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ENGINEERS ACADEMY

RPSC - A.En.

Assistant Engineering

ELECTRICAL

Rajasthan Public Service Commission (RPSC)

Volume - 12

Communication

SIERRA INNOVATIONS

FOURIER SERIES AND FOURIER TRANSFORM

THEORY

1. FOURIER SERIES :

A fourier series is a representation of a function f(t) by the linear combination of elements of a complete set of infinite mutually orthogonal functions.

These elements must be mutuallly orhtogonal.

Note: (i) **Mutually orthogonal functions:**

Two functions are said to be mutually orthogonal over an interval between t_1 and t_2, if the integral of their product over this interval is zero.

i.e. $$\int_{t_1}^{t_2} f(t)h(t) = 0$$

In general $$\int_{t_1}^{t_2} f_i(t)f_k(t)dt = 0 \qquad (i \neq k)$$

(ii) It assure that one function f(t) does not have any component of other function g(t).

(iii) The example of orthogonal functions are: Lengendre polynomial, Jacobi polynomials, Trigonometric and exponential function.

(iv) Orthogonalities in complex functions

$$\int_{t_1}^{t_2} f_1(t)f_2^*(t)dt = \int_{t_1}^{t_2} f_1^*(t)f_2(t)dt = 0$$

where, f_1^* and f_2^* are complex conjugate of $f_1(t)$ and $f_2(t)$ respectively.

2. DIRICHLET'S CONDITIONS

There are sufficient conditions that needs to be satisfied by a function f(t) for its fourier series representation within the interval (t_1, t_2). These are follows:

(i) x(t) is absolutely integrable, i.e. $\int_{-\infty}^{+\infty} |x(t)|dt < \infty$

(ii) x(t) is single valued and has only finite number of maxima and minima within any finite interval.

(iii) x(t) has a finite number of finite discontinuities within any finite interval.

Note: Fourier series is valid for periodic signals only.

3. TRIGONOMETRIC FOURIER SERIES :

A function f(t) can be represented by a fourier series comprising the following sine and cosine functions:

$$f(t) = a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t) + \ldots\ldots + a_n \cos(n\omega_0 t) + \ldots\ldots + b_1 \sin(\omega_0 t) + b_2 \sin(2\omega_0 t) + \ldots\ldots + b_n \sin(n\omega_0 t) + \ldots\ldots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right], \quad t_0 \le t \le t_0 + T$$

Where

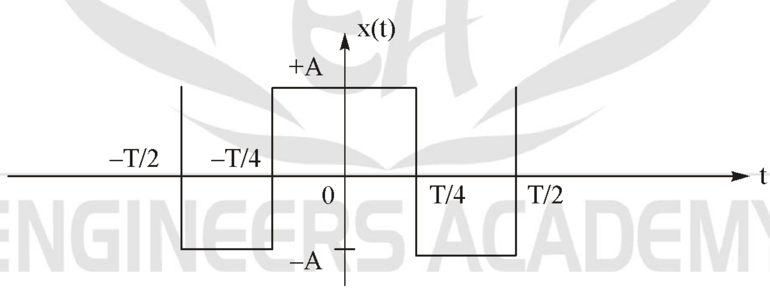
$$T = \frac{2\pi}{\omega_0}$$

$$a_0 = \frac{1}{T} \int_{\text{Over a one time period}} f(t)\,dt = \text{D.C. or average value}$$

$$a_n = \frac{2}{T} \int_{\text{over an one time period}} f(t) \cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T} \int_{\text{over an one time period}} f(t) \sin(n\omega_0 t)\,dt$$

Example: Find the trigonometric fourier series representation of following figure



Solution:

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right] \quad \ldots(i)$$

Signal x(t) is given by

$$x(t) = \begin{cases} -A & ; -\frac{T}{2} < t < -\frac{T}{4} \\ +A & ; -\frac{T}{4} < t < +\frac{T}{4} \\ -A & ; \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} x(t)\cos(n\omega_0 t)\,dt$$

$$a_n = \frac{2}{T}\left[\int_{-\frac{T}{2}}^{-\frac{T}{4}} -A\cos(n\omega_0 t)\,dt + \int_{-\frac{T}{4}}^{\frac{T}{4}} A\cos(n\omega_0 t)\,dt + \int_{\frac{T}{4}}^{\frac{T}{2}} -A\cos(n\omega_0 t)\,dt\right]$$

$$= \frac{2A}{T}\left[-\int_{-\frac{T}{2}}^{-\frac{T}{4}} \cos(n\omega_0 t)\,dt + \int_{-\frac{T}{4}}^{+\frac{T}{4}} \cos(n\omega_0 t)\,dt - \int_{\frac{T}{4}}^{\frac{T}{2}} \cos(n\omega_0 t)\,dt\right]$$

$$a_n = \frac{2A}{T}\left[\left[-\frac{\sin(n\omega_0 t)}{n\omega_0}\right]_{-\frac{T}{2}}^{-\frac{T}{4}} + \left[\frac{\sin(n\omega_0 t)}{n\omega_0}\right]_{-\frac{T}{4}}^{+\frac{T}{4}} + \left[-\frac{\sin(n\omega_0 t)}{n\omega_0}\right]_{\frac{T}{4}}^{\frac{T}{2}}\right]$$

By solving this, we get

$$a_n = \frac{8A}{n\omega_0 T}\sin\left(\frac{n\omega_0 T}{4}\right) - \frac{4A}{n\omega_0 T}\sin\left(\frac{n\omega_0 T}{2}\right)$$

$$a_n = \frac{8A}{2n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{4A}{2n\pi}\sin(n\pi) \;;\; \omega_0 T = 2\pi$$

$$a_n = \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right) - 0$$

The second term in above expression is zero for all integer values of n.

$$a_n = \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

and $b_n = 0$ because given waveform is even function. and $a_0 = 0$ because given wave form is symmetrical about horizontal axis.

Putting the values of a_n in equation (i)

$$x(t) = \frac{4A}{\pi}\left[\cos(\omega_0 t) - \frac{1}{3}\cos(3\omega_0 t) + \frac{1}{5}\cos(5\omega_0 t)\ldots\ldots\right] \textbf{\textit{Ans.}}$$

Note: 1. Symmetry conditions

(i) If x(t) = even function ; then, $b_n = 0$.

(ii) If x(t) = odd function ; then $a_0 = 0$ and $a_n = 0$.

(iii) If x(t) is symmetrical about horizontal axis, then $a_0 = 0$.

2. Some trigonometric identity

(i) $\sin\left(\frac{n\pi}{2}\right) = \left.\begin{matrix} -1; & n = 3,7,11 \\ +1; & n = 1,5,9 \\ 0; & n = \text{even} \end{matrix}\right] n = 0,1,2,3,.......$

(ii) $\cos\left(\frac{n\pi}{2}\right) = \left.\begin{matrix} 0; & n = \text{odd} \\ (-1)^{n/2}; & n = \text{even} \end{matrix}\right] n = 0,1,2,3,4,.......$

(iii) $\tan\left(\frac{n\pi}{2}\right) = \infty$; n = 1,2,3,......

(iv) $\sin(n\pi) = 0$; n = 0,1,.......

(v) $\cos(n\pi) = (-1)^n$; n = 0,1,.......

(vi) $\tan(n\pi) = 0$; n = 0,1,.......

4. POLAR FOURIER SERIES REPRESENTATION :

A function x(t) can be represented by a polar fourier series

$$x(t) = D_0 + \sum_{n=1}^{\infty} D_n \left(n\omega_0 t - \phi_n\right)$$

where,

$$D_0 = a_0 = \frac{1}{T}\int_{\substack{\text{over a} \\ \text{time period}}} x(t)\,dt = \text{D.C. value or average value.}$$

$$D_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

5. COMPLEX / EXPONENTIAL FOURIER SERIES

A function x(t) with period T can be represented by complex / exponential fourier series.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \exp\left(jn\omega_0 t\right) \qquad ...(ii)$$

where,

$$\omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{T}\int_{\substack{\text{over a time} \\ \text{period}}} x(t)\exp\left(-jn\omega_0 t\right)dt$$

Coefficient C_n are in general complex form, hence

$$C_n = |C_n|\exp(j\phi_n) \quad \text{...(iii)}$$

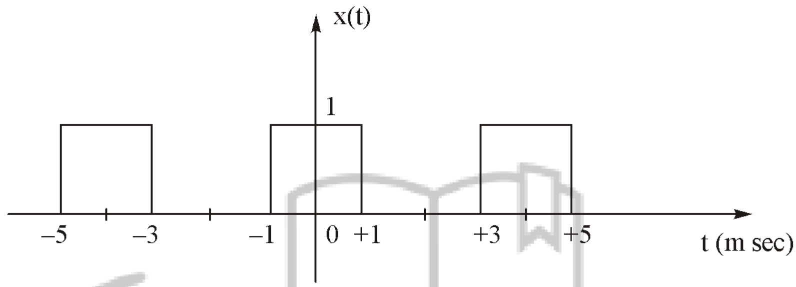
So, using equation (iii) in equation (ii)

$$x(t) = \sum_{n=-\infty}^{\infty} |C_n|\exp\left(j\left(n\omega_0 t + \phi_n\right)\right)$$

The plot between $|C_n|$ versus n (or $n\omega_0$) is called magnitude spectrum and ϕ_n versus n (or $n\omega_0$) is called phase spectrum.

It is important to note that the spectrum of a periodic signal exists only at discrete frequency.

Example: For the unit amplitude reactangular pulse train shown in figure below, compute the fourier series coefficient.



Solution: Signal x(t) has a period T = 4 millisecond and it is ON for half the period and OFF during the remaining half.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \exp\left(jn\omega_0 t\right)dt$$

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

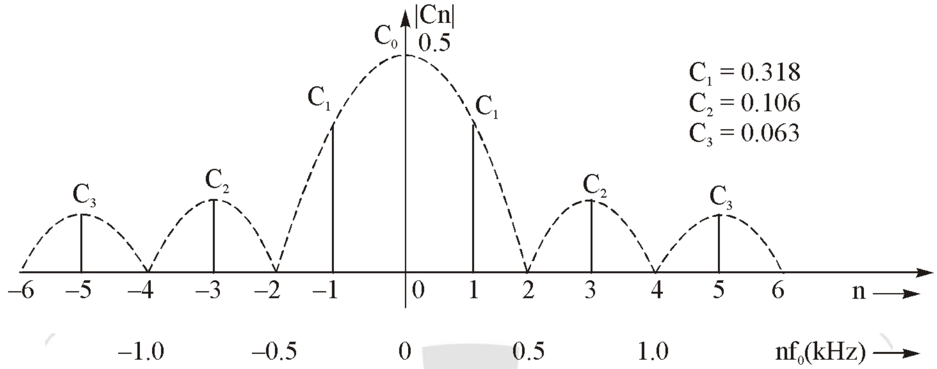
$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\exp(-jn\omega_0 t)\,dt$$

$$= \frac{1}{T}\int_{-T/4}^{T/4} \exp(-jn\omega_0 t)\,dt$$

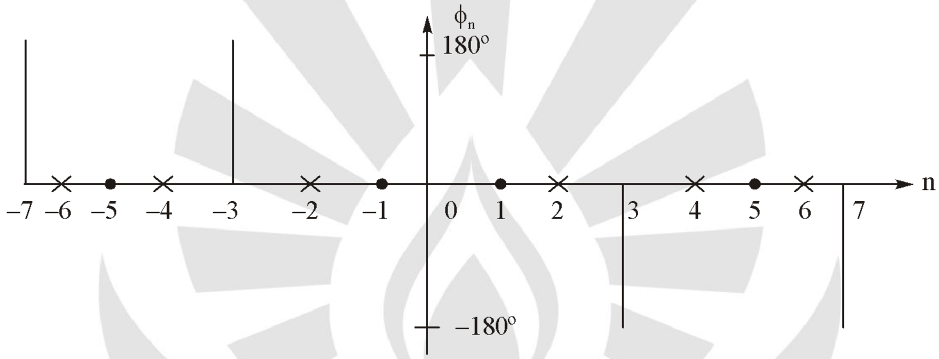
$$= \frac{1}{T}\frac{2\sin\left(\frac{n\pi}{2}\right)}{n\omega_0}$$

$$= \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

All the fourier coefficients are real but could be positive and negative. Hence ϕ_n is either zero or $\pm\pi$ for all n.



(a) Megnitude spectrum.



(b) Phase spectrum.

- C_0, the vaerage or the DC value of pulse train is $\frac{1}{2}$.
- Spectrum exists only at discrete frequencies, $f = nf_0$ with $f_0 = 250$ Hz. Such a spectrum is called the discrete spectrum
- The envelope consists of several lobes and the magnitude of each lobe keeps decreasing with increase in frequency.
- The magnitude spectrum is symmetric and phase spectrum is antisymmetric. This is because x(t) is real.
- ϕ_n at $n = \pm 2, \pm 4$ etc. is undefined at $|C_n| = 0$ for these n. This is indicated with a cross on the phase spectrum plot.

6. SINC FUNCTION

$$\text{sin c } \lambda = \text{sin c}(\lambda) = \frac{\sin(\pi\lambda)}{(\pi\lambda)}$$

7. SAMPLING FUNCTION

$$S_a(\lambda) = \frac{\sin\lambda}{\lambda}$$

8. FOURIER TRANSFORM

Fourier transform is used to find the frequency component in time domain signal.

Signal x(t)

- Periodic
  - → (Fourier series is used)
  - → Spectrum will be discrete spectrum
- Aperiodic
  - → (Fourier transform is used)
  - → Spectrum will be continuous spectrum.

Fourier transform of x(t) is X(f) . X(f) is defined as

$$x(t) \rightleftharpoons X(f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \qquad \text{(Analysis equation)}$$

$$X(f) = F[x(t)]$$

X(f) is, in general, a complex quantitiy.

$$X(f) = X_R(f) + jX_I(f)$$

$$= |X(f)|e^{j\phi(f)}$$

where, $X_R(f)$ = Real part of X(f)

$X_I(f)$ = Imaginary part of X(f)

$|X(f)|$ = Magnitude of X(f)

$$= \sqrt{(X_R(f))^2 + (X_I(f))^2}$$

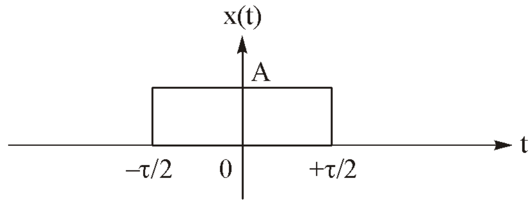
$$\theta(f) = \arg[X(f)] = \tan^{-1}\left[\frac{X_I(f)}{X_R(f)}\right]$$

The plot between |X(f)| versus f, is known as magnitude spectrum and θ(f) versus f, is known as the phase spectrum.

Inverse fourier transform (IFT) is defined as

$$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}dt \qquad \text{(Synthesis equation)}$$

Example: Find the fourier transform of figure.



Solution:

$$x(t) \rightleftharpoons X(f)$$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$= \int_{-\tau/2}^{+\tau/2} A \cdot e^{-j2\pi ft}dt$$

$$= A\left[\frac{e^{-j2\pi ft}}{-j2\pi f}\right]_{-\tau/2}^{\tau/2}$$

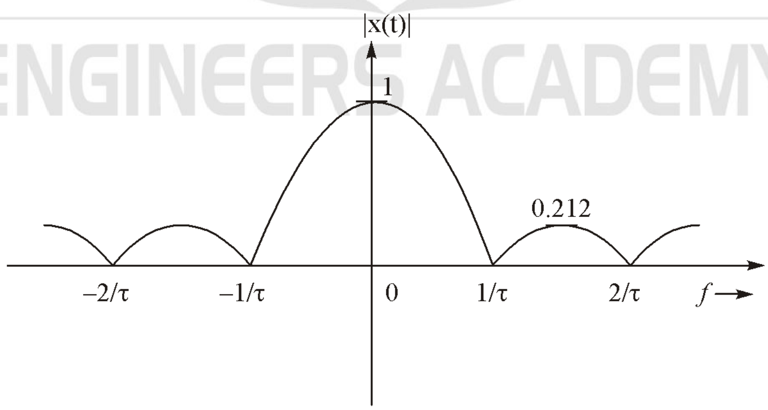
$$= A\tau \ \text{sinc}(\tau f)$$

Note:

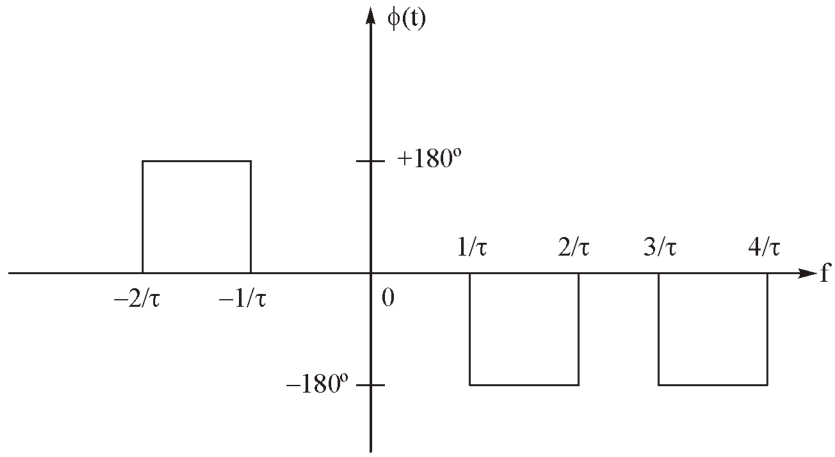
$$\sin c[x] = \frac{\sin(\pi x)}{\pi x} = \begin{cases} 1 \ ; & x = 0 \\ 0 \ ; & x = \pm 1, \pm 2, ... \end{cases}$$

$$\sin c(f\tau) = \begin{cases} 1 \ ; & f\tau = 0 \Rightarrow f = 0 \\ 0 \ ; & f\tau = \pm 1, \pm 2, \pm 3, ... \\ & f = \pm\frac{1}{\tau}, \pm\frac{2}{\tau}, \pm\frac{3}{\tau}, ... \end{cases}$$

$$X(f) = |X(f)|e^{j\theta(f)}$$



(a) Magnitude spectrum



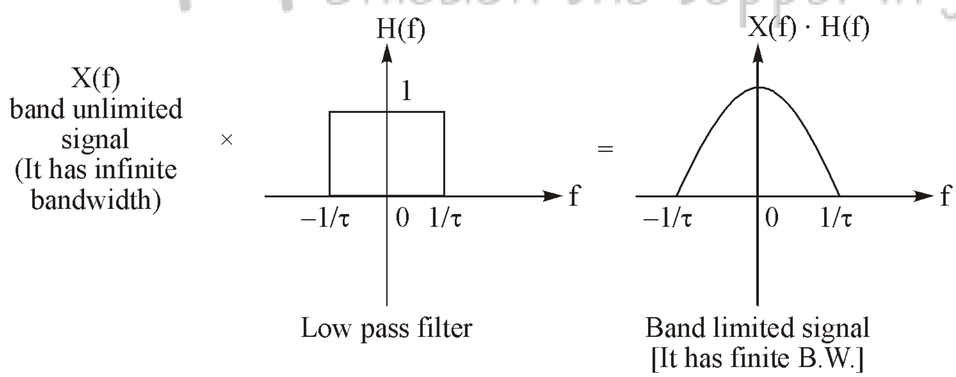
(b) Phase spectrum

- The fourier transform of x(t), X(f) contains all the possible frequencies.
- During the interval, $\frac{m}{\tau} < |f| < \frac{m+1}{\tau}$, with m odd, $\sin c(f\tau)$ is negative. As the magnitude spectrum is always positive, negative value of $\sin c(f\tau)$ are taken care of by making $\theta(f) = \pm 180^{\circ}$.
- Signal bandwidth (B.W) = Highest positive frequency − Lowest positive frequency

$$= \infty - 0 = \infty$$

for proper transmission of a signal, channel B.W. > signal B.W.

- Signal B.W. is ∞. So, it is not possible to transmit the signal. Signal B.W. should be finite.
- Before transmission, the above signal should be band limited by band limiting process.
- To band limit a signal, all the its significant frequency components has to be retained and insignificant frequency component has to be eliminated.
- Significant frequency contains almost of 95% to 99% of total strength of given signal.



$$\text{B.W.} = \frac{1}{\tau}$$

$$\tau \neq 0$$

For transmission, significant frequency are given high importance for effective utilization of available channel bandwidth.

9. PROPERTIES OF FOURIER TRANSFORM

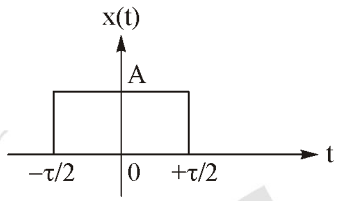
(i) Duality property :

If $x(t) \rightleftharpoons X(f)$

then, $X(t) \rightleftharpoons x(-f)$

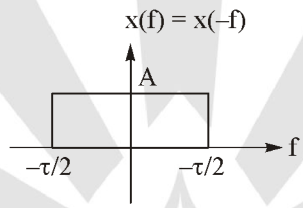
Example: Find out the fourier transform of sin c (100t).

Solution :



$\rightleftharpoons A\tau \sin(f\tau)$

$A\tau \sin(t\tau) \rightleftharpoons$

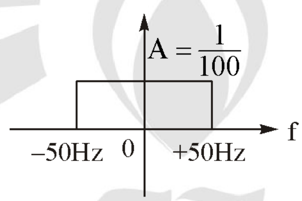


(By using duality)

Put $\tau = 100$

and $A = 1$

sin c (100t) $\rightleftharpoons$



Example: Find out the fourier transform of δ(t).

Solution:

$$F[\delta(t)] = \int_{-\infty}^{+\infty} \delta(t) e^{-j2\pi ft} dt$$

$$F[\delta(t)] = \int_{-\infty}^{+\infty} \delta(t) dt \qquad [x(t)\delta(t) = x(0)\delta(t)]$$

$$F[\delta(t)] = 1 \qquad \left[\int_{-\infty}^{\infty} \delta(t)\, dt = \text{Area under curve} = 1\right]$$

$$\delta(t) \rightleftharpoons 1$$

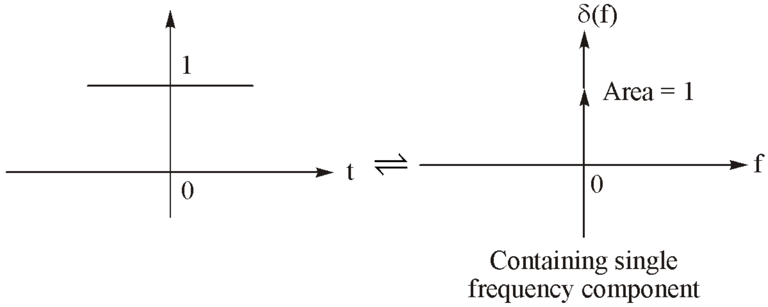
Example: Find the fourier transform of 1.

Solution:

$$\delta(t) \rightleftharpoons 1$$

$$1 \rightleftharpoons \delta(-f) \qquad \text{[by using duality property]}$$

$$1 \rightleftharpoons \delta(f) \qquad [\delta(f) \text{ is an even function}]$$



(ii) Frequency Shifting Property :

If $$x(t) \rightleftharpoons X(f)$$

then, $$x(t)e^{j2\pi f_0 t} \rightleftharpoons X(f-f_0)$$

and $$x(t)e^{-j2\pi f_0 t} \rightleftharpoons X(f+f_0)$$

Example: Find out the fourier transform of $e^{j2\pi f_0 t}$ and $e^{-j2\pi f_0 t}$.

Solution:

$$1 \rightleftharpoons \delta(f)$$

$$1\cdot e^{j2\pi f_0 t} \rightleftharpoons \delta(f-f_0)$$

$$1\cdot e^{-j2\pi f_0 t} \rightleftharpoons \delta(f+f_0)$$ [by using frequency shifting property]

$$F\left[e^{j2\pi f_0 t}\right] = \delta(f-f_0)$$

$$F\left[e^{-j2\pi f_0 t}\right] = \delta(f+f_0)$$

Example: Find the fourier transform of $A\cos(2\pi f_0 t)$.

Solution.

$$e^{j2\pi f_0 t} \rightleftharpoons \delta(f-f_0)$$

$$e^{-j2\pi f_0 t} \rightleftharpoons \delta(f+f_0)$$

By adding both equation

$$e^{j2\pi f_0 t} + e^{-j2\pi f_0 t} \rightleftharpoons \delta(f-f_0)+\delta(f+f_0)$$

$$2\cos(2\pi f_0 t) \rightleftharpoons \delta(f-f_0)+\delta(f+f_0)$$

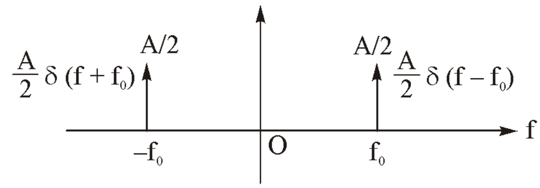
$$\cos(2\pi f_0 t) \rightleftharpoons \frac{1}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$$

By multiplying A on both side,

$$A\cos(2\pi f_0 t) \rightleftharpoons \frac{A}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$$

$$A\cos(2\pi f_0 t) \rightleftharpoons \frac{A}{2}\delta(f-f_0)+\frac{A}{2}\delta(f+f_0)$$

$$A\cos(2\pi f_0 t) \rightleftharpoons$$



Fourier transform of $A\cos(2\pi f_0 t)$ has two frequency component at $+f_0$ and $-f_0$.

Example: Find the fourier transform of $x(t)\cos(2\pi f_0 t)$ when fourier transform of $x(t)$ is X(f).

Solution: Given that $x(t) \rightleftharpoons X(f)$

$$x(t)e^{j2\pi f_0 t} \rightleftharpoons X(f-f_0) \quad \text{[by using shifting property]}$$

$$x(t)e^{-j2\pi f_0 t} \rightleftharpoons X(f+f_0)$$

By adding both,

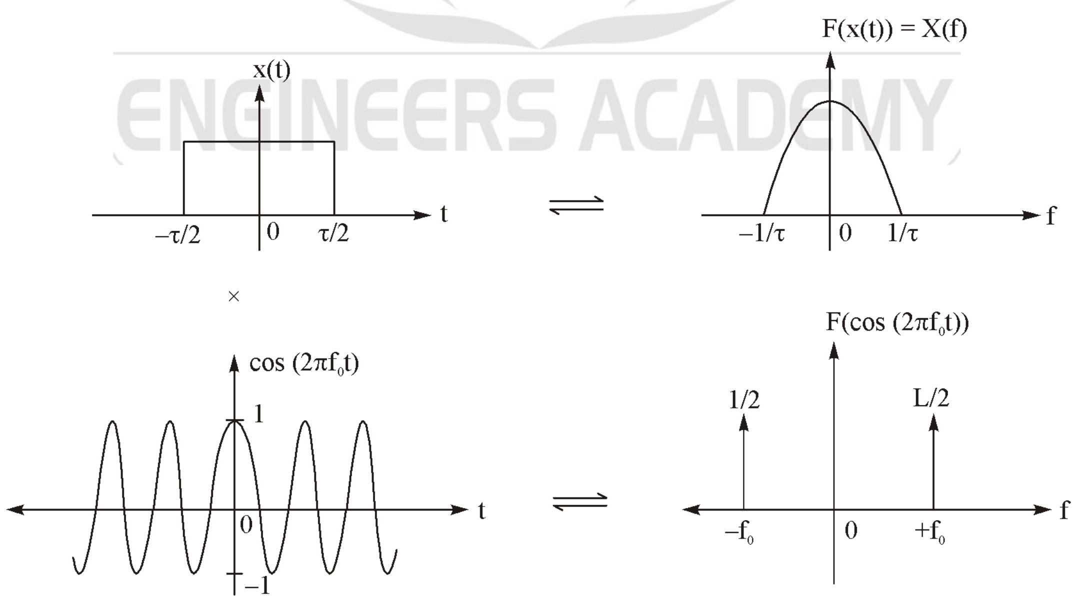
$$x(t)e^{j2\pi f_0 t} + x(t)e^{-j2\pi f_0 t} \rightleftharpoons X(f-f_0)+X(f+f_0)$$

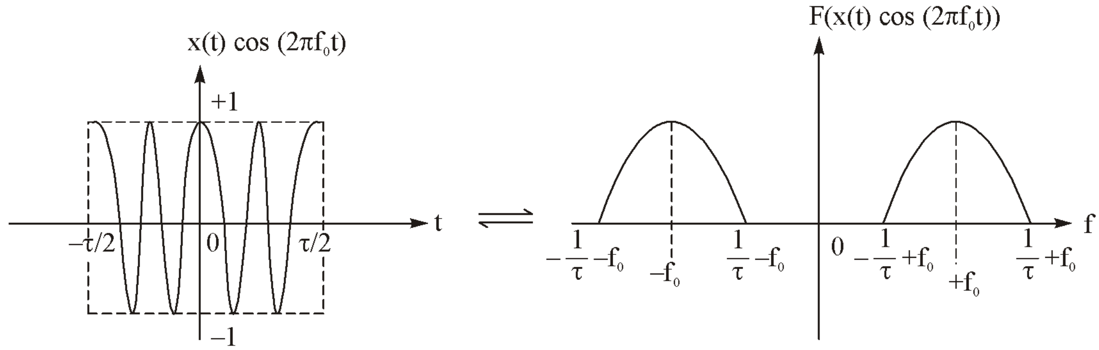
$$x(t)\left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right] \rightleftharpoons X(f-f_0)+X(f+f_0)$$

$$x(t)\left[2\cos(2\pi f_0 t)\right] \rightleftharpoons X(f-f_0)+X(f+f_0)$$

$$x(t)\cos(2\pi f_0 t) \rightleftharpoons \frac{1}{2}\left[X(f-f_0)+X(f+f_0)\right]$$

It means that when the signal x(t) is multiplied with cosine signal, then fourier transform of x(t) is shifted by $+f_0$ and $-f_0$.





10. RESPONSE OF A LINEAR SYSTEM

$$x(t) \longrightarrow \boxed{\text{Filter } H(\omega_n)} \longrightarrow y(t)$$

$$H(\omega_n) = |H(\omega_n)|e^{-j\theta(\omega_n)}$$

$$x(t) = \sum_{n=-\infty}^{+\infty} x_n e^{j2\pi nt/T_0}$$ (in terms of fourier series)

then $$\boxed{y(t) = \sum_{n=-\infty}^{\infty} H(\omega_n) x_n e^{j2\pi nt/T_0}}$$

11. NORMALIZED POWER IN FOURIER SERIES

(i) Normalized power in trigonometric fourier series

$$S = a_0^2 + \sum_{n=1}^{\infty} \frac{a_n^2}{2} + \sum_{n=1}^{\infty} \frac{b_n^2}{2}$$

(ii) Normalized power in polar fourier series

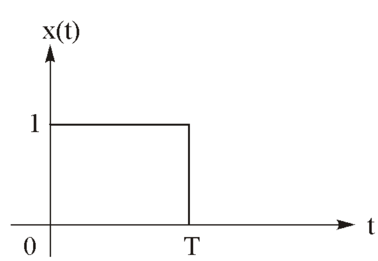
$$S = C_0^2 + \sum_{n=1}^{\infty} \frac{C_n^2}{2}$$

(iii) Normalized power in complex / exponential fourier series

$$S = \sum_{n=-\infty}^{\infty} D_n D_n^*$$

Example: Find the fourier transform of rectangular pulse.

Solution: Rectangular pulse is defined as



$$x(t) = \begin{cases} 1 & ; \quad 0 < t < T \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Fourier transform of $x(t) = x(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$

$$= \int_{0}^{T} e^{-j2\pi ft}dt = \left[\frac{e^{-j2\pi ft}}{-j2\pi f}\right]_0^T$$

$$= -\frac{1}{j2\pi f}\left[e^{-j2\pi fT} - 1\right]$$

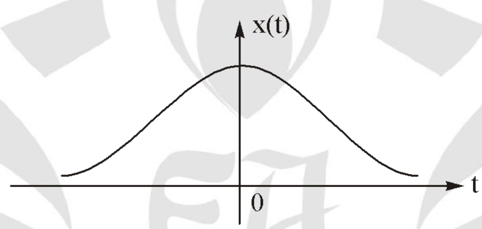
$$= -\frac{1}{j2\pi f}\frac{\left[e^{-j2\pi fT/2} - e^{j2\pi fT/2}\right]}{e^{j2\pi fT/2}}$$

$$= Te^{-j\omega T/2}\sin c\left(\frac{\omega T}{2\pi}\right)$$

Example: Find the fourier transform of Gaussian pulse.

Solution: A Gaussian pulse is defined as

$$x(t) = e^{-\pi t^2}$$



Fourier transform of $x(t) = F[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

$$F\left[x(t)\right] = \int_{-\infty}^{\infty} e^{-\pi t^2}e^{-j\omega t}dt = \int_{-\infty}^{\infty} e^{-\left[\pi t^2 + j\omega t\right]}dt$$

Put $\pi t^2 + j\omega t = \left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}}\right)^2 + \frac{\omega^2}{4\pi}$

$$F\left[x(t)\right] = \int_{-\infty}^{\infty} e^{-\left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}}\right)^2} e^{-\omega^2/4\pi}dt$$

$$= e^{-\pi f^2}\int_{-\infty}^{\infty} e^{-\left(\sqrt{\pi}t + \frac{j\omega}{2\sqrt{\pi}}\right)^2} dt$$

Assuming $\sqrt{\pi}\,t + \dfrac{j\omega}{2\sqrt{\pi}} = y$

$$\sqrt{\pi}\,dt = dy$$

$$dt = \frac{dy}{\sqrt{\pi}}$$

$$F[x(t)] = e^{-\pi f^2}\int_{-\infty}^{\infty} e^{-y^2}\,dy/\sqrt{\pi} = \frac{e^{-\pi f^2}}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-y^2}dy$$

$$= \frac{e^{-\pi f^2}}{\sqrt{\pi}}2\int_{0}^{\infty} e^{-y^2}dy$$ (because e^{-y^2} is an even function)

$$= \frac{2}{\sqrt{\pi}}e^{-\pi f^2}\int_{0}^{\infty} e^{-y^2}dy$$

$$= \frac{\cancel{2}}{\cancel{\sqrt{\pi}}}e^{-\pi f^2}\cdot\frac{\cancel{\sqrt{\pi}}}{\cancel{2}} \qquad \left[\because \int_{0}^{\infty} e^{-y^2}dy = \frac{\sqrt{\pi}}{2}\right]$$

$$F\left[e^{-\pi t^2}\right] = e^{-\pi f^2}$$

$$e^{-\pi t^2} \rightleftharpoons e^{-\pi f^2}$$

12. SOME IMPORTANT PROPERTIES OF FOURIER TRANSFORM

(i) Time Scaling Property

If $x(t) \rightleftharpoons X(f)$

Then $x(at) \rightleftharpoons \dfrac{1}{|a|}X\left(\dfrac{f}{a}\right)$

where, a = any real constant.

(ii) Time Shifting Property

If $x(t) \rightleftharpoons X(f)$

Then $x(t-b) \rightleftharpoons X(f)e^{-j2\pi fb}$

(iii) Time Differentiation Property

If $x(t) \rightleftharpoons X(f)$

Then $\dfrac{d}{dt}x(t) \rightleftharpoons (j2\pi f)X(f)$

(iv) Area Under the Curve

If $x(t) \rightleftharpoons X(f)$

then $\int_{-\infty}^{\infty} x(t)\,dt = X(0)$ = Area under the curve x(t)

and $\int_{-\infty}^{\infty} X(f)\,df = x(0)$ = Area under the curve X(f)

13. CONVOLUTION

It is a mathematical operation which is used to express the input / output relationship in a linear time invariant system.

It is represented by *

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)\,d\tau$$

(a) Convolution Theorem in Time Domain

If $x_1(t) \rightleftharpoons X_1(f)$

$x_2(t) \rightleftharpoons X_2(f)$

then $x_1(t) * x_2(t) \rightleftharpoons X_1(f)X_2(f)$

(b) Convolution Theorem in Frequency Domain

If $x_1(t) \rightleftharpoons X_1(f)$

$x_2(t) \rightleftharpoons X_2(f)$

then $x_1(t)x_2(t) \rightleftharpoons X_1(f) * X_2(f)$

14. ENERGY SIGNAL

Energy signal has finite energy and zero average power.

x(t) is non periodic signal or time limited signal, then energy is expressed as

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

If $x(t) \rightleftharpoons X(f)$, then

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$$

$$|X(f)|^2 = \frac{E}{\Delta f} = \text{Energy spectral density} = \text{Energy density spectrum.}$$

$= \text{energy per unit B.W.} = \psi(f)$

15. POWER SIGNAL

A power signal has finite power and infinite energy.

All periodic signal is power signal.

$$0 < P < \infty, \ E = \infty$$

where, P = average power and E is energy of signal.

If x(t) is periodic signal, then

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

If complex fourier series ofx(t) = $\sum_{n=-\infty}^{\infty} C_n \exp(jn\omega_0 t)$

then power (in terms of complex fourier series coefficient.

$$= \sum_{n=-\infty}^{\infty} |C_n|^2$$

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