omega L)}$$

$\Rightarrow$ $$\alpha + j\beta = j\omega\sqrt{LC}$$

Attenuation constant:

$\therefore$ $$\alpha = 0$$

and phase constant $$\beta = \omega\sqrt{LC}$$

Velocity of wave travel i.e. $v_c$

$$v_c = f\,\lambda$$

$\Rightarrow$ $$v_c = f \times \frac{2\pi}{\beta} = \frac{2\pi f}{\beta} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}}$$

$\therefore$ $$v_c = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

In air or vacuum $$v_c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8 \text{ m/s} \quad \text{(i.e. velocity of light)}$$

$\therefore$ For lossless line

$$\gamma = j\beta,\ \alpha = 0,\ \beta = \omega\sqrt{LC}$$

$$V = V_R\cosh(j\beta l) + Z_0 I_R \sinh(j\beta l)$$

$$I = \frac{V_R}{Z_0}\sinh(j\beta l) + I_R\cosh(j\beta l)$$

Where $$\cosh(j\beta l) = \frac{e^{j\beta l} + e^{-j\beta l}}{2} = \cos\beta l$$

and $$\sinh(j\beta l) = j\left[\frac{e^{j\beta l} - e^{-j\beta l}}{2j}\right] = j\sin\beta l$$

$\therefore$ $$\bar{V} = V_R\cos\beta l + jZ_0 I_R\sin\beta l$$

and $$\bar{I} = j\frac{V_R}{Z_0}\sin\beta l + I_R\cos\beta l$$

For SIL $$Z_L = Z_0 \quad \therefore V_R = Z_0 I_R$$

$\therefore$ $$\bar{V} = \bar{V}_R[\cos\beta l + j\sin\beta l]$$

$\Rightarrow$ $$\bar{V} = \bar{V}_R e^{j\beta l}$$