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ENGINEERS ACADEMY

# RPSC - A.En.

## Assistant Engineering

## ELECTRICAL

**Rajasthan Public Service Commission (RPSC)**

## Volume - 2

## Electrical Machines

SIERRA INNOVATIONS

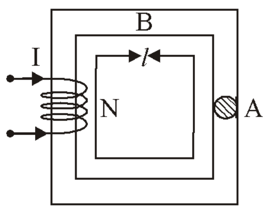
# TRANSFORMERS

## THEORY

A Transformer is a static device comprising coupled coils (Primary and Secondary) wound on common magnetic Core.

MMF (F)

$$F = NI \text{ Amp. turns}$$




Let the mean core length $= l$

Flux density $= B$, Flux $\phi = BA$

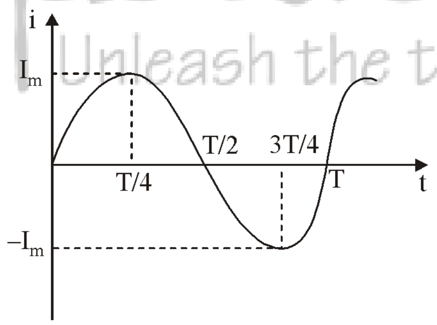
$A$ = Area of cross-section of core.

Magnetization force or magnetic field intensity (H)

$$H = \frac{NI}{l} AT^s / m$$

## 1.1 Magnetic Hysteresis Curve

If ac current is made to flow through coil in the magnetic circuit shown above $i = I_m \sin \omega t$




Time period $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Where $\omega = 2\pi f$ is frequency

Let initially $B = 0$ (i.e. residual magnetism is absent)

For $0 < t < \frac{T}{4}$ where i increases from zero to $I_m$ initially B increases linearly with H (or i) and after a certain value of H, B doesn't increases significantly i.e. B remains almost constant i.e. saturation.

For linear magnetic circuit

$$B \propto H$$

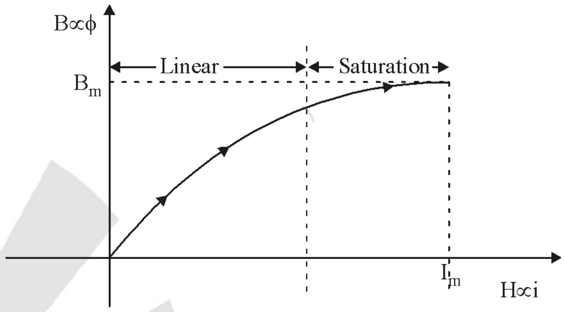
$$\Rightarrow \quad B = \mu H$$

$$\Rightarrow \quad B = \mu_0 \mu_r H$$

Where $\mu$ = Permeability of iron core

$\mu_r$ = Relative permeability of core

$\mu_0$ = Permeability of air




Flux linking $\phi = BA = \mu HA = \mu A \times \frac{Ni}{l} \quad (H = \frac{Ni}{l})$

$$\Rightarrow \quad \phi = \frac{Ni}{l} = \frac{F}{Rl}$$

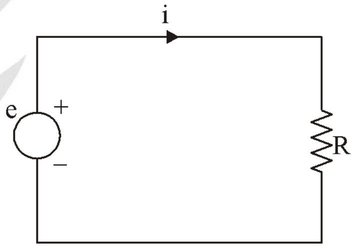
Where F = Ni (mmf in Amp-Turns) applied

$R_l = \frac{l}{\mu A}$ reluctance of core

mmf = Magneto-Motive Force

**The equation $\phi = \frac{F}{R_l}$ is developed on basis of the analogy of electrical circuit (force voltage analogy) shown below :**

$$\text{Current} = \frac{\text{EMF}}{\text{Resistance}}$$




$$\Rightarrow \quad i = \frac{e}{R}$$

Where resistance $R = \frac{l}{\sigma A} \quad \left( R = \frac{1}{G} = \frac{1}{\sigma A / l} = \frac{l}{\sigma A} \right)$

$l$ & A are the length & area of cross-section while $\sigma$ is the conductivity of material.

$$\text{Flux} = \frac{\text{mmf}}{\text{Reluctance}}$$

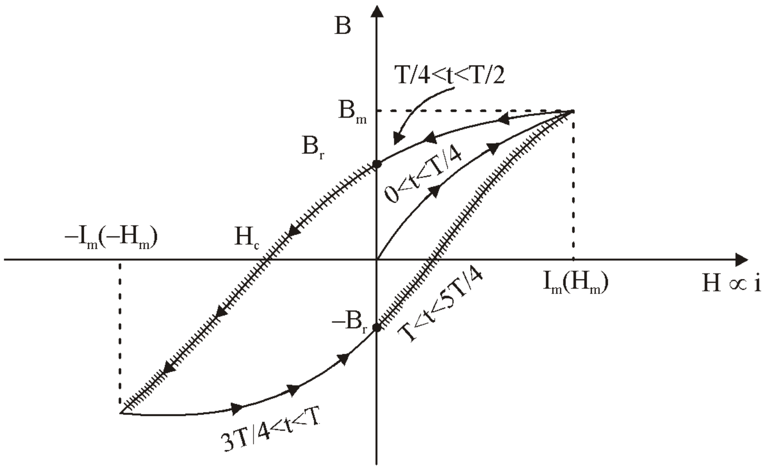
$$\Rightarrow \quad \phi = \frac{F}{R_l}$$

**Note :** For high permeability material e.g. Iron, $\mu_r$ is high & $R_l$ is low it is said to be magnetic conductor or magnetic material. For low permeability material $\mu_r \simeq 1$ e.g. Cu etc. $R_l$ is high so it is said to be non-magnetic material or magnetic insulator.

After $\frac{T}{4}$ i.e. $\frac{T}{4} < t < \frac{T}{2}$ where i decreases from $I_m$, B also decreases but not in the same manner.

At $t = \frac{T}{2}$, i = 0 but $B = B_r \neq 0$ i.e. some residual magnetism is left.

$B_r$ = Retentivity or Residual flux density.




After $\frac{T}{2}$ i.e. $\frac{T}{2} < t < \frac{3T}{4}$ the direction of current i (& H) gets reversed so magnetization is going on decreasing and at a particular value of current say $I_c \left( \& \; H_c = \frac{NI_c}{l} \right)$ B becomes zero i.e. residual magnetism is lost due to $H_c$.

$H_c$ = Coercivity or Coercive force

As i increases further (in –ve direction) B gets reversed & becomes max at $t = \frac{3T}{4}$.

$\Rightarrow \quad P_h \propto B_m^x f$

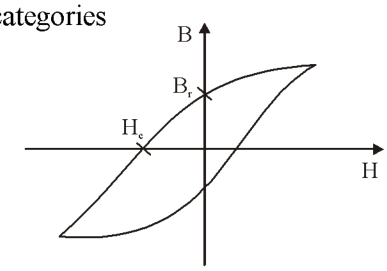
$$P_h = K_h B_m^x f$$



Where x = Steinmetz constant (x = 1.6), $K_h$ = Hystresis coefficient

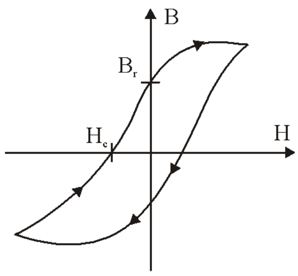
According to B-H loop we categorize magnetic material broadly into two categories

**Hard Magnetic Material**




- Wider B-H loop
- $B_r$, $H_c$ (Both high)
- Hysteresis loss Higher
- Suitable for d.c applications & permanent magnet etc.

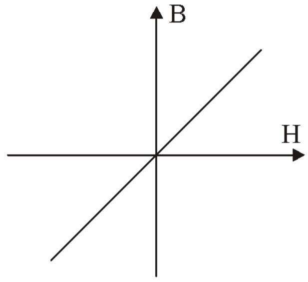
**Soft Magnetic Material**




- Narrow B-H loop
- $B_r$, $H_c$ (both low)
- Hysteresis loss small
- Used for a.c applications e.g. Transformer, AC machines.

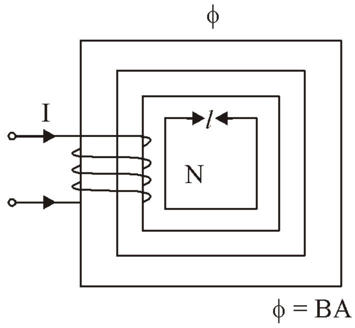
Let us neglect hysteresis & saturation, the B-H loop will be linear (as in case of air)

According to B-H loop




$$B \propto H$$
$$B = \mu H$$
$\mu$ = Permeability of core
$$\mu = \mu_0 \mu_r$$
$\mu_0$ = Absolute Permeability
$\mu_r$ = Relative Permeability

For linear magnetic circuit $\phi \propto I$




$$\phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{F}{R_l}$$

$$\phi = BA \quad \left(\because B = \frac{\mu NI}{l}\right)$$

$$\phi = \frac{\mu NI}{l} \cdot A$$

$$\phi = \frac{NI}{(l/\mu A)} = \frac{\text{mmf}}{\text{Reluctance}}$$

$$\text{Reluctance} = \frac{l}{\mu A}$$

$$\phi = \frac{\mu NIA}{l}$$

Flux linkage $\psi = N\phi$

$$N\phi \propto I$$
$$N\phi = LI$$

Inductance $L = \dfrac{N\phi}{I}$ i.e. flux linkage per unit current.

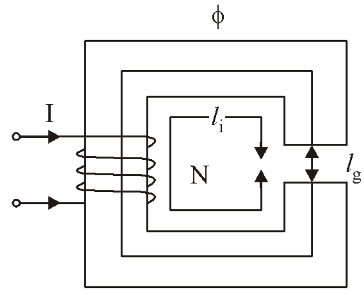
$$L = \frac{N\phi}{I} = \frac{N}{I}\left(\frac{\mu NIA}{l}\right)$$

$$L = \frac{\mu N^2 A}{l}$$

$$\Rightarrow \quad L = \frac{N^2}{(l/\mu A)} = \frac{N^2}{R_l}$$

$R_l$ = Reluctance

$\because$ $\boxed{L \propto \frac{1}{R_l}}$




Air gap length $= l_g$

$Rl_i$ = Reluctance of iron path

$Rl_g$ = Reluctance of air path

Total Reluctance in the path of flux, $\phi$

$$Rl = Rl_i + Rl_g$$

For iron path $$Rl_i = \frac{l_i}{\mu_0 \mu_r A}$$

$\mu_r \rightarrow$ Relative Permeability of iron.

For airgap, $$Rl_g = \frac{l_g}{\mu_0 A}, \text{ As } (\mu_r = 1) \text{ for air}$$

As Permeability of iron is much greater than permeability of air ($\mu_r = 1$)

i.e $\mu_r >> 1$

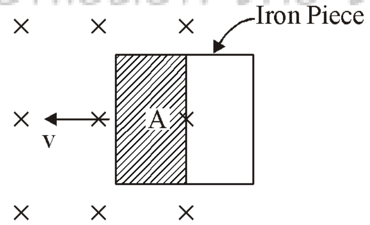
So, there fore we can say Reluctance of air gap will be more.

i.e. $Rl_g >> Rl_i$

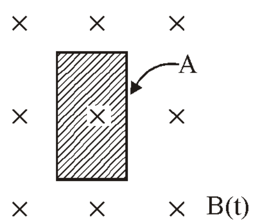
i.e. Total reluctance $Rl \simeq Rl_g$ (air gap reluctance).

## 1.2 Eddy Currents

If an iron piece is lying in the magnetic field, the flux linking $\phi = BA \cos\theta$ , so $\phi$ can be changed if either B, A or $\theta$ changes.




**Case-I :** B is constant but area 'A' of iron piece linking with B is changing (e.g. in dc machines) i.e. $\phi = BA$ also changing with time.

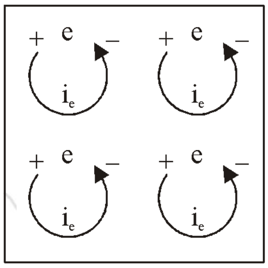


**Case-II :** Iron piece is stationary but B is changing w.r.t time (e.g. transformers), so $\phi = BA$ is changing w.r.t time.

As flux linking $\phi(t)$ is changing (in both the cases) there is induced emf in the iron piece i.e.

$$e \propto -\frac{d\phi}{dt}$$

Due to the induced emfs, there are induced currents in the iron i.e. eddy currents $i_e$




$$i_e = \frac{e}{R_e}$$

Where $R_e$ is the resistance in the path of eddy currents i.e. resistance of iron.

Eddy current loss i.e. power loss due to eddy currents

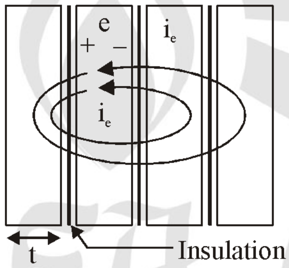
$$P_e = i_e^2 R_e = \frac{e^2}{R_e}$$

As 'e' is independent of $R_e$

$$\Rightarrow \quad P_e \propto \frac{1}{R_e}$$

So $P_e$ can be reduced by increasing $R_e$ i.e. using high resistivity iron.

$R_e$ can also be increased if instead of thick iron, laminated iron is used i.e. thin layers of iron pieces are separated by very thin layers of insulation.

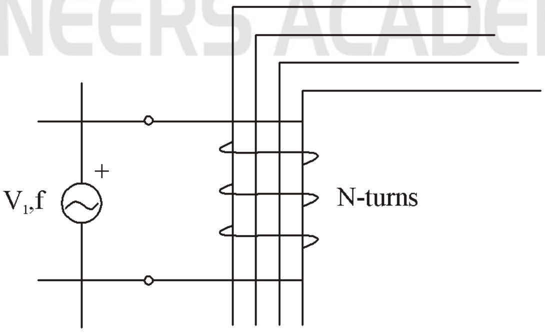



As resistance is introduced in the path of eddy currents so resistance $R_e$ increases & hence power loss decreases.

Where 't' is thickness of lamination.

Consider the laminated iron core of transformer






The eddy current losses are

$$P_e \propto \frac{\pi^2 B_m^2 f^2 t^2}{\rho_e \beta}$$

Where $B_m$ is peak flux density in the core

f is frequency

t is thickness of lamination

$\rho_e$ is resistivity of iron core

$\beta$ is constant (depending upon the shape & size of lamination).

$P_e$ can also be reduced by using high resistance of iron core.

$$P_e \propto B_m^2 f^2 t^2$$

or $$P_e \propto B_m^2 f^2$$

$\Rightarrow$ $$P_e = K_e B_m^2 f^2$$

Where $K_e$ is constant

The combination of hysteresis loss $P_h$ & eddy current loss $P_e$ is said to be iron loss.

$$P_i = P_h + P_e = K_h B_m^{1.6} f + K_e B_m^2 f^2$$

## 1.3 Transformer Equation

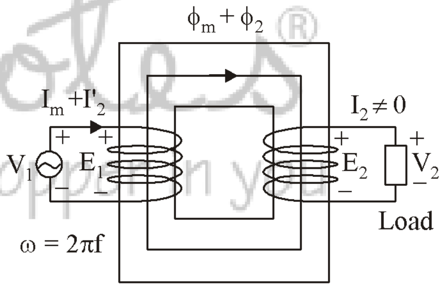
**Primary :** Where source is connected.

**Secondary :** Where load is connected.

**At No load.** Due to magnetising current $I_m$ magnetising flux $\phi_m$ is produced.

Let $\phi_m = \phi_{max} \sin\omega t$

Induced emf in Primary and secondary is $e_1$ & $e_2$




$$e_1 = \frac{-N_1 d\phi_m}{dt}, \; e_2 = \frac{-N_2 d\phi_m}{dt}$$

$$e_1 = -N_1 \frac{d}{dt}(\phi_{max} \sin\omega t)$$

$$e_1 = -N_1 \omega \phi_{max} \cos\omega t$$

$$e_1 = -N_1 \omega \phi_{max} \sin(90 - \omega t),$$

As $$e_1 = N_1 \omega \phi_{max} \sin(\omega t - 90°)$$

peak emf $$E_m = N_1 \omega \phi_{max}$$

We can say induced emf $\bar{E}$ lags behind the corresponding flux $\phi_m$ by 90°

$$e_i = N_1 \omega \phi_{max} \sin(\omega t - 90°)$$

$$e_i = E_m \sin(\omega t - 90°)$$

Peak emf $$E_m = N_1 \omega \phi_{max}$$

$$= N_1 (2\pi f) \phi_{max} = 2\pi f N_1 \phi_{max}$$

Let r.m.s value of $E_m$ is $E_1$ $$E_1 = \frac{E_m}{\sqrt{2}} = \sqrt{2}\pi f N_1 \phi_{max}$$

Similarly $$E_2 = \sqrt{2}\pi N_2 f\,\phi_{max}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{N_2}{N_1} = K$$

Turns ratio or Transformation ratio

As $$V_1 \approx E_1$$

$$V_1 \simeq \sqrt{2}\pi f\, N_1\phi_{max}$$

$$\phi_{max} = \frac{1}{\sqrt{2}\pi N_1}\frac{V_1}{f}$$

$$\phi_{max} = \frac{1}{\sqrt{2}\pi N_1}\left(\frac{V_1}{f}\right)$$

$$\phi_{max} \propto \frac{V_1}{f}$$

$$\phi_{max} = \text{constant}$$

$$\phi_m = \frac{N_1 I_m}{Rl} \text{ \& Secondary flux } \phi_2 = \frac{N_2 I_2}{Rl}$$

According to Lenz's law the flux $\phi_2$ of current $I_2$ will oppose $\phi_m$.

**Lenz's law :** The induced current flows in such a direction so as to oppose very cause of its production.

So net flux in core = $\phi_m - \phi_2$

Due to flux $\phi_2$ net flux in the core decreased, $(\phi_m - \phi_2)$

However $$\phi_m \propto \frac{V_1}{f} = \text{constant},$$

That's why to maintain flux $\phi_m$ constant the primary winding produces additional flux $\phi_2$, so it takes additional current $I'_2$

Secondary flux $$\phi_2 = \frac{N_2 I_2}{R_l}$$

Additional flux by Primary $$\phi_2 = \frac{N_1 I'_2}{R_l}$$

$\therefore$ $$\phi_2 = \frac{N_1 I'_2}{R_l} = \frac{N_1 I_2}{R_l}$$

$$N_1 I'_2 = N_2 I_2$$

Primary Current $\bar{I}_1 = \bar{I}_m + \bar{I}_2'$

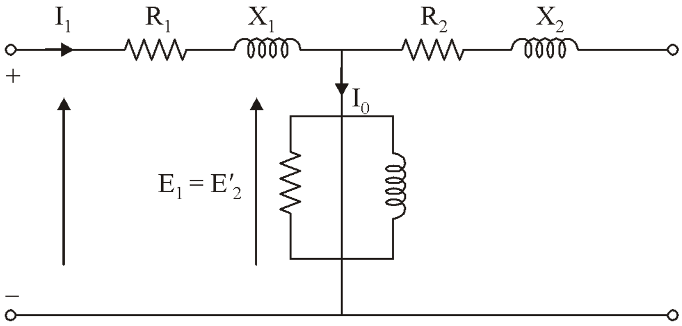
As $\bar{I}_m << \bar{I}_2'$

$I_1 \simeq I_2'$

so $N_1 I_1 \simeq N_2 I_2$

This equation is valid only when magnetising current is negligible.

**Example 1**




**Note :** $E_1 \simeq V_1$

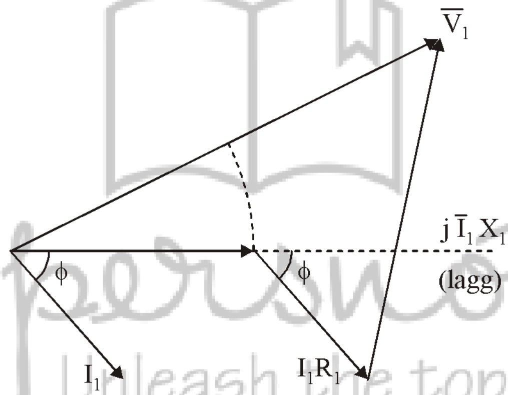
$V_1 = \bar{E}_m + \bar{I}_m(R_1 + jX_1)$

***Solution***

Let at lag pf cos $\phi$

$\bar{I}_1$ lags $\bar{E}_1$ by angle $\phi$

At lagging

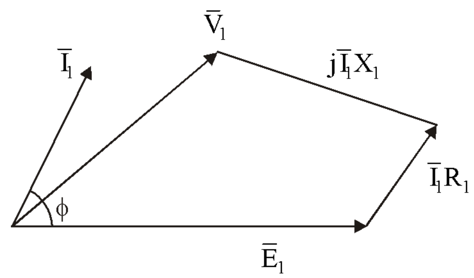



So $E_1 \leq V_1$ (As leakage impedance $R_1$, $X_1$ very low)

At leading pf cos $\phi$

$\bar{I}_1$ leads $\bar{E}_1$ by angle $\phi$

$E_1 \geq V_1$



**Example 2 :** The useful flux of a Transformer is 1Wb. when it is loaded at 0.8pf lag, then its mutual flux

(a) May decrease to 0.8Wb (b) May increase to 1.01Wb

(c) Remains constant (d) May decrease to 0.99Wb

**Ans.(d)**

***Solution:*** At no load (I = 0)

$$V_1 \approx E_1$$

$$E_1 = \sqrt{2}\pi f N_1 \phi_m$$

$$\phi_m = \frac{1}{\sqrt{2}\pi N_1}\frac{E_1}{f}$$

At no load $E_1 \approx V_1$

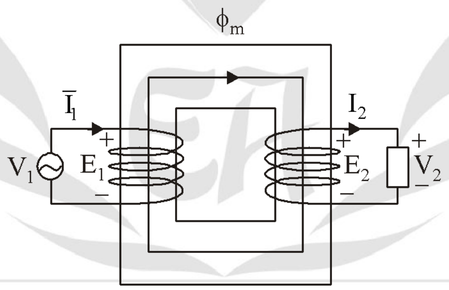
$$\phi_{mo} = \frac{1}{\sqrt{2}N_1\pi}\left(\frac{V_1}{f}\right) = 1\text{Wb}$$

At lag pf $E_1$ decreases slightly ie $E_1 \le V_1$

So $$\phi'_m = \frac{1}{\sqrt{2}\pi N_1}\left(\frac{E_1}{f}\right) \le \phi_{mo}$$

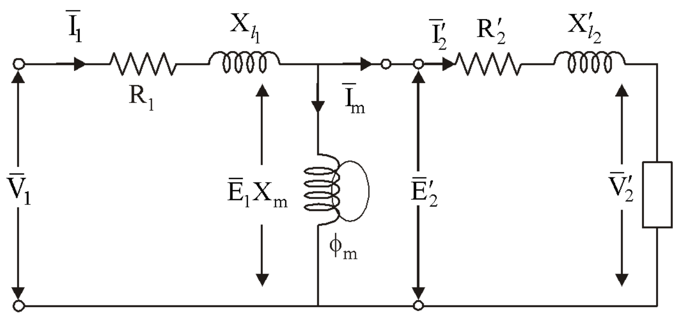
So magnetising flux decreases slightly ie approx 0.99 Wb

**Equivalent Circuit of Transformer**




$\phi_m$ = magnetising flux or useful flux






$R_1$ = Resistance of Primary

$R_2$ = Resistance of Secondary

$X_{l_1}$ = Leakage Reactance of Primary

$X_{l_2}$ = Leakage Reactance of Secondary

$X_m$ = Magnetising Reactance

Leakage flux through air $= \phi_l$

$\because \quad \phi_m >> \phi_l$

Total flux $\quad \phi = \phi_m + \phi_l$

$L = L_m + L_l$ (because $N\phi = LI$)

$\phi \propto L$

Where $\quad L_m$ = Magnetising inductance

$L_l$ = Leakage inductance

$L = L_m + L_l$

Multiplying both side by ω

$$\omega L = \omega L_m + \omega L_l$$

$$X = X_m + X_l$$

$X_m$ = Leakage Reactance

$X_l$ = Magnetising Reactance

Magnitude of e

$$e = \underbrace{N\frac{d\phi}{dt}}_{\text{induced emf}} = \underbrace{L\frac{di}{dt}}_{\text{voltage drop across L}}$$

From the equivalent circuit we can write

$$E_2 = V_2 + I_2(R_2 + jX_{12})$$

$$\frac{E_2}{K} = \frac{V_2}{K} + (KI_2)\left(\frac{R_2}{K^2} + j\frac{X_{12}}{K^2}\right)$$

$$E_2' = V_2' + I_2'(R_2' + jX_{l_2}')$$

Compare above equations :

$E_2' = \frac{E_2}{K}$ = Secondary emf referred to Primary

$V_2' = \frac{V_2}{K}$ = Secondary voltage referred to Primary

$I_2' = KI_2$ = Secondary current referred to Primary

$R_2' = \frac{R_2}{K^2}$ = Secondary resistance referred to Primary

$$X'_{l_2} = \frac{X_{l_2}}{K^2} = \text{Secondary leakage Reactance referred to Primary.}$$

The iron losses or core losses of iron core

$$P_i = P_h + P_e$$

Where $P_h \rightarrow$ Hysteresis loss

$P_e \rightarrow$ Eddy current loss

$$P_h = K_h B_m^{1.6} f \;\&\; P_e = K_e B_m^2 f^2$$

$B_m \rightarrow$ Peak flux density,

$f \rightarrow$ Frequency

Magnetising flux

$$\phi_m = \frac{1}{\sqrt{2}\pi N_1}\left(\frac{E_1}{f}\right)$$

$\Rightarrow$
$$B_m = \frac{1}{\sqrt{2}\pi N_1 A}\left(\frac{E_1}{f}\right)$$

Let $P_e \propto E_1^2$ (approximated)

& $P_e \propto E_1^2$

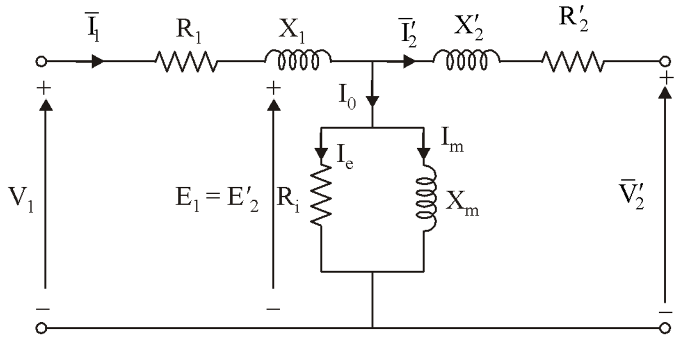
So $P_i \propto E_1^2$

Hence
$$P_i = \frac{E_1^2}{R_i}$$

where $R_i \rightarrow$ is constant

$R_i \rightarrow$ core loss resistor

By connecting $R_i$ across voltage $E_1$ in equivalent circuit, iron losses can be represented




$$\bar{I}_1 = \bar{I}_0 + \bar{I}'_2$$

Where no load current $\bar{I}_0 = \bar{I}_e + \bar{I}_m$

$\bar{I}_0$ is approximately 4-5% of rated or full load current.

$$\bar{E}_1 = \bar{V}_1 - \bar{I}_1(R_1 + jX_1)$$

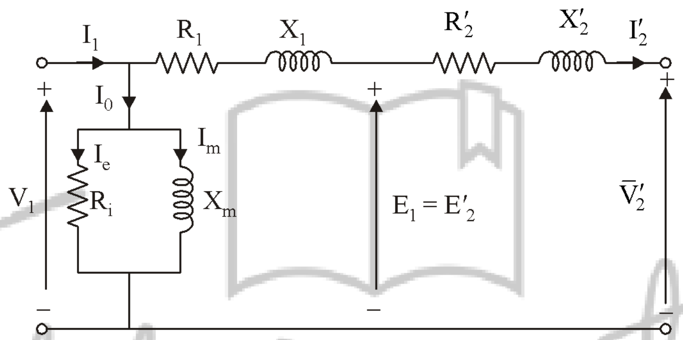
Let primary leakage impedance drop is neglected

$$E_1 \simeq V_1 \text{ so no load current}$$

$$\bar{I}_0 = \frac{\bar{E}_1}{\bar{Z}_0} \simeq \frac{\bar{V}_1}{\bar{Z}_0}$$

Where $\bar{Z}_0$ is the impedance of magnetising branch

Hence $\bar{Z}_0$ can also be connected across $\bar{V}_1$ so approximated equivalent circuit is



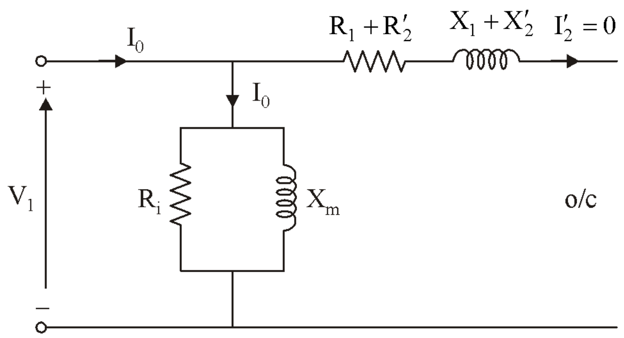


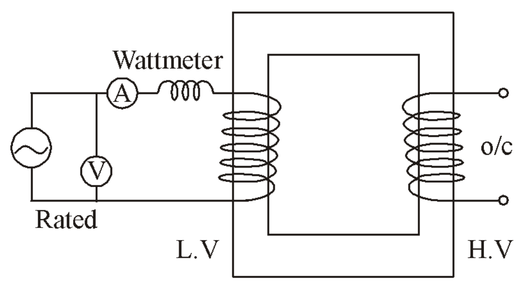

## 1.4 Open Circuit and Short Circuit Test

These test are performed to determine the circuit constants, efficiency and regulation without actually loading the Transformer.

**Open Circuit Test or No Load Test**

To determine iron loss, As iron loss depends upon the applied voltage so it is performed at rated voltage hence it is performed on L.V side



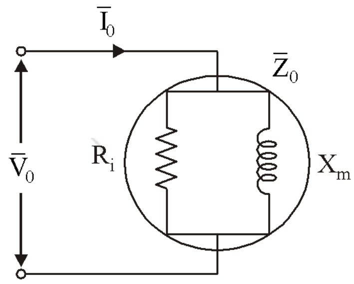



**Readings**

$V_0 \quad I_0 \quad P_i$

Where $V_0$ is the rated voltage applied.

$$P_i = \frac{V_0^2}{R_i}$$




$$\Rightarrow \quad R_i = \frac{V_0^2}{P_i}$$

$$G_i = \frac{1}{R_i}$$

$$\frac{1}{Z_0} = \frac{1}{R_i} + \frac{1}{jX_m} = \frac{1}{R_i} - j\frac{1}{X_m}$$

$$\bar{Y}_0 = G_i - jB_m$$

$\bar{Y}_0$ admittance

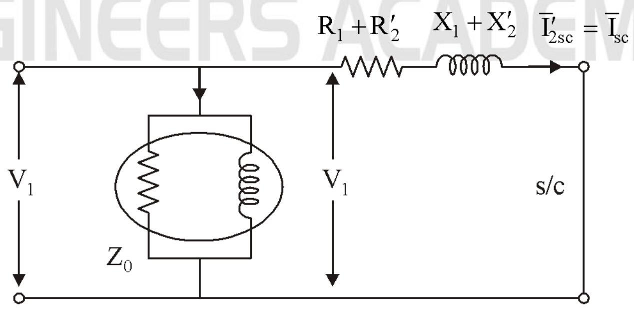
$$|Y_0| = \sqrt{G_i^2 + B_m^2}$$

$$Z_0 = \frac{V_0}{I_0} \text{ and } Y = \frac{I_0}{V_0}$$

Susceptance $\quad B_m = \sqrt{Y_0^2 - G_i^2}$

Reactance $\quad X_m = \frac{1}{B_m}$

## 1.5 Short Circuit Test or S.C. Test



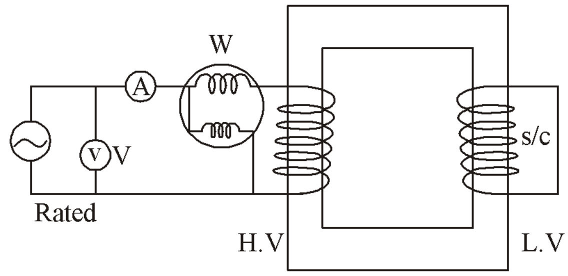

**Short** circuit test is performed to determine Cu-loss, As Cu-loss depends upon load current so short circuit test is performed at rated current.

To flow the rated current in short circuit condition reduced voltage upto 5% of rated voltage is required.

So short circuit test is performed at H.V side or low current side.

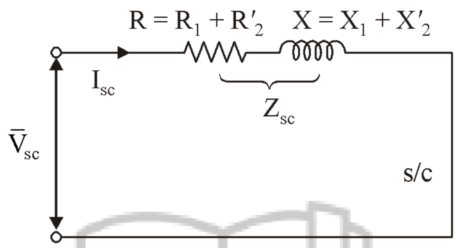
**Reduced Voltage is Applied for SC Test**

At reduced voltage the no load current $I_0$ will be very very small, so it can be neglected.




$$Z_{sc} = R + jX$$

$$Z_{sc} = \sqrt{R^2 + X^2}$$




**Reading**

$V_{sc}$ $\quad$ $I_{sc}$ (rated value) $\quad$ $P_{sc}$ (Cu-loss)

$$P_{sc} = I_{sc}^2 R \quad \Rightarrow \quad R = \frac{P_{sc}}{I_{sc}^2}$$

$$R = \frac{P_{sc}}{I_{sc}^2}$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

$$X = \sqrt{Z_{sc}^2 - R^2}$$

## 1.6 Transformer Efficiency

The ratio of output power to the input power in a machine is known as the efficiency.

$$\eta = \frac{P_0}{P_{in}} = \frac{P_0}{P_0 + P_L}$$

Power input = Power output + Losses

$$P_{in} = P_0 + \text{Losses}$$

$$\% \, \eta = \frac{\text{Output}}{\text{Output} + \text{Losses}} \times 100$$

**Losses**

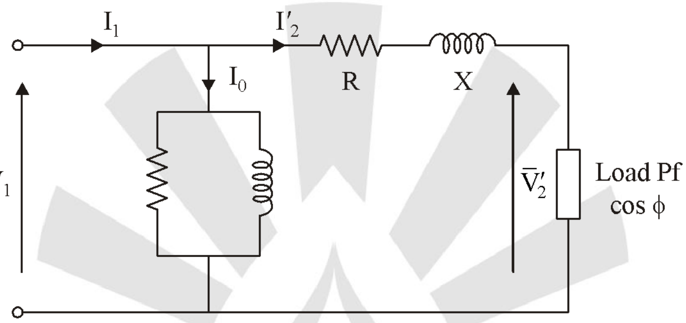
**1. Iron loss or core loss**

This is constant with respect to load current

$$P_i = \frac{V_1^2}{R_i}$$

**2. Cu loss or load loss**

Varies with respect to load current $I_2'^2(R_1 + R_2')$




Power output

$$P_0 = V_2' I_2' \cos\phi$$

$$V_2' = \frac{V_2}{K}, \quad I_2' = K I_2, \quad K = \frac{N_2}{N_1}$$

$$P_0 = V_2' I_2' \cos\phi = \frac{V_2}{K}(K I_2)\cos\phi$$

**At rated or full load current**

So $= V_2 I_{2fl}$

$x = 0.5 = 50\%$ of rated load or half full load

$P_0 = V_2 I_2 \cos\phi$

∵ At a load current

∴ $I_2 = x I_{2fl}$

$P_0 = V_2 (x I_{2fl}) \cos\phi \; x \le 1$

$P_0 = x V_2 I_{2fl} \cos\phi$

Load VA $S_L = V_2 I_2 = V_2 (x I_{2fl}) = x S_0$

Where $[S_0 = V_2 I_{2fl}]$

**1. Iron Loss i.e. constant loss ($P_i$)**

**2. Cu-losses**

$$P_{Cu} = I_2'^2(R_1 + R_2')$$

$$= (KI_2)^2\left[R_1 + \frac{R_2}{K^2}\right] = \frac{(KI_2)^2}{K^2}[R_1K^2 + R_2]$$

$$= I_2^2[R_1' + R_2] = I_2^2 R_{02}$$

$R_1' = K^2R_1$ ($R_1'$ = Primary resistance referred to secondary side)

$R_{01} = R_1 + R_2'$

$R_{01}$ = Total resistance reffered to primary side

$R_{02} = R_1' + R_2$

$R_{02}$ = Total resistance referred to secondary side

$P_{Cu} = (x\,I_{2fl})^2$

$R_{02} = x^2 I_{2fl}^2 R_{02} = x^2 P_{cF}$

Where $P_{cF}$ = Full load Cu-losses

$P_{CF} = I_{2fl}^2 R$

**Efficiency**

$$\eta = \frac{x S_0 \cos\phi}{xS_0\cos\phi + P_i + x^2P_{cF}} = \frac{\text{output}}{\text{Output} + \text{losses}}$$

for maximum efficiency, Denominator should be minimum.

i.e. For $\eta_{max}$

i.e. $$\frac{d}{dx}\left[S_0\cos\phi + \frac{P_i}{x} + x\,P_{cF}\right] = 0$$

$$0 - \frac{P_i}{x^2} + P_{cF} = 0$$

$$P_i = x^2\,P_{cF}$$

i.e. Iron loss = Cu loss

$$x = \sqrt{\frac{P_i}{P_{CF}}}$$



## 1.7 Voltage Regulation of a Transformer

### Voltage Regulation

The voltage regulation of a transformer is the arithmetical difference in the secondary terminal voltage between no load and full load at a given power factor with the same value of primary voltage for both no load and full load.

$$V{\cdot}R = \frac{V_{nl} - V_{fl}}{V_{nl}} \times 100$$

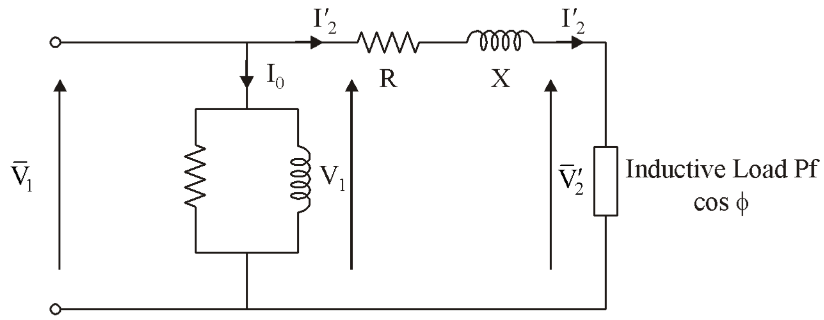
Where $V_{nl}$ = Voltage at no load i.e. no load voltage

$V_{fl}$ = full load voltage

$$\left(\text{Either } \%\, V{\cdot}R = \frac{E_2 - V_2}{E_2} \times 100\right)$$

Where $E_2$ = No load secondary terminal voltage

$V_2$ = Full load secondary terminal voltage



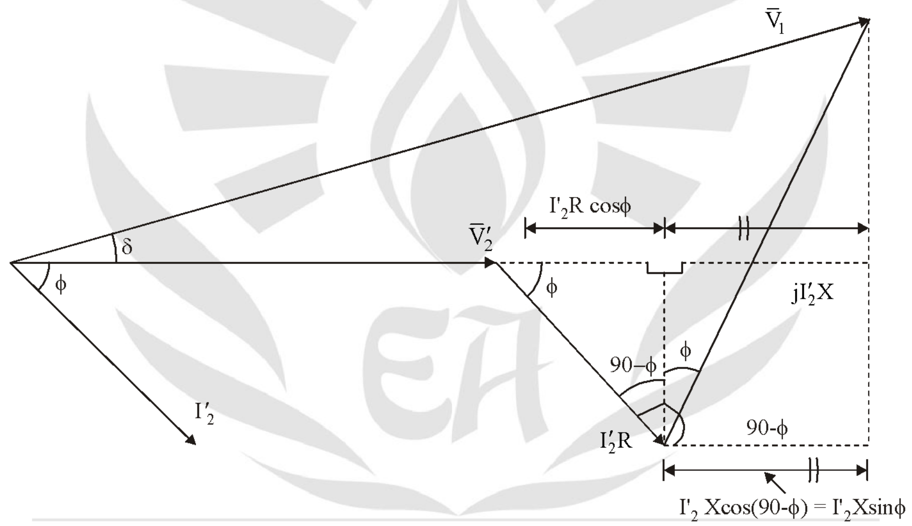

Load pf cos $\phi$ lagg.

i.e. $I_2'$ lags behind $\bar{V}_2'$ by $\phi$

$$\bar{V}_1 = \bar{V}_2' + I_2'(R + jX) \quad \ldots(1)$$

$$= \bar{V}_2' + I_2'R + jI_2'X$$

**Phasor Diagram :** $I_2'$ lags behind $\bar{V}_2'$ by angle $\phi$






From Phasor Diagram

$$V_1 \cos \delta = V_2' + I_2'R\cos\phi + I_2'X\sin\phi$$

As $\delta$ is small, $\cos \delta = 1$

$$V_1 \approx V_2' + I_2'(R\cos\phi + X\sin\phi)$$

At No load $\quad I_2' = 0$

$$V_1 = V_2' + 0$$

$$V_1 = V_2'$$

or $\quad V_{2nl}' = V_1$

$$V.R = \frac{V'_{2nl} - V'_{2fl}}{V'_{2nl}} = \frac{V_{2nl}/K - V_{2fl}/K}{V_{2nl}/K}$$

$$= \frac{V_{2nl} - V_{2nl}}{V_{2nl}}$$

$$= \frac{V_1 - V'_2}{V'_2} \text{ because } (V_{2nl} = V_1)$$

As $V_1 = V'_2 + I'_2(R\cos\phi + X\sin\phi)$

$$V_1 = V'_2 + I'_2(R\cos\phi + X\sin\phi)$$

From $V.R = \dfrac{I'_2(R\cos\phi + X\sin\phi)}{V'_2}$

$$X = X_1 + X'_2 = X_{01}$$

& $R = R_1 + R'_2 = R_{01}$

$$V.R. = \frac{KI_2}{(V_2/K)}(R_{01}\cos\phi + X_{01}\sin\phi)$$

$$V.R. = \frac{I_2}{V_2}\cdot K^2(R_{01}\cos\phi + X_{01}\sin\phi)$$

$$= \frac{I_2}{V_2}(K^2R_{01}\cos\phi + K^2X_{01}\sin\phi)$$

$$K^2R_{01} = K^2(R_1 + R_2/K^2) = K^2R_1 + R_2 = R'_1 + R_2$$

$$\simeq R_{02}$$

$$V.R. = \frac{I_2}{V_2}(R_{02}\cos\phi + X_{02}\sin\phi)$$

At a load $I_2 = xI_{2fl}$

$$V.R. = \frac{xI_{2fl}}{V_2}[R_{02}\cos\phi + X_{02}\sin\phi]$$

$$\frac{V_2}{V_{2fl}} = \frac{V_{2\text{rated}}}{I_{2\text{rated}}} = \frac{V_{2B}}{I_{2B}} = Z_{2B}$$

$Z_{2B}$ = Base impedance on secondary side

$$V.R. = x\left[\frac{I_{2fl}}{V_2}R_{02}\cos\phi + \frac{I_{2fl}}{V_2}X_{02}\sin\phi\right]$$

$$V.R. = x\left[\frac{R_{02}}{V_2 / I_{2fl}}\cos\phi + \frac{X_{02}}{V_2 / I_{2fl}}\sin\phi\right]$$

$\because$ $$\frac{V_2}{I_{2fl}} = Z_{2B}$$

$$V.R. = x\left[\frac{R_{02}}{Z_{2B}}\cos\phi + \frac{X_{02}}{Z_{2B}}\sin\phi\right]$$

$$V.R. = x[R_{pu}\cdot\cos\phi + X_{pu}\sin\phi]$$

$$X_{pu} = \frac{X_{02}}{X_{2B}}$$

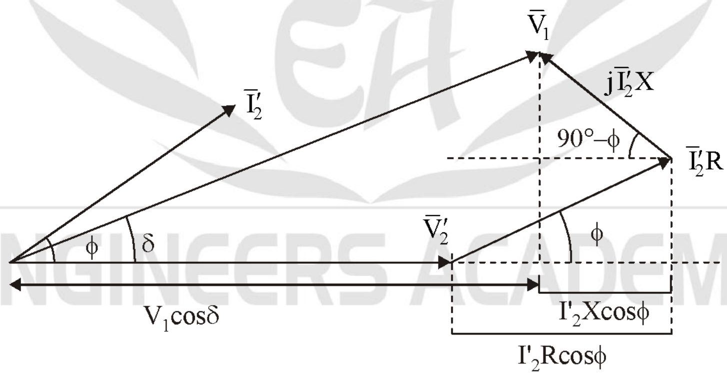
$$R_{pu} = \frac{R_{02}}{Z_{2B}}$$

At full load $x = 1$

$$V.R. = R_{pu}\cos\phi + X_{pu}\sin\phi$$

**At load pf lead i.e. capacitive load**

From Phasor Diagram




$$V_1 \cos\delta = V'_2 + I'_2(R\cos\phi) - I'_2(X\sin\phi)$$

As δ is small, $\cos\delta \simeq 1$

V.R. (similarly as from lagg loads)

Here for lead load pf

V.R. will be $V.R. = R_{pu}\cos\phi - X_{pu}\sin\phi$ lead

At UPF $\cos\phi = 1$

$$V.R = R_{pu} \cdot 1 - 0 = R_{pu}$$

$$V.R = R_{pu}$$

At leading power factor $\quad V.R = R_{pu} \cos\phi - X_{pu} \sin\phi$

For zero voltage regulation $\quad 0 = R_{pu} \cos\phi - X_{pu} \sin\phi$

$$\tan\phi = \frac{R_{pu}}{X_{pu}}$$

$$\frac{R_{pu}}{X_{pu}} = \frac{R_{02} / Z_{2B}}{X_{02} / Z_{2B}} = \frac{R_{02}}{X_{02}}$$

or
$$\frac{R_{pu}}{X_{pu}} = \frac{R_{01} / Z_{1B}}{X_{01} / Z_{1B}} = \frac{R_{01}}{X_{01}}$$

i.e.
$$\tan\phi = \frac{R_{pu}}{X_{pu}} = \frac{R}{X}$$

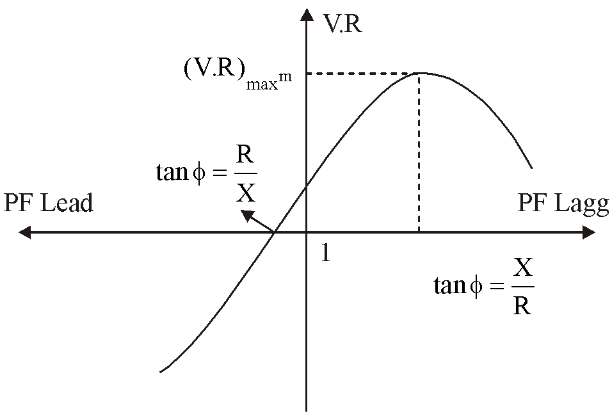
At lagg, for maximum V.R $\quad \frac{d}{d\phi}(V.R) = 0$

$$\Rightarrow \quad \frac{d}{d\phi}(R_{pu} \cos\phi + X_{pu} \sin\phi) = 0$$

$$\Rightarrow \quad -R_{pu} \sin\phi + X_{pu} \cos\phi = 0$$

$$R_{pu} \sin\phi + X_{pu} \cos\phi$$

$$\tan\phi = \frac{X_{pu}}{R_{pu}} = \frac{X}{R}$$



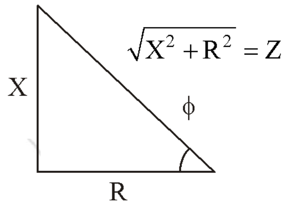
**Example 3 :** A transformer has leakage impedance of $Z_e = r_e + jX_e$. Its maximum voltage regulation occures at a power factor of :

(a) $\frac{r_e}{x_e}$ leading (b) $\frac{r_e}{Z_e}$ leading (c) $\frac{x_e}{Z_e}$ leading (d) $\frac{r_e}{Z_e}$ lagging

**Ans. (d)**

***Solution***

At pf cosϕ lagg.




$$\tan\phi = \frac{X}{R}$$

$$\cos\phi = \frac{R}{\sqrt{X^2+R^2}} = \frac{R}{Z}\text{lagg}$$

$$\cos\phi = \frac{R}{Z} = \frac{r_e}{Z_e}\text{lagg.}$$

**Example 4 :** A 20kVA, 800/400V, 1 – ϕ transformer with percentage resistance & reactance of 4% and 6% respectively is supplying a current of 50A to an inductive load such that load resistance and reactance are equal. If source voltage is maintained constant at 800V the load voltage is :

(a) 374V (b) 428V (c) 406V (d) 394V

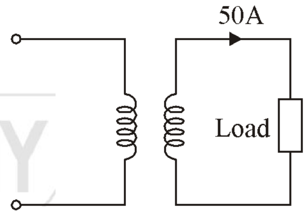
**Ans.(a)**

***Solution***

20 kVA

800V/400V

Rated current $V_1I_1 = V_2I_2 = 20 \times 10^3$



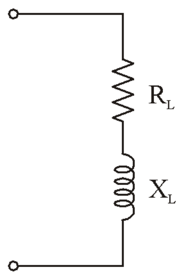

$$I_1 = \frac{20\times10^3}{800} = 25\text{A}$$

$$I_2 = \frac{20\times10^3}{400} = 50\text{A}$$

Rated current 25A/50A

Supplying rated current or operates at full load.

Inductive load i.e. cosϕ lagg




$$Z_L = R_L + jX_L,\ X_L = R_L$$

$$= R_L + jR_L = R_L(1 + j)$$

⇒ $\sqrt{2}R_L\angle45°,\ Z_L = R_L + jR_L = Z_L\angle\theta_L$

$$R_{pu} = 0.04$$

$$X_{pu} = 0.06$$

$$\text{Load pf} = \cos\phi = \cos\phi_L = \frac{1}{\sqrt{2}}$$

V.R at full load and lagg pf $\cos\phi$

$$V.R = R_{pu}\cos\phi + X_{pu}\sin\phi$$

$$\cos\phi = \frac{1}{\sqrt{2}}$$

$$\sin\phi = \frac{1}{\sqrt{2}}$$

$$V.R = 0.04 \times \frac{1}{\sqrt{2}} + 0.06 \times \frac{1}{\sqrt{2}} = 0.0707$$

$$V.R\% = 7.07\%$$

$$V.R = \frac{V_1 - V_2'}{V_2'},\ V_1 = 800V$$

$$0.707 = \frac{800V - V_2'}{V_2'}$$

$$V_2' = 747.17V$$

Transformation ratio $K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{1}{2} = 0.5$

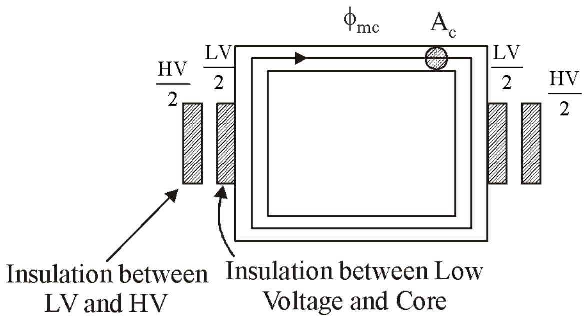


$$V_2' = \frac{V_2}{K}$$

$$V_2 = KV_2' = (0.5)\,(747.17)$$

$$V_2 = 373.6V$$

## 1.8 Types of Transformer




1. **Core type**
   (a) Cost of insulation is less
   (b) To reduced the cost of insulation LV is kept inside.
2. **Shell type**
   (a) Cost of insulation is more Area of cross-section of core

$$A_c < A_s$$

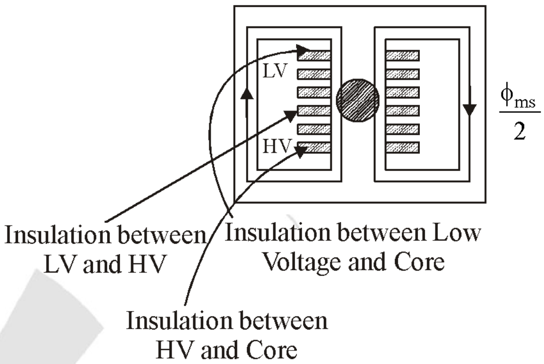
for same mechanical strength the area of cross-section of core can be designed higher in the shell type than that in core type

For same peak flux density $B_m$

$$A_c < A_s$$

$$B_m A_c < B_m A_s$$

$$\phi_{mc} < \phi_{ms}$$




Let $V_1 \simeq E_1 = \sqrt{2}\pi f N_1\phi_m$

$\therefore$ Induced voltage per turn $T_{TC} < V_{TS}$

Number of turn required for particular voltage rating V

$$N = \frac{V}{V_T}; \; N_{core} > N_{shell}$$

**1. In Core Type**

For a particular voltage rating more number of turns are required in the core type.

(c) Less iron required, and more Cu (Conductor materials).

(d) For HV rating core type is preferred

**2. In Shell Type**

(c) More iron required & less Cu.

(d) For LV rating shell type is preferred.

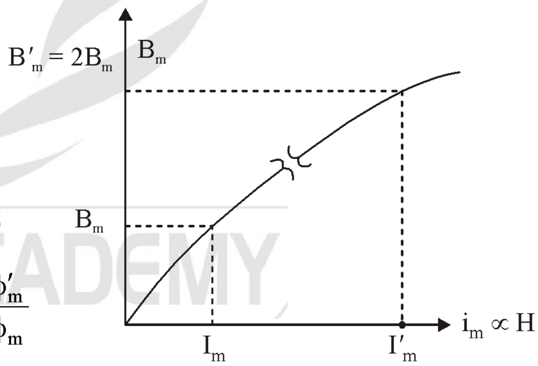
The additional advantage of core type is that the cost of insulation is less that's why power transformer are designed with core type.

**Performance of Transformer at different frequencies**

**Transformer Designed for 240V, 50Hz.**

**Low Frequency Operation**

It is used at 240V, 25Hz




$I'_m \approx 100\ I_m$

$$V_1 = 240V, \quad f_1 = 50Hz$$

$$V_1' = 240V, \quad f_1' = 25Hz$$

$$\frac{V_1'}{V_1} = \frac{f'\phi_m'}{f\phi_m} = \frac{240}{240} = \frac{25\phi_m'}{50\phi_m}$$

$$V_1 = \sqrt{2}\pi f N_1\phi_m$$

$$V_1' = \sqrt{2}\pi f' N_1\phi_m'$$

$$\Rightarrow \quad \phi_m' = 2\phi_m$$

$$B_m' A = 2\, B_m A$$

$$B_m' = 2B_m \quad \text{Peak flux density}$$

Due to saturation mode if the flux density is doubled, the magnetising current required will be extremely high around hundred times of normal magnetising current.

$$I_m = 4 \rightarrow 5\% \text{ of full load } (I_{fl})$$

$$I_m = 0.05 I_{fl}$$

$$I'_m = 100 I_m = 5 I_{fl}$$

For same $B_m$ (i.e. same $I_m$) $\quad B_m A' = 2 B_m A$

$$A' = 2A$$

Area of cross-section of core required is doubled i.e. larger size

If $\quad V'_1 = 120V$

Then $\quad \frac{V'_1}{V_1} = \frac{120}{240} = \frac{25\phi'_m}{50\phi_m}$

$$\phi'_m = \phi_m$$

**High Frequency Operation**

It is used at 240V, 100Hz

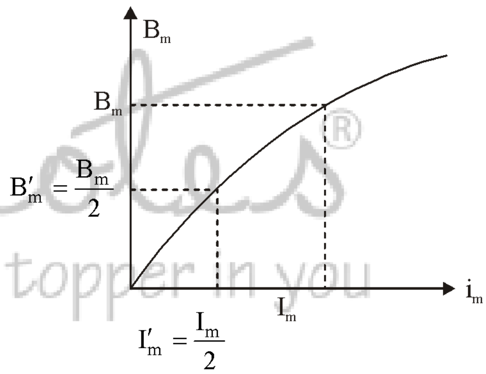
$$V'_1 = 240, \quad f' = 100\text{Hz}$$

$$\frac{V'_1}{V_1} = \frac{f'\phi'_m}{f\phi_m} = \frac{240}{240} = \frac{100\phi_m}{50\phi_m}$$

$$\phi'_m = \frac{\phi_m}{2}$$

$$B'_m A = B_m \frac{A}{2}$$

$$B'_m = \frac{B_m}{2}$$




For same $B_m$ $\quad A' = \frac{A}{2}$ i.e. small size

$$A' = \frac{A}{2}$$

## 1.9 Auto Transformer

When the primary and secondary windings are electrically connected so that a part of the winding is common to both high voltage and low voltage sides, the transformer is known as auto transformer.

Let a two winding transformer of 10 kVA, 400/100V Rated currents

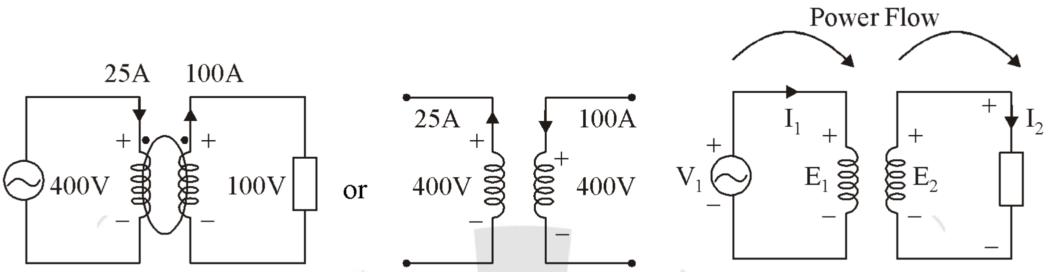
$$V_1 I_1 = V_2 I_2 = 10 \times 10^3$$

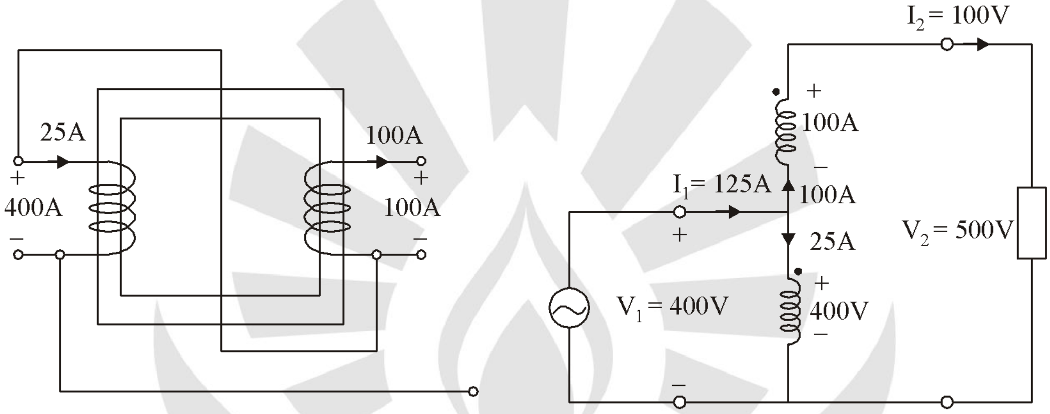
$$I_1 = \frac{10 \times 10^3}{V_1} = \frac{10 \times 10^3}{400} = 25A$$

$$I_2 = \frac{10 \times 10^3}{V_2} = \frac{10 \times 10^3}{400} = 100A$$

$$\text{Rated currents} = 25A/100A$$

VA rating as two winding transformer $S_{TW} = 10$ kVA




VA rating of auto transformer $V_1I_1 = 400 \times 125 = 50$ kVA

$V_2I_2 = 500 \times 100 = 50$ kVA

$S_{auto} = V_1I_1 = V_2I_2 = 50$ kVA

In case of auto transformer the power is transferred via both inductively and conductively while in case of two winding configuration the power is transferred only via induction.

**In Auto Transformer**

VA Transferred inductively $S_{ind} = 10$ kVA (same as that in the two winding configuration)

VA Transferred conductively $S_{cond} = 50 - 10 = 40$ kVA

In the two winding configuration there is electrical isolation between primary and secondary but not in auto transformer connection.

In both two winding and auto transformer configuration there is same voltage across each winding so iron losses are same as well as same current in each winding so Cu-loss will also be same.

**Example 5 :** When tested as a two winding transformer 10 kVA, at rated load, 0.85 lagg its efficiency is 97%. Find its efficiency as an auto transformer of rating 50 kVA.

***Solution***

$$S_{T.W} = 10 \text{ kVA}$$

$$\eta_{T.W} = \frac{xS_{TW}\cos\phi}{xS_{TW}\cos\phi + P_i + x^2PC_F}$$

At rated load i.e. $x = 1$, pf $\cos\phi = 0.85$

$$0.97 = \frac{1\times10\times0.85}{1\times10\times0.85+P_i+(1)^2P_{CF}}$$

$$P_i + P_{CF} = 0.263 \text{ kW}$$

$$S_{auto} = 50 \text{ kVA}$$

$$\eta_{auto} = \frac{xS_{auto}\cos\phi}{xS_{auto}\cos\phi+P_i+x^2P_{CF}}$$

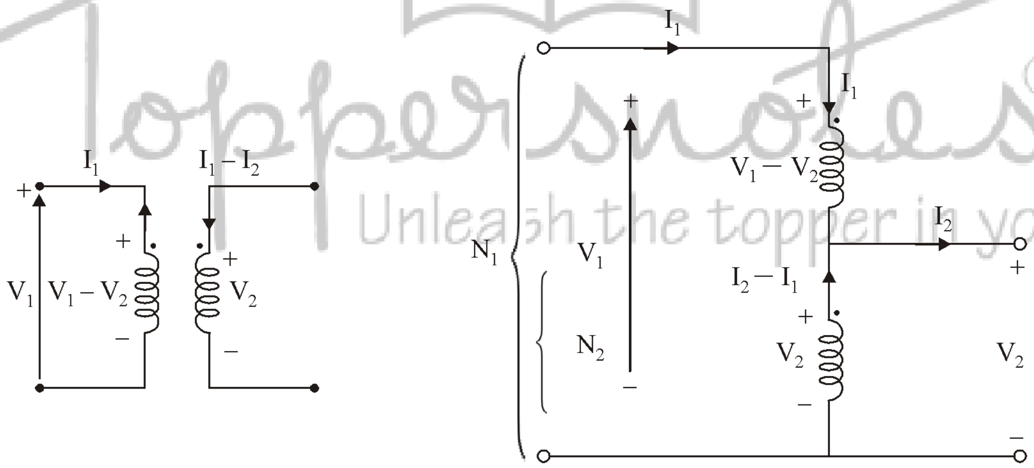
At rated load $x = 1$, pf $\cos\phi = 0.85$

$$\eta_{auto} = \frac{1\times50\times0.85}{1\times50\times0.85+P_i+(1)^2P_{CF}}$$

$$\eta_{auto} = \frac{1\times50\times0.85}{1\times50\times0.85+P_i+P_{CF}}$$

$$\eta_{auto} = \frac{1\times50\times0.85}{1\times50\times0.85+0.263} = 99.38\%$$

## 1.10 Comparison of Ratings of Two Winding and Auto Transformer






Let auto transformer ratio $a = \frac{V_1}{V_2} = \frac{N_1}{N_2} > 1$

$$S_{auto} = V_1I_1 = V_2I_2$$

$$S_{T.W} = (V_1 - V_2)\, I_1 = V_2(I_2 - I_1)$$

$$S_{T.W} = (V_1 - V_2)\, I_1 = \left(1-\frac{V_2}{V_1}\right)(V_1I_1)$$

$$S_{T.W} = \left(1 - \frac{1}{a}\right) S_{auto}$$

$$S_{T.W} = \frac{a-1}{a} S_{auto}$$

$$S_{T.W} = \frac{a}{a-1} S_{T.W}$$

If $a >> 1$

$$S_{auto} \simeq S_{T.W}$$

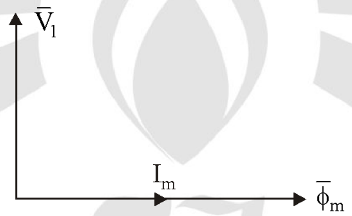
$$a < 2 \begin{cases} \text{If } a = 1.1 & S_{auto} = \dfrac{1.1}{1.1-1} = 11 S_{T.W} \\ \text{If } a = 1.5 & S_{auto} = 3S_{T.W} \\ \text{If } a = 2 & S_{auto} = 2S_{T.W} \end{cases}$$

$$a = 10,\ S_{auto} = 1.11\ S_{T.W}$$

$$a = 100,\ S_{auto} = 1.01\ S_{T.W}$$

There is significant increment in the VA rating using auto transformer only if $a < 2$ that's auto transformer are used mainly when $a < 2$.

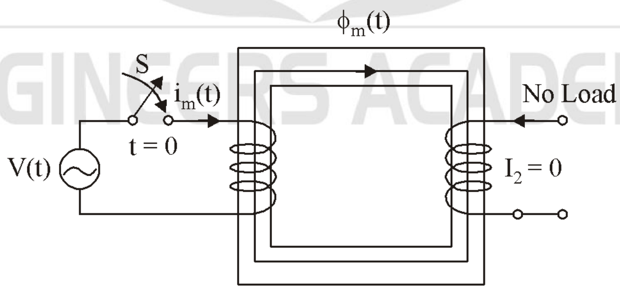
**Switching Transient in Transformer i.e. Transient Magnetising Current of Transformer**




In steady state $\phi_m$ lags behind the applied voltage is by 90°

OR

Steady state magnetising flux $\phi_{m(ss)}$ lags behind applied voltage V by 90°.





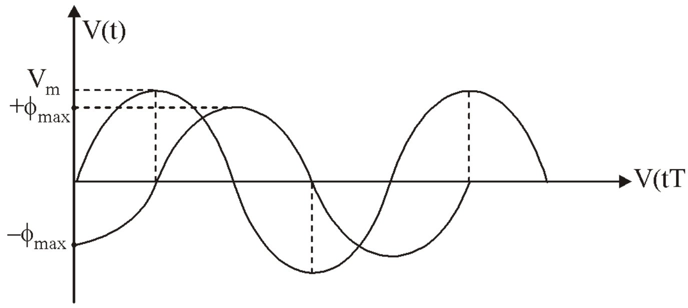

Let the transformer is switched-ON to the supply by closing S at t = 0,. at no load

At no load $i_m$ is drawn form supply

$$v(t) = V_m \sin \omega t$$

Initial flux i.e. at $t = 0$ or

Total flux $\phi_m = \phi_{res} \simeq 0$




$$\phi_{m(ss)} = \phi_{max} \sin(\omega t - 90°)$$

At $t = 0$, $\phi_{m(ss)} = -\phi_{max}$

At $t \geq 0$, Total magnetising flux

$$\phi_{m(t)} = \phi_{mtr}(t) + \phi_{m(ss)}(t)$$

$\phi_{mtr}(t) \rightarrow$ Transient magnetising flux

$\phi_{m(ss)}(t) \rightarrow$ Steady state magnetising flux

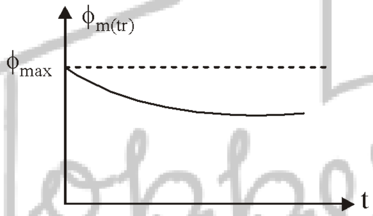
At $t = 0$, $\phi_m(0) = 0$

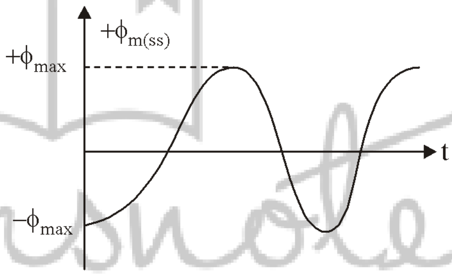
$$\phi_m(t) = \phi_{m(tr)}(t) + \phi_{m(ss)}(t)$$

As $\phi_{m(ss)}(0) = -\phi_{max}$

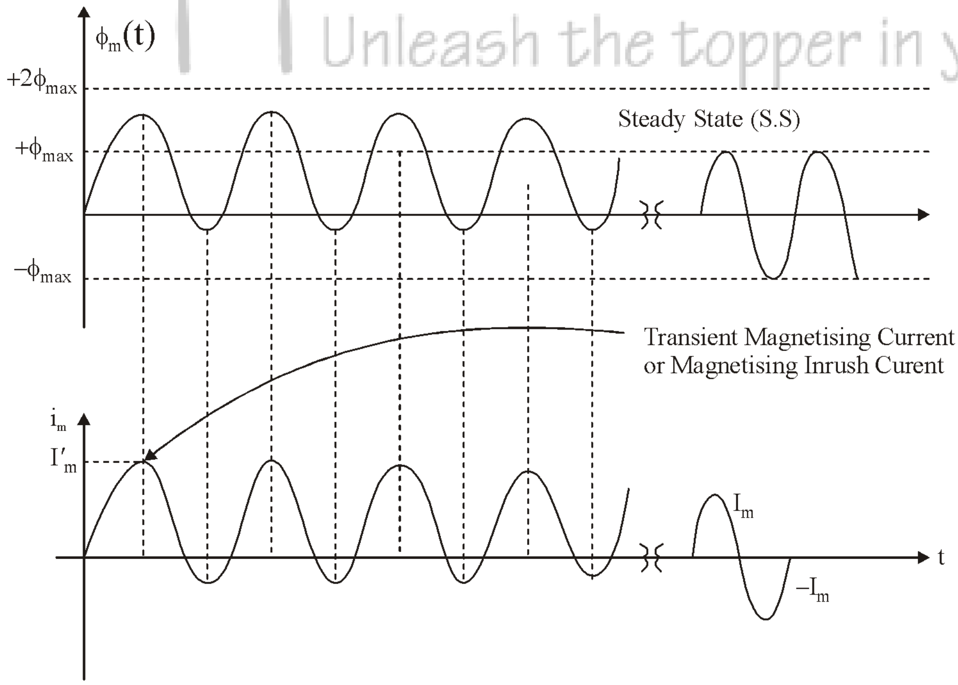
$\Rightarrow$ $0 = \phi_{mtr}(0) - \phi_{max}$

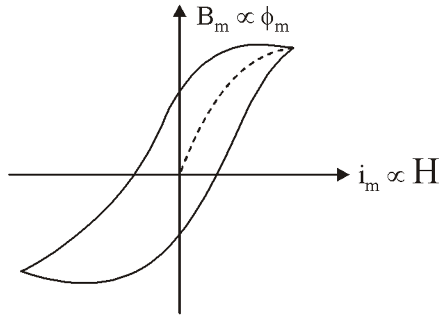
i.e. $\phi_{mtr}(0) = \phi_{max}$



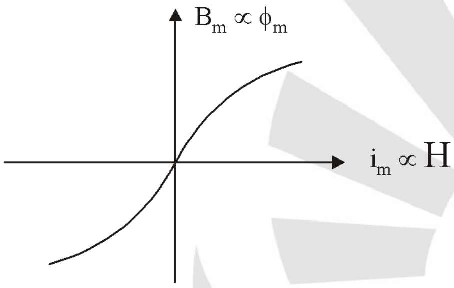



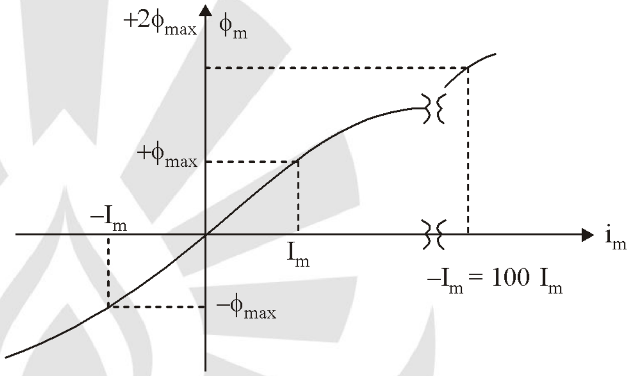





Soft iron core of transformer i.e. narrow loop. If hysteresis neglected the approximate curve



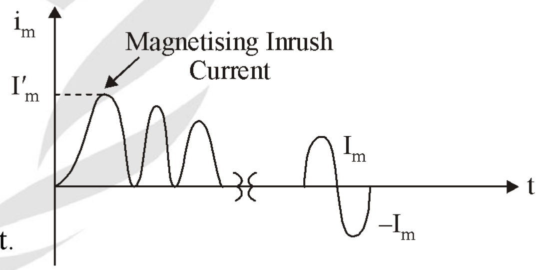




$$I_{inrush} = I'_m$$

$$I'_m = 100\ I_m$$

$$I_m = 5\%\ \text{of}\ I_{fl} = 0.05\ I_{fl}$$

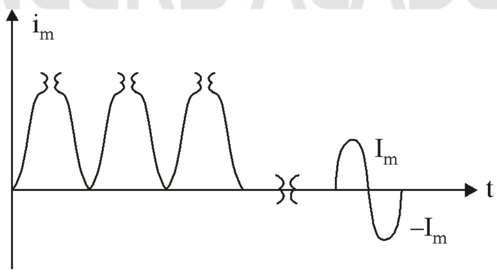
$$I_{inrush} \simeq 5\ I_{fl}$$




In general inrush current is 6 to 10 times of $I_{fl}$ or rated current.

Due to such a large inrush current there is false tripping of relay and circuit breaker so, harmonic restraint relay is used.

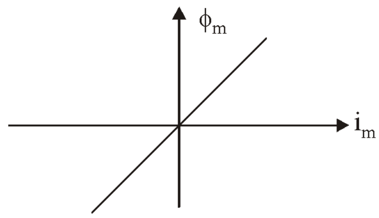





**The Nature of Steady State Magnetising Current**

$$\phi_m = \frac{1}{\sqrt{2}\pi N_1}\frac{V_1}{f}$$

$$\phi_m \propto \frac{V_1}{f}$$




As applied voltage V is sinusoidal so $\phi_m$ also sinusoidal

Let $\phi_m = \phi_{max} \sin\omega t$

If magnetising circuit is linear

i.e. $\phi_m \propto i_m$

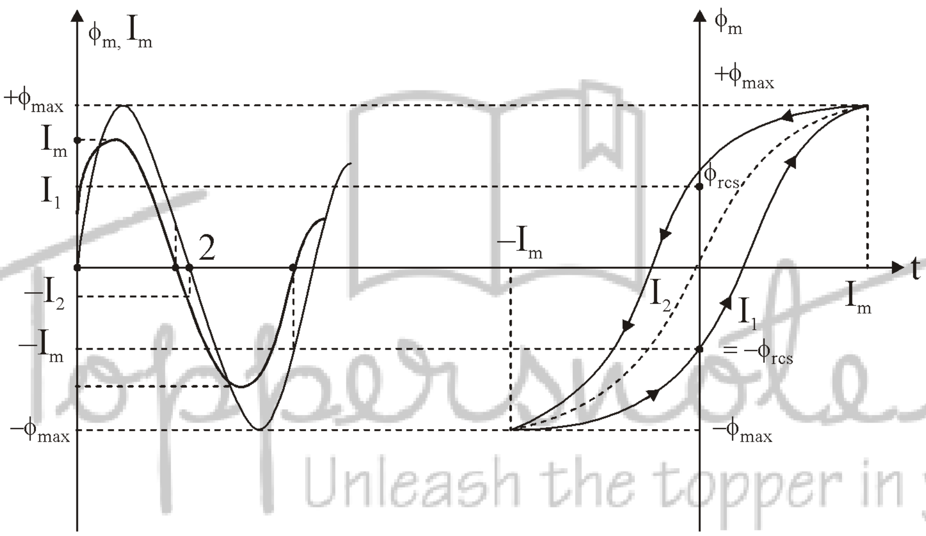
$\phi_m = K\, i_m$ where K is constant

$\phi_m = \phi_{max} \sin\omega t$

$$i_m = \frac{\phi_{max}}{K} \sin \omega t$$

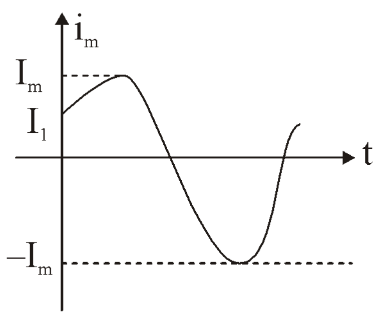
$\Rightarrow$ $i_m$ also sinusoidal

If magnetic circuit is not linear i.e. hysteresis and saturation






The magnetic flux is sinusoidal but the magnetising current is not sinusoidal due to hysteresis and saturation

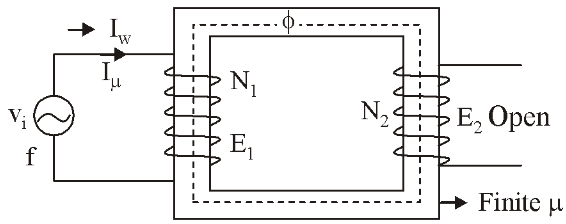



$i_m$ contains harmonics

According to the Fourier series $3^{rd}$ harmonic is the strongest i.e. upto 40% of fundamental component

## 1.11 Operation of Transformer with Iron Losses

Due to Iron losses in transformer core, the transformer draws an additional component of current from source which is called Iron loss component of current.

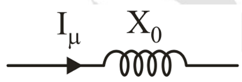



$I_w$ = Iron loss component of current. (It is drawn from the source to supply the power required for iron losses).

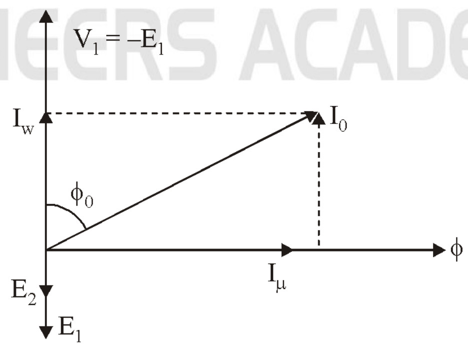
$$\bar{I}_0 = \bar{I}_\mu + \bar{I}_w \rightarrow \text{No load current of transformer.}$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2}$$

**Difference Between Reactive Component and Active Component**

| $I_\mu$ | $I_w$ |
|---|---|
| 1. Reactive component of current (or) wattless component of current. | 1. Active component of current (or) wattful component of current. |
| 2. It is always quadrature with applied voltage. | 2. It is always in phase with applied voltage. |
| 3. Its magnitude is about 4 to 5% of full load current. | 3. Its magnitude is about 1 to 2% of full load current. |
| $(I_\mu >> I_w)$ ∴ $I_0$ is approximately 5% of full load current | |
| 4. Representing parameter for $I_\mu$ is inductor, since $I_\mu$ is quadrature with applied voltage.   | 4. Representing parameter for $I_w$ is resistor, since $I_w$ is inphase with applied voltage.   |

**Vector Diagram of a Transformer under NO Load Condition**




$\phi_0$ = No load phase angle of transformer and the range is 70° to 75°.
$\text{Cos}\phi_0$ = No load power factor of transformer and the range is 0.2 to 0.25 lag.

Transformer has poor no load power factor of order 0.2 to 0.25 lag, because its magnatising component of current is very high when compared with Iron loss component of current. $(I_\mu >> I_w)$.

By keeping voltage constant if frequency of operation is reduced during operation of transformer then :

1. $I_\mu$ current increases due to deep saturation of transformer core.
2. Due to increased $I_\mu$ current, no load power factor of transformer decreases.

**Polar form of no-load Current**

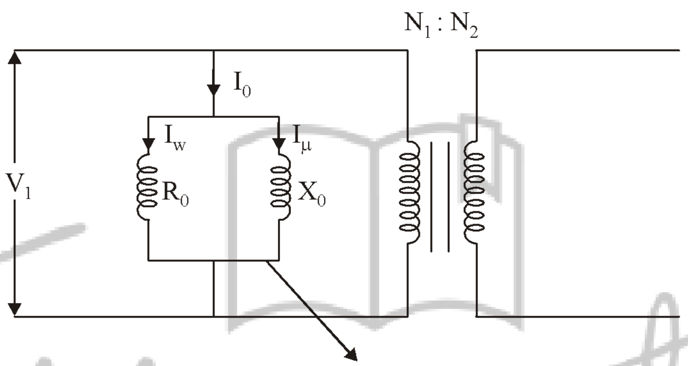
$$I_0 \angle -\phi_0$$
$$I_\mu = I_0\cos\phi_0$$
$$\text{No-load power} = V_1 I_0 \cos\phi_0 = V_1 I_w = \text{Iron loss}$$

No load primary copper loss = $I_0^2 R_1$. This loss is very low because $I_0$ is 5 to 6% of full load current.

$\therefore$ No load power $\approx$ Iron loss

Power consumed by transformer under no load condition is approximately equal to iron loss in transformer (However small amount of no load primary copper loss takes place inaddition to iron loss if winding resistance of primary is considered.)

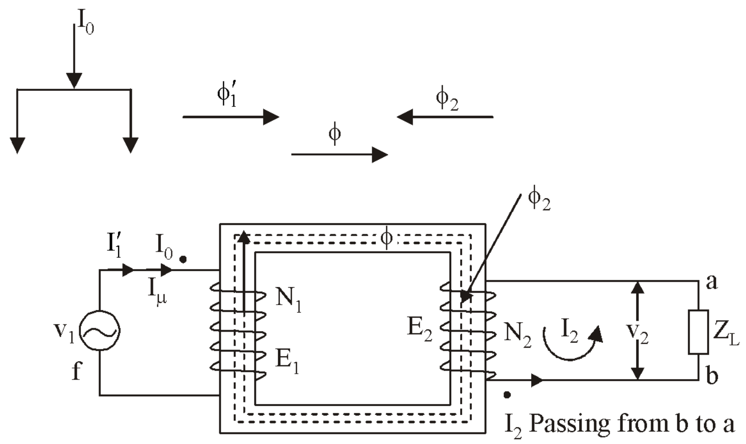
## 1.12 Equivalent Circuit of Transformer Under No-Load Condition




$$R_0 = \frac{V_1}{I_w}; \quad X_0 = \frac{V_1}{I_\mu}$$

$R_0$ and $X_0$ should be in parallel, and should be in primary circuit. This parallel branch called no-load branch or shunt branch.

**Operation of Transformer Under Load Condition**



- $N_1I_\mu$ = Primary mmf and the corresponding flux due to this mmf is $\phi$ (working flux or main flux).
- Whenever transformer is loaded, load current will flow through secondary circuit and the direction of secondary load current can be found by using Lenz's law. According to lenz's law, direction of secondary current $I_2$ should be such that secondary mmf opposites to the change in mutual flux in the core.

The process is as follows, $V_2$ = Secondary terminal voltage.

$I_2$ = Secondary load current.

$N_2I_2$ = Load component of secondary mmf.

$\phi_2$ = Load component of secondary flux.

Here $\phi_2$ is opposes the changes in mainfield flux($\phi$) to satisfy Lenz's Law.

In this process current in primary winding will be increased and additional current $I_1'$ in primary is called load component of primary current.

Total primary current under loaded condition, $\bar{I}_1 = \bar{I}_0 + \bar{I}_1'$.

$N_1I_1'$ = Load component of primary mmf.

$\phi_1'$ = Load component of primary flux.

To satisfy transformer action, $\phi_1'$ is used to nullify $\phi_2$ so that resultant load component of flux in core is zero and therefore flux in the core is main flux only even under load condition. That means amount of flux in transformer core is always maintained constant irrespective of load across its secondary terminals, hence transformer can be treated as constant flux device.

**Power Transformer Condition**

$$\phi_1' = \phi_2$$

$$\Rightarrow \quad N_1I_1' = N_2I_2$$

Load component of primary AT = load component of secondary AT

$$I_2' = \frac{N_2}{N_1}I_2 = KI_2$$

(The total primary current is the phasor sum of $I_2'$ and $I_0$)

So $$I_1 = I_2' + I_0$$

Where $$I_2' = -KI_2 \text{ } (I_2' \text{ is } 180° \text{ out of phase with } I_2)$$

$$\frac{N_2}{N_1} = \frac{I_1'}{I_2}$$

But $$\frac{N_2}{N_1} = \frac{E_2}{E_1}$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \frac{I_1'}{I_2}$$

$$E_1I_1' = E_2I_2$$

Load component of primary VA = load component of secondary VA.

**Approximate by,**

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_1'$$

If $I_0$ is neglected,

$$\Rightarrow \qquad \bar{I}_1 = \bar{I}_1'$$

$$\Rightarrow \qquad N_1 I_1 \approx N_2 I_2$$

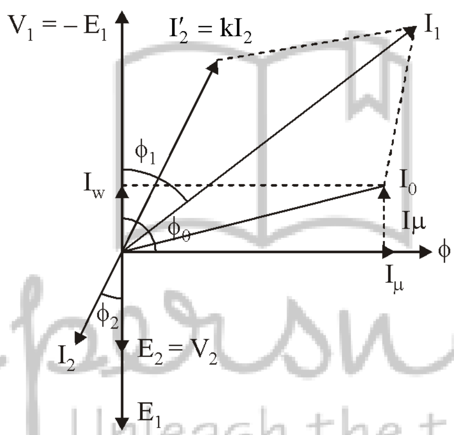
$$K \approx \frac{I_1}{I_2}$$

But
$$K = \frac{E_2}{E_1}$$

$$\Rightarrow \qquad \frac{E_2}{E_1} \approx \frac{I_1}{I_2}$$

$$E_2 I_2 \approx E_1 I_1$$

**Vector Diagram for Lagging Power Factor Load**






$\cos\phi_1$ = Primary power factor

$\cos\phi_0$ = No load power factor

$\cos\phi_2$ = load power factor

Here $\qquad \phi_1 > \phi_2$

Then $\qquad \cos\phi_1 < \cos\phi_2$

$E_2 = V_2$, since winding resistance is zero and this is voltage equation without voltage drop.

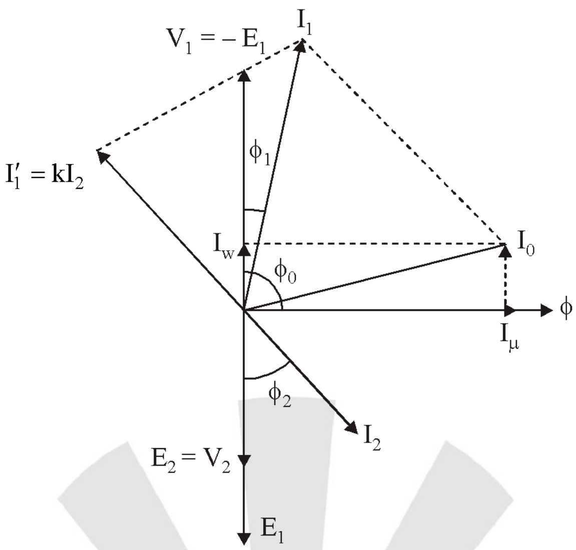
Similarly $\qquad V_1 = -E_1$

The phase angle of $I_2$ w.r.t. $V_2$ depends on the nature of load.

**Vector Diagram for Leading Power Factor Load**

Here $\qquad \phi_1 < \phi_2$

Then $\qquad \cos\phi_1 > \cos\phi_2$.



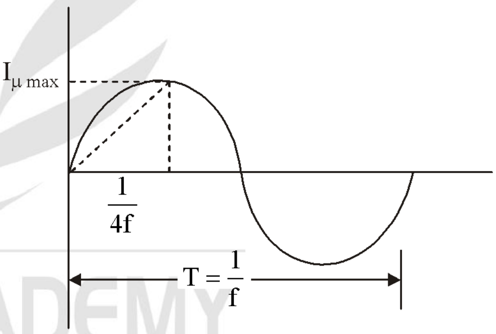

When p.f. of lagging load on secondary side increases, then primary side p.f will increases.

When p.f. of leading load on secondary side increases, then primary p.f. decreases as well as increases depending up on the load power factor.

**Example 6 :** A 5 kVA, 50 Hz single phase transformer has ratio 200/400V. The data taken on the LV side at rated voltage shows the open circuited wattage as 100W. The mutual inductance between the primary and secondary windings is 2H. Neglect winding resistance and leakage reactance. What value will be the current taken by the transformer, if the no load test conducted on H.V side?

(a) 0.4A (b) 0.318A (c) 0.25A (d) 0.296A

***Solution:*** Given data, 5 kVA, 50Hz.




$$I_0 = \sqrt{I_\mu^2 + I_w^2}$$

$$\text{Iron loss } V_1 I_w = 100\text{W}$$

$$I_w = \frac{100}{200} = 0.5\text{A}$$

Secondary induced emf

$$e_2 = M\frac{di_\mu}{dt}$$

$$400 = 2 \times \frac{I_{\mu m} - 0}{\frac{1}{4}f}$$

$$400 = 2 \times I_{\mu m} \times 4 \times 50$$

$$I_{\mu m} = 1\text{A}$$

$$I_{\mu(rms)} = \frac{I_{\mu\,max}}{\sqrt{2}} = 0.707\text{A}$$

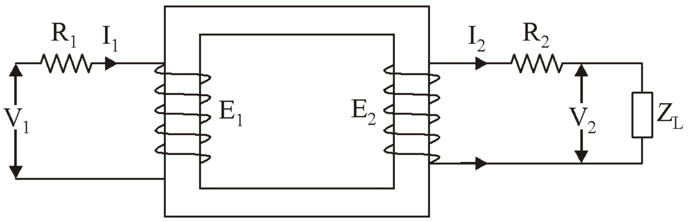
$$\therefore \quad I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{(0.707)^2 + (0.5)^2} = 0.86\text{A}$$

No load current towards H.V side

$$I_{0HV} = \frac{I_0(LV)}{K} = \frac{0.86}{2} = 0.43A$$

## 1.13 Effect of Winding Resistance on the Operation of Transformer

The Equivalent circuit of transformer with winding resistance is as follows :




Where $R_1$ = Winding resistance of primary

$R_2$ = Winding resistance of secondary

No load current $I_0$ is neglected here.

Because of winding resistance, IR drop and $I^2R$ drops are exists in transformer winding. Therefore the voltage equations are :

$$V_1 = -E_1 + I_1R_1$$

$$E_2 = V_2 + I_2R_2$$

$$\text{Total copper losses in transformer} = I_1^2R_1 + I_2^2R_2$$

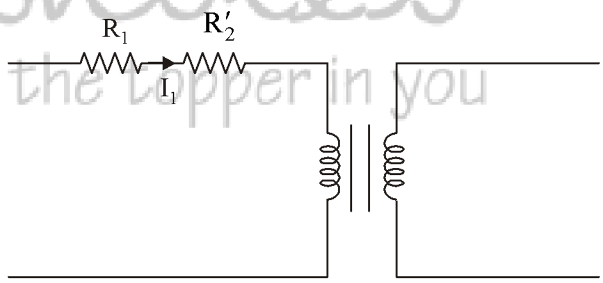
The condition that must be satisfied while transferring winding resistance from one side to another side is copper loss of that resistance should be maintained constant so that the performance of transformer should not be affected.

**Transfer Resistance from Secondary Side to Primary Side**

$$I_2^2R_2 = I_1^2R_2'$$

$$R_2' = R_2\left(\frac{I_2}{I_1}\right)^2$$

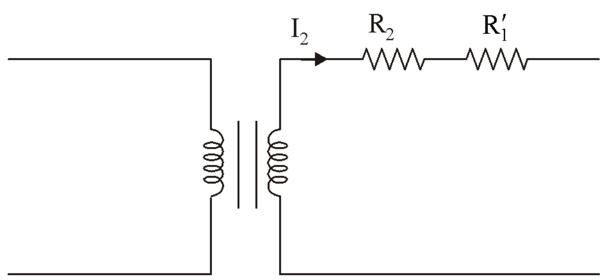
$$R_2' = \frac{R_2}{K^2}$$




**Transfer Resistance from Primary Side to Secondary Side**

$$I_1^2R_1 = I_2^2R_1'$$

$$R_1' = R_1K^2$$




Total resistance of transformer w.r.t. primary side

$$R_{01} = R_1 + R_2'$$

Total resistance of transformer w.r.t secondary side

$$R_{02} = R_2 + R_1'$$

Total Cu-loss w.r.t primary side = $I_1^2R_{01}$

Total Cu-loss w.r.t secondary side = $I_2^2R_{02}$

Total Cu-loss of transformer = $I_2^2R_1 + I_2^2R_2 = I_2^2R_{02} = I_1^2R_{01}$

**Per Unit Resistance Drop**

$$\text{Per unit primary resistance drop} = \frac{\text{Resistance drop in primary}}{\text{Induced emf (base voltage)}} = \frac{I_1R_1}{E_1}$$

$$\text{Similarly p.u secondary resistance drop} = \frac{I_2R_2}{E_2}$$

$$\text{Total p.u resistance w.r.t primary side} = \frac{I_1R_{01}}{E_1}$$

$$\text{Total p.u resistance w.r.t secondary side} = \frac{I_2R_{02}}{E_2}$$

P.u resistance drop w.r.t primary = p.u resistance drop w.r.t secondary.

In P.U system P.U unit resistance drop is also simply called as P.U resistance of the transformer.

$$\text{\% resistance of transformer w.r.t primary} = \frac{I_1R_{01}}{E_1} \times 100.$$

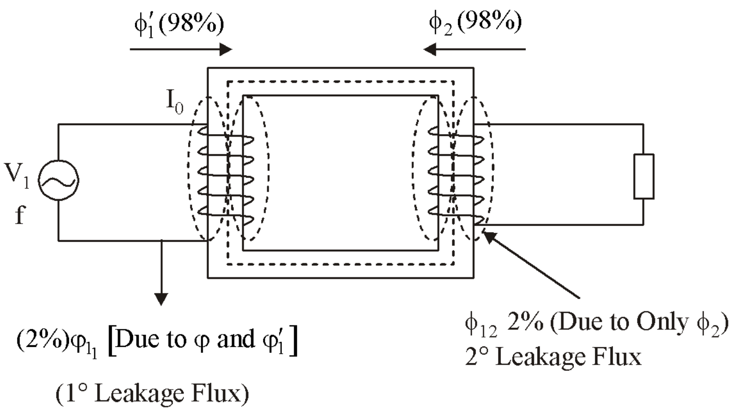
$$\text{\% resistance of transformer w.r.t secondary} = \frac{I_2R_{02}}{E_2} \times 100.$$

% R of transformer w.r.t primary = % R of transformer w.r.t secondary.

As % resistance on both sides of transformer is same, it is easy to transfer % resistance from one side to another side when compared to resistance in ohmic values.

Hence, the transformer resistance is expressed in p.u values rather than ohmic values.

## 1.14 Effect of Magnetic Leakage Flux on Operation of Transformer



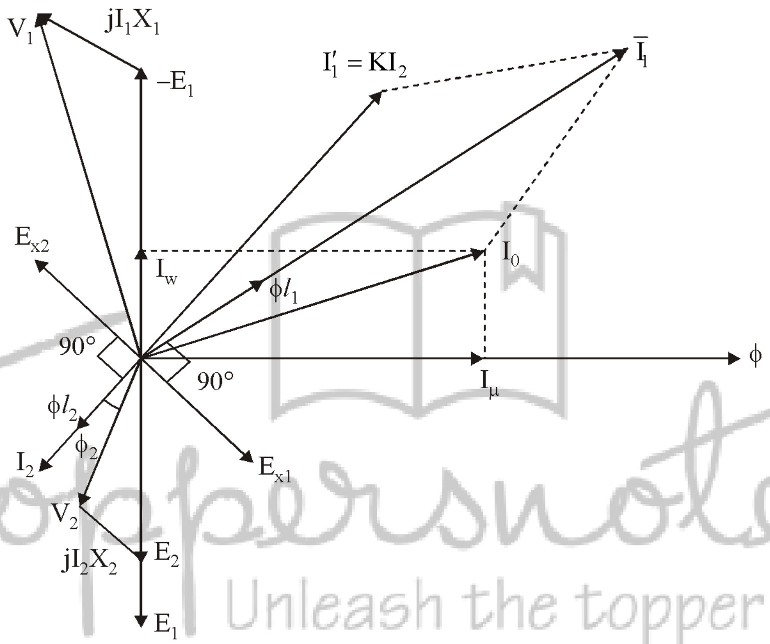
- Magnetic leakage flux is the part of the load component flux which links either HV winding or LV winding, but not both and it is not having any role in transferring the power from one circuit to another circuit.
- More the magnetic leakage flux in transformer, lesser will be the power transfer capability of transformer.
- Magnetic leakage flux at primary and secondary increases with increase in load current in the windings. That means magnetic leakage flux depends on load current but is independent on applied voltage. Whereas main field flux depends on applied voltage but is independent on load current.

Magnetic leakage fluxes available at primary and secondary are always in phase with respective currents in the windings.

Whenever leakage flux considered at primary and secondary, some additional emf's are induced in both windings ($E_{x1}$ and $E_{x2}$) which are lag behind the respective currents by exactly 90°.

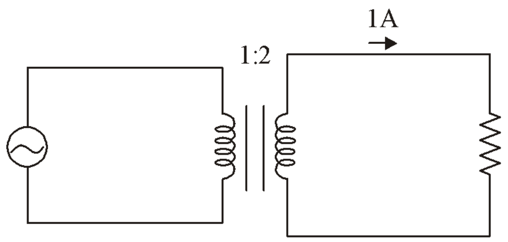
$E_{x1}$, $E_{x2} \rightarrow$ lags $\phi_{l1}$, $\phi_{l2}$ respectively by 90° due to Lenz's law.

**Vector Diagram**






**Example 7 :** A single-phase transformer has a turns ratio of 1:2, and is connected to a purely resistive load as shown in the figure. The magnetizing current drawn is 1A, and the secondary current is 1A. If core losses and leakage reactance are neglected, the primary current is :




(a) 1.41A (b) 2A (c) 2.24A (d) 3A

***Solution***

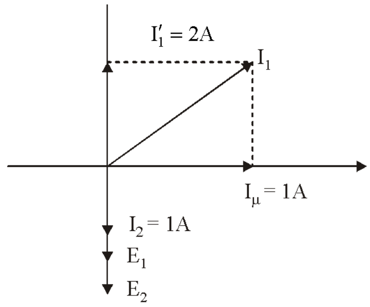
$$\frac{I_1'}{I_2} = \frac{N_2}{N_1} = 2$$

$$I_1' = 2I_2 = 2A.$$

From phase or diagram

$$I_1 = \sqrt{(I_1')^2 + (I_\mu)^2 + 2I_1'I_\mu \cos 90}$$

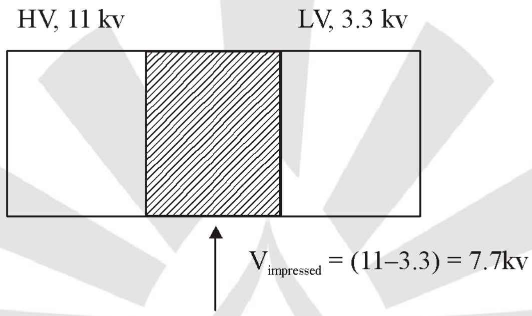
$$= \sqrt{1^2 + 2^2} = 2.24A$$



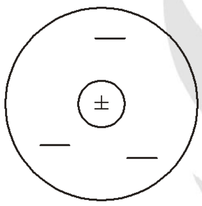

## 1.15 Dielectric Losses

Dielectric Loss normally takes place in insulating materials of transformer such as in solid and liquid insulations.

In insulating material, no free electrons available, however voltage is applied, a small amount of current exists due to the conversion of all the atoms into electric dipoles because of which there is displacement of charges and the corresponding current is called displacement current.



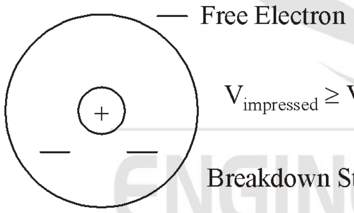

**Atomic Behaviour of Insulating Material**

1. 

$V_{impressed} = 0$

Neutral State

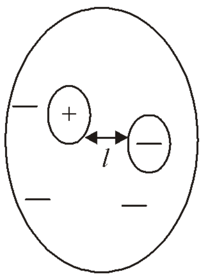
As outermost electrons are symmetrically distributed and effective (–ve) charge coinsides with (+ve) charge.

2. 


$V_{impressed} \geq V_{breakdown}$

Breakdown Stage

If applied voltage is more than breakdown voltage, free electrons are available in the dielectric. the voltage required to the outermost electron from its parent atom is called Breakdown voltage (or) dielectric strength of insulating material.

3. 


$0 < V_{impressed} < V_{breakdown}$

Polarized State

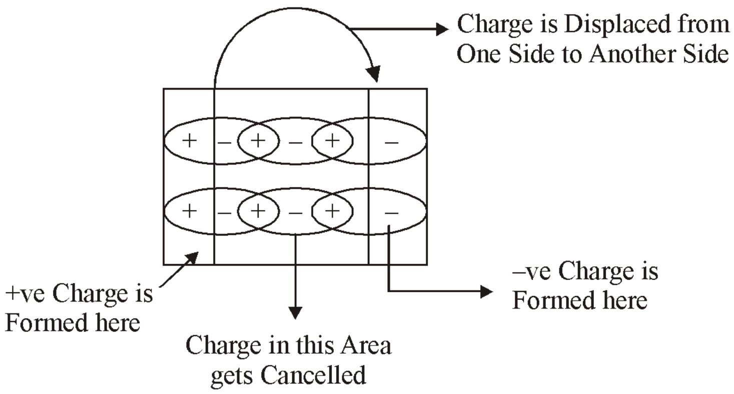
This voltage is sufficient to disturb the symmetry of electrons distribution, but no free electrons are available.

Effective (–ve) charge does not coinside with the (+ve) charge and these two charges are displaced by a length *l*.Thus it results in the formation of a electric dipole.

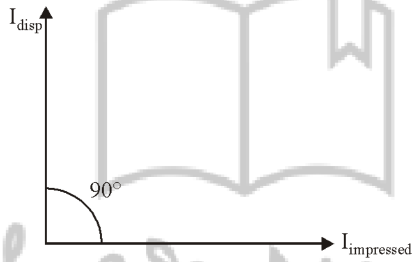
The process of conversion of all the atoms into electric dipoles is called polarization and dielectric is said to be in polarized state.

Due to polarization, the charge displacement takes place in insulating material. This current is called displacement current.

**Displacement Current**




**Pure Dielectric Material**

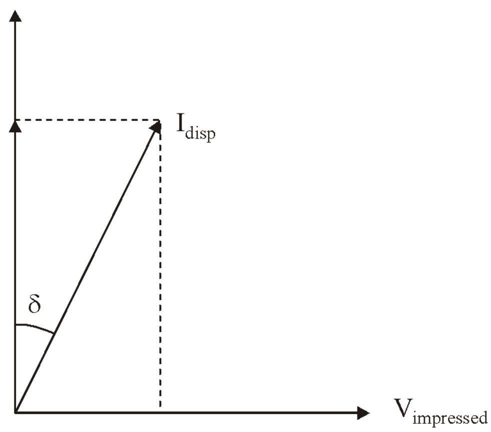



- In pure dielectric, there is no dielectric loss as component of I displace along $V_{impressed}$ is 0°.

**Practical Dielectric Material**

δ = loss angle of dielectric material

tanδ = dissipation factor, which indicates the quality of dielectric material.




tanδ = 0, then it indicates the perfectness of insulating material.

tanδ test → This test is conducted regularly to know the quality of insulating material.

$$\text{Dielectric loss} = V_{impressed} \times I_{disp} \sin\delta.$$

$$\text{Dielectric loss} \propto \sin\delta$$

Dielectric loss depends on applied voltage but it is independent on load current. Hence this loss can be treated as constant loss.

Dielectric loss is approx 0.25% of full load output.

**Example 8 :** A Transformer has hysteresis loss of 30 watts at 240V, 60Hz supply. the Hysteresis loss at 200V, 50Hz supply is :

(a) 30W (b) 25W (c) 60W (d) 100W

***Solution :*** $V_1 = 240V$, $f_1 = 60Hz$ and $V_2 = 200V$, $f_2 = 50Hz$

$$\frac{V_1}{f_1} = \frac{V_2}{f_2} = 4$$

$$\frac{v}{f} = \text{constant}$$

$$w_h \propto f$$

$$\frac{w_{h2}}{w_{h1}} = \frac{f_2}{f_1}$$

$$w_{h2} = \frac{50}{60} \times 30 = 25W$$

**Operational and Design Difference Between Power and distribution Transformers.**

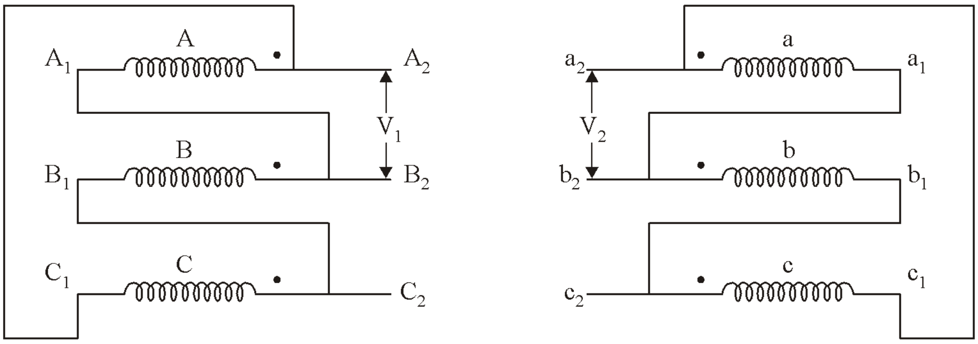
| Power Transformer | Distribution Transformer |
|---|---|
| 1. Available in transmission network voltage level > 33kv. | 1. Available in distribution network of power system. voltage level < 33kv. |
| 2. Consumers are not directly connected to power transformer. | 2. The distribution transformers are directly connected to consumers. |
| 3. Load fluctuations are less | 3. Load fluctuations are more. |
| 4. Steadily loaded at full load through out 24 hrs. | 4. Loaded based on load cycle of Consumers. |
| 5. Cu-losses and Iron losses takes place steadily through out 24 hrs. | 5. Cu-losses take place based on load cycle of Consumer and Iron losses takes place throughout 24 hrs. |
| 6. Copper losses are kept minimum while designing. | 6. Iron losses are kept minimum while designing. |
| 7. Specific weight is more. | 7. Specific weight is less. |
| 8. Full load copper loss ≈ iron loss. | 8. Full load copper loss = 2 × iron loss. |

## 1.16 3-ϕ Transformer Connections

1. Delta-Delta (Δ – Δ)
2. Star-Star (Y – Y)
3. Delta-Star (Δ – Y)
4. Star-Delta (Y – Δ)

**Δ – Δ Connection**

(a) Identification of vector group : Identifying the phase displacement from primary to secondary.



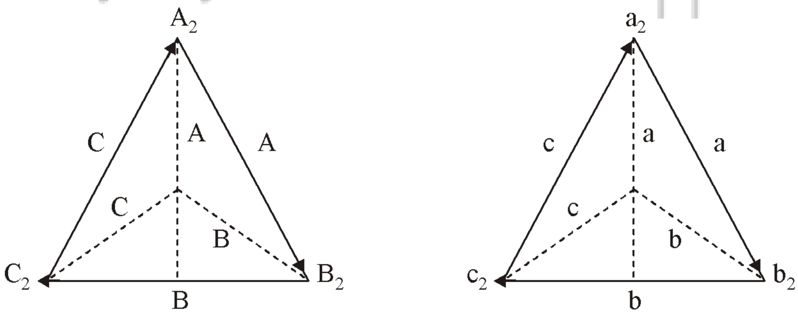

Before making terminal connection, conduct polarity test on windings for getting polarities. According to Bureau of Indian standards $A_2$, $B_2$, $C_2$ are dotted terminals and $A_1$, $B_1$, $C_1$ undotted terminals.

According to transformer action the displacement for dot to dot winding is zero. The displacement for dot to undoted winding is 180°.

**Rules to Draw Vector Diagrams**

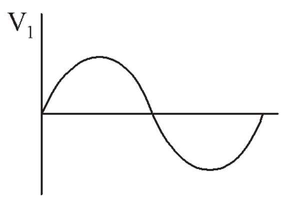
- The primary side vector diagram should be drawn in any manner with particular phase sequence.
- The secondary side vector should be drawn parallel to primary side vectors such that the phase sequence on both sides is same.
- If out coming terminals on both sides is similar, then secondary side vector should be drawn in same direction to primary.
- If out coming terminal on both sides is dissimilar, the secondary side vector should be drawn in opposite direction.



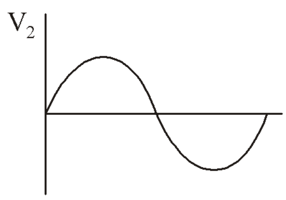


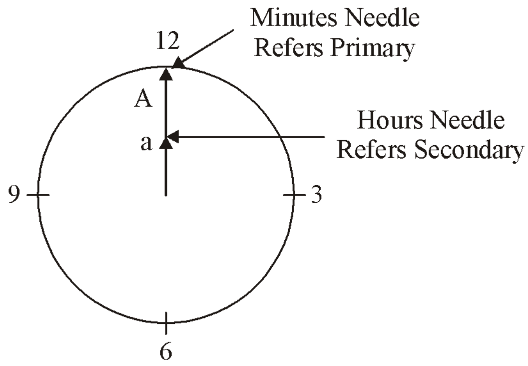

- If primary side phase sequence is ABC, then secondary side is also abc.

**Primary Side Voltage**



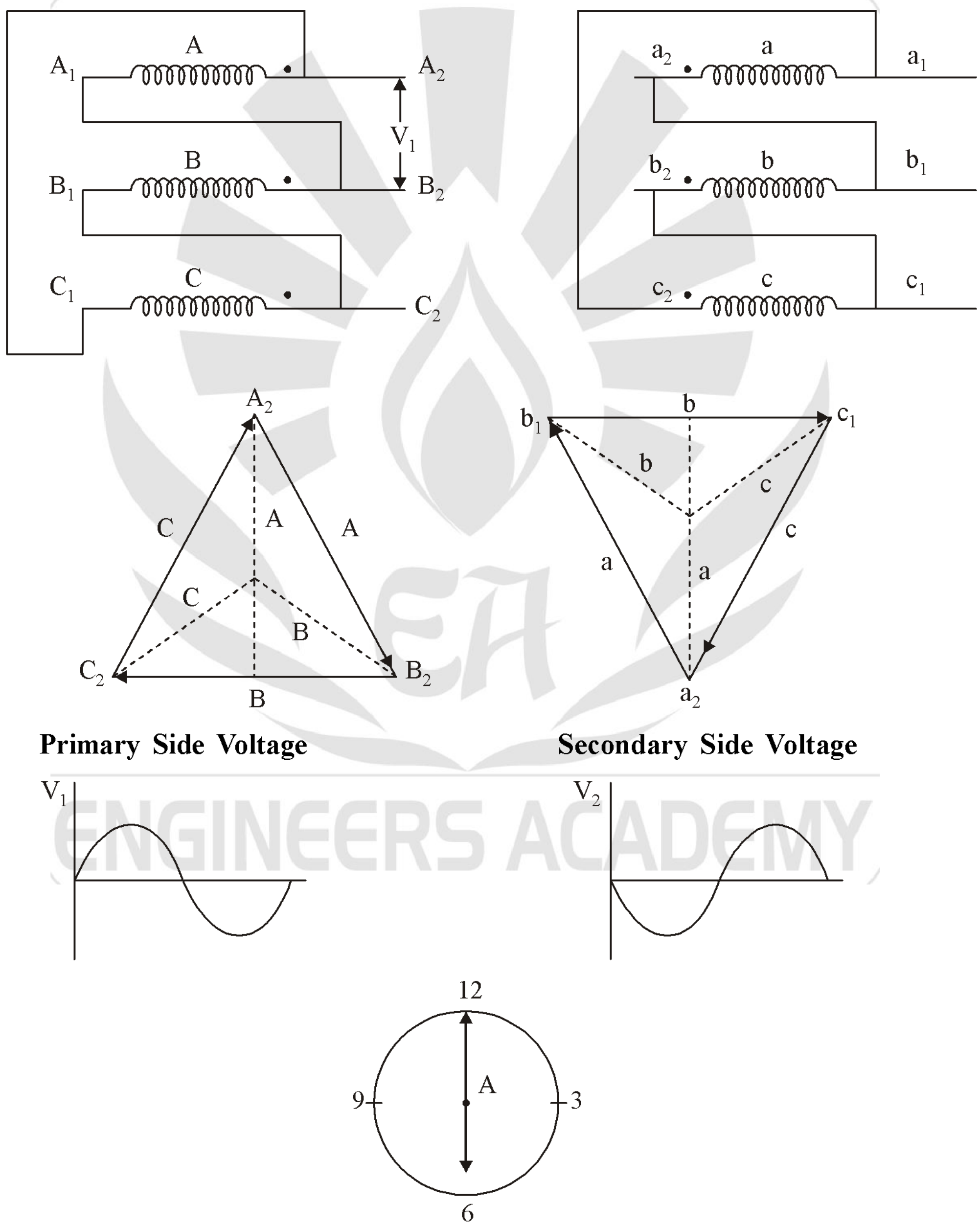

**Secondary Side Voltage**






This connection is called as Dd12 (or) Dd0

(b)

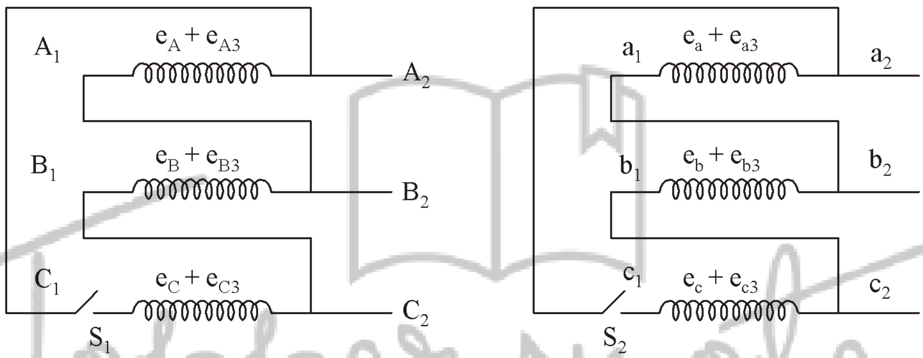



This connection is called Dd6 connection.

## 1.17 Features of Δ-Δ Connection

1. If x : 1 is phase turns ratio then line turns ratio is x : 1.
2. As phase voltage is equal to line voltage in Δ/Δ, this connection requires more no of turns per phase when compared to Y/Y of same voltage rating.
3. As $V_{ph} = V_L$ in Δ/Δ, it requires more amount of insulation as compared to Y/Y of same voltage rating.
4. As $I_{ph} = \frac{I_L}{\sqrt{3}}$ in Δ/Δ it requires 57.7% of cross section area of conductor when compared to Y/Y of same current rating.
5. Y/Y → suitable for High voltage and low current rating transformer i.e. small KVA rating.
   Δ/Δ → Suitable for Low voltage and high current rating transformer i.e. large KVA rating.
6. Both sides Δ connection offer closed path for the flow of 3rd harmonic currents, therefore the shape of emf in this connection is always sinusoidal irrespective of type of core.

**Δ-Δ Transformer with Different Switching Operations**




**If switches both $s_1$ $s_2$ are open :**

(i) Shape of $I_\mu$ on primary side is → Sinusoidal.

(ii) Shape of $I_\mu$ on secondary side → No magnetizing current in secondary.

(iii) Shape of flux in Transformer core → Flat topped wave with 3rd harmonic flux.

(iv) Shape of emf in winding → Peak wave with 3rd harmonic emf.

**If switch $s_1$ is closed and $s_2$ is open :**

(i) Shape of $I_\mu$ on primary side is → Peak waveform with 3td harmonic current.

(ii) Shape of $I_\mu$ on secondary side → No magnetizing current in secondary.

(iii) Shape of flux in Transformer core → Sinusoidal.

(iv) Shape of emf in winding → Sinusoidal.

**If switch $s_1$ is opened and $s_2$ is closed**

(i) Shape of $I_\mu$ on primary side is → Sinusoidal.

(ii) Shape of $I_\mu$ on secondary side → Sinusoidal with frequency 3f. (only 3rd harmonic).

(iii) Shape of flux in Transformer core → Sinusoidal.

(iv) Shape of emf in winding → Sinusoidal.

**If switch $s_1$ is closed and $s_2$ is closed :**

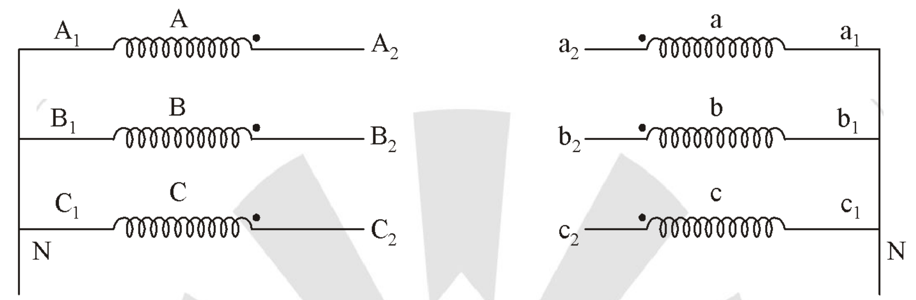
(i) Shape of $I_\mu$ on primary side is → Peak waveform with 3rd harmonic current.

(ii) Shape of $I_\mu$ on secondary side → sinusoidal with frequency 3f. (only 3rd harmonic).

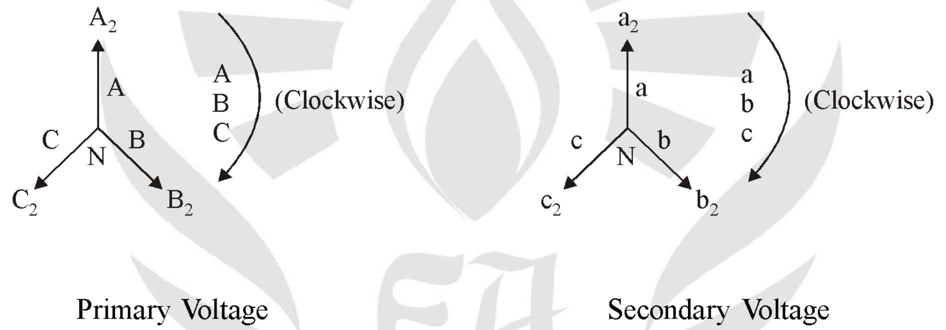
(iii) Shape of flux in Transformer core → Sinusoidal.

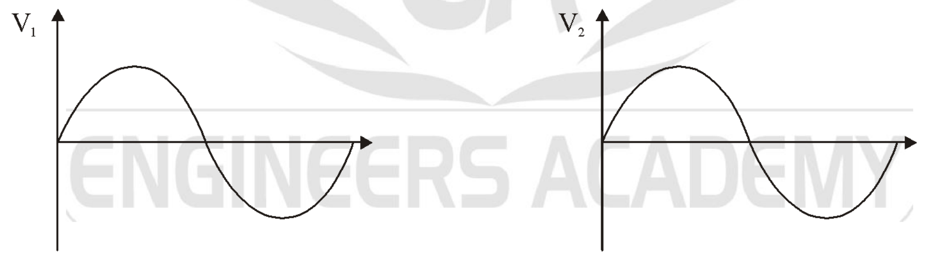
(iv) Shape of emf in winding → Sinusoidal.

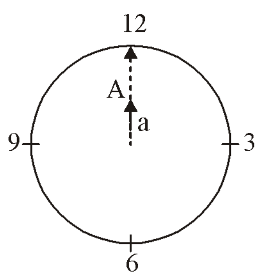
## 1.18 Y – Y- TRANSFORMER CONNECTION




Outcoming terminals on both sides with dot, so the displacement angle is zero and draw secondary vectors parallel to primary side vectors in same direction.

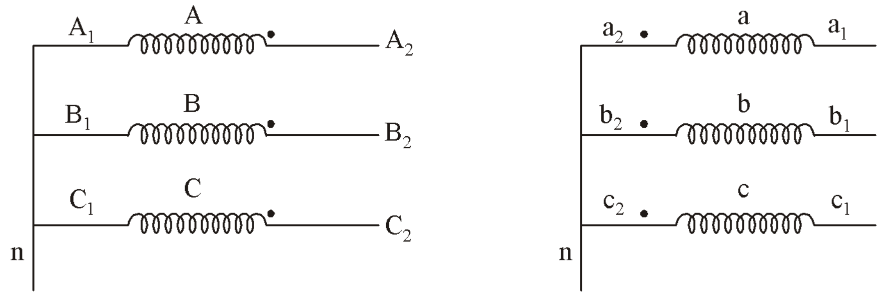


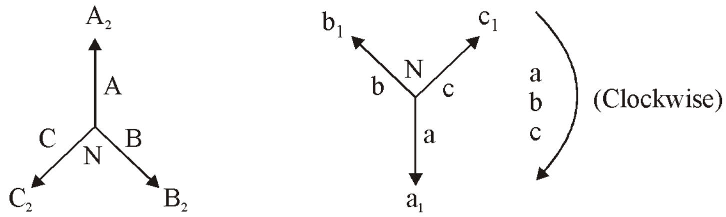



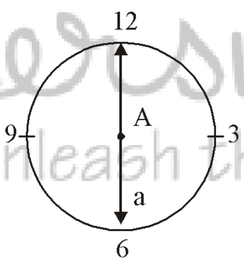

This connection is also known as Y y0 or Y y12.

If out coming terminals of secondary are changed :




**Primary Voltage** **Secondary Voltage**

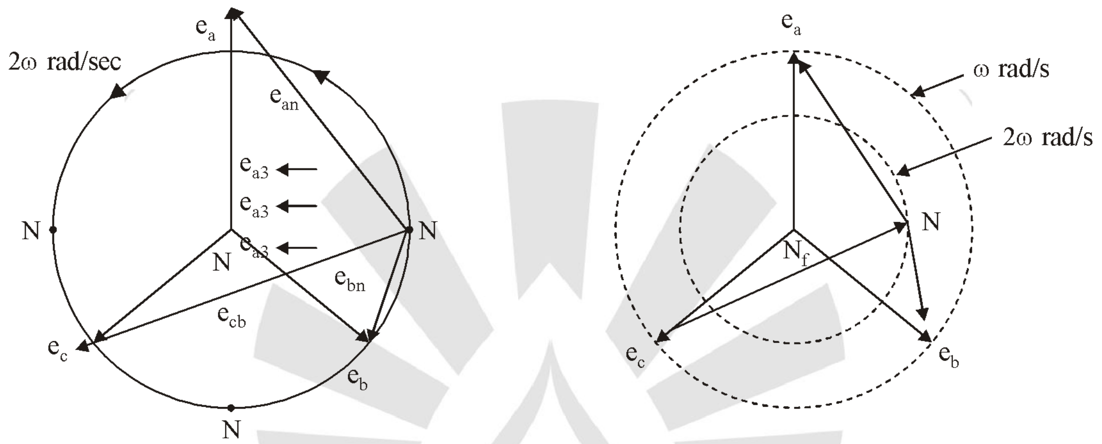

This connection offers 180° phase displacement and this connection is known as Y y6.

**Features of Y/Y Connection**

1. In this transformer, line turns ratio is equal phase turns ratio.
2. Y/Y requires only 57.7% of total no of turns per phase and 57.7% of total amount of insulation when compared to Δ/Δ of same voltage rating.
3. Y/Y requires more cross section area of conductor when compared to Δ/Δ of same current rating.
4. Due to above reasons Y/Y connection is economical for high voltage, small KVA rating Transformer's.
5. Y/Y transformer connection doesn't provide any closed path within the phases for the flow of $3^{rd}$ harmonic current. Therefore the shape of emf in this connection is sinusoidal only when 3 limbed core type core is employed. On the other hand shape of emf in the connection is non sinusoidal if 5 limbed shell type core is employed.

**Drawbacks of Y/Y Transformer with 5 Limbed Shell Type Core or 3-ϕ Bank Construction**

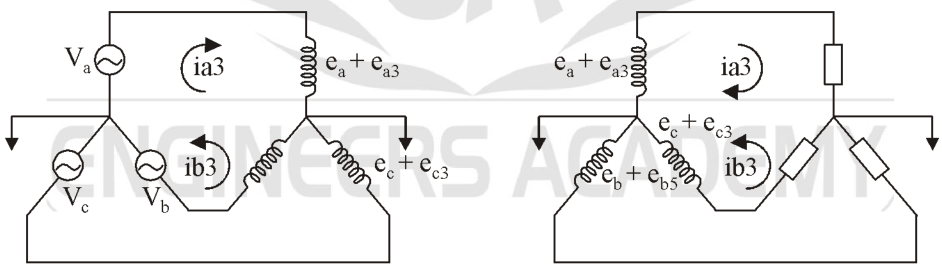
1. Shape of emf in Y/Y transformer is non-sinusoidal.
2. As each phase contains 3rd harmonic emf's in addition to fundamental, the neutral point is shift from its original (fundamental) position and produce unbalanced phase voltages.
3. Speed of rotation of 3rd harmonic is 3ω, but fundamental emf frequency is 'ω'. Therefore the relative speed between these two vectors is 2ω rad/sec, so the neutral point of Y is not only shifted but also rotates at a speed of 2ω rad/sec. This type of neutral in Y/Y transformer is called oscillating neutral (or) floating point neutral.




4. Due to oscillation of neutral point, the different phase voltages are not maintained constant and the transformer is not able to supply single-phase loads. However, it is able to supply 3-ϕ loads since Line voltages are free from 3rd harmonic emfs as the 2 phases between the lines are connected in series subtractive polarity and 3rd harmonic voltages get cancelled.

The above drawbacks in Y/Y transformer can be eliminated by providing closed path within the phases for the flow of 3rd harmonic currents by using following techniques :

**1. Neutral Grounding :** In this method path for the flow of 3rd harmonic currents can be provided by grounding neutrals are transformer along with source and load neutrals.




The main drawback of neutral grounding method is 3rd harmonic impedance offered by closed path for the flow of 3rd harmonic currents in this method is very high so that 3rd harmonic currents are weak and hence the drawbacks of Y/Y transformer cannot be eliminated completely.

**2. Tertiary Winding Method :** In order to eliminated the drawbacks of Y-Y transformer completely, the system should be provided low 3rd harmonic impedance are for the flow of 3rd harmonic currents.

This can be practically achieved by introducing a 3rd winding on same transformer core which is called as tertiary winding.

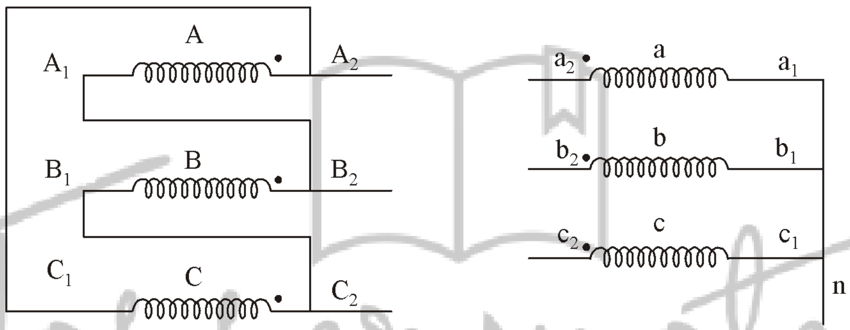
By using Δ connected tertiary winding 3rd harmonic impedance for flow of 3rd harmonic current can be provided and all the drawbacks can be completely eliminated.

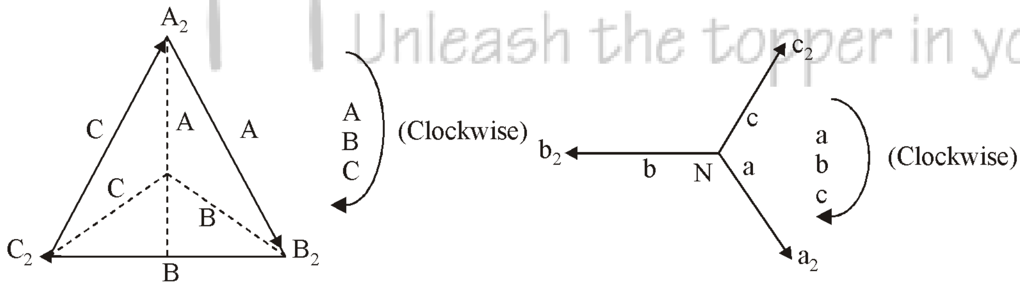
This can be practically achieved by introducing a 3rd winding on same transformer core which is called as tertiary winding.

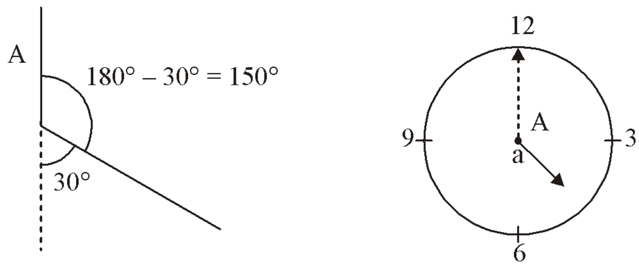
Functions of Δ- connected tertiary winding in Y/Y transformer with 5 limbed shell type core :

1. It makes the shape of emf as sinusoidal in Y/Y transformer by eliminating 3rd harmonic emf's from individual phases.
2. It reduces voltage unbalance in Y/Y transformer by shifting back the neutral point to the fundamental location.
3. It stabilizes the neutral point in Y/Y transformer that's why this winding is also called stabilizing winding.
4. It helps the Y-Y transformer to supply single phase loads also.
5. It provides path for zero sequence currents under unbalanced load condition and helps the Y-Y transformer to supply unbalanced loads also along with balanced loads.
6. It helps the earth fault relay to activate promptly under earth fault condition to trip the Transformer in time.

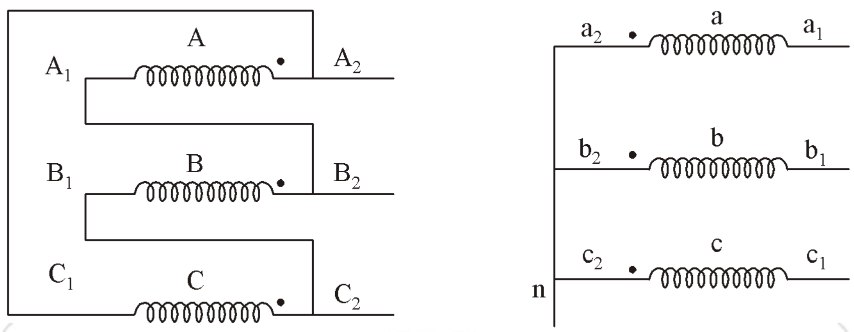
## 1.19 Δ/Y Transformer Connection

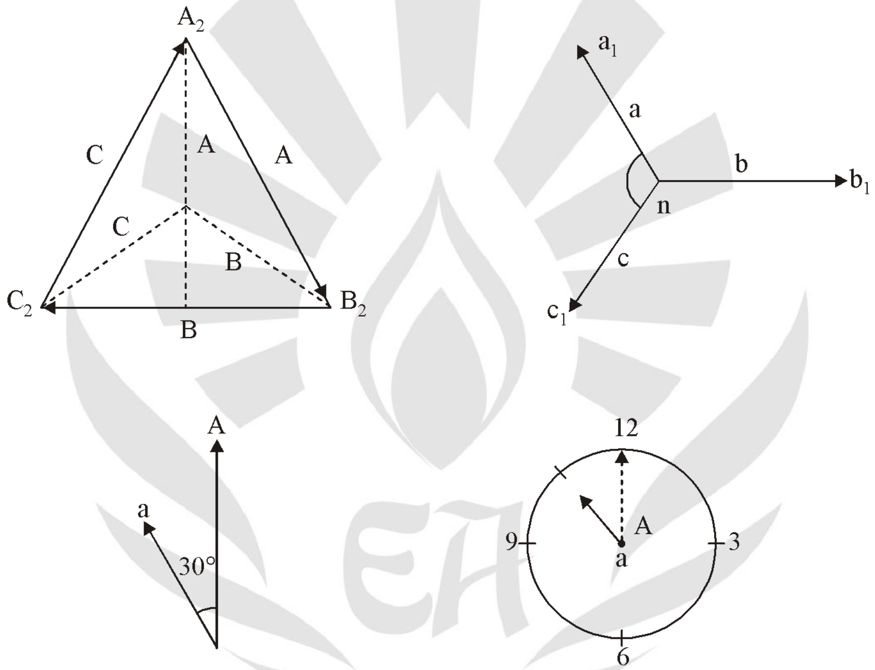


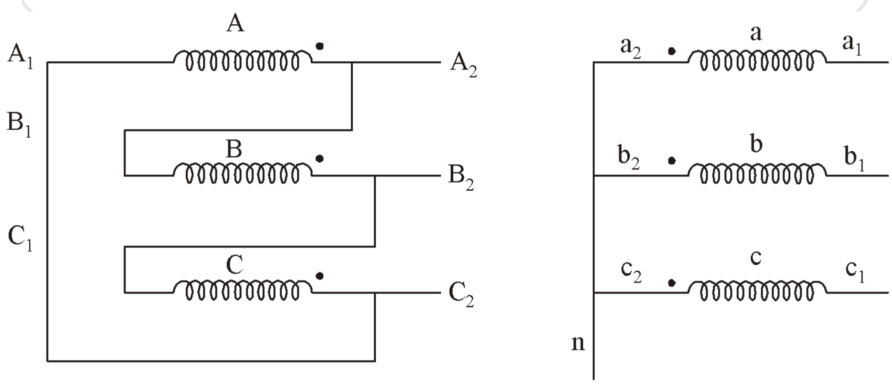
This connection offers 150° phase displacement between primary and secondary and this connection is also called Dy5 connection.

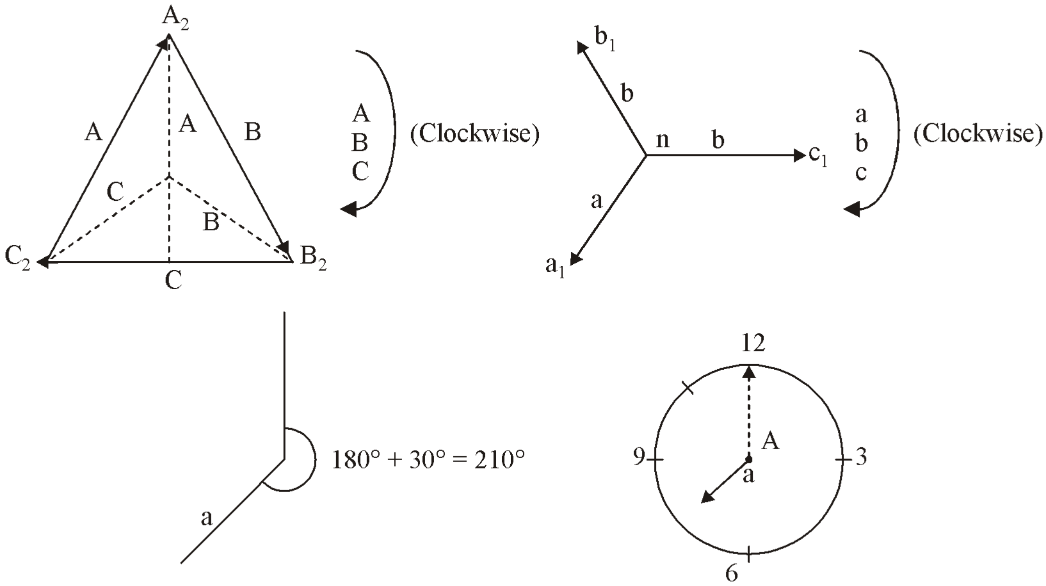


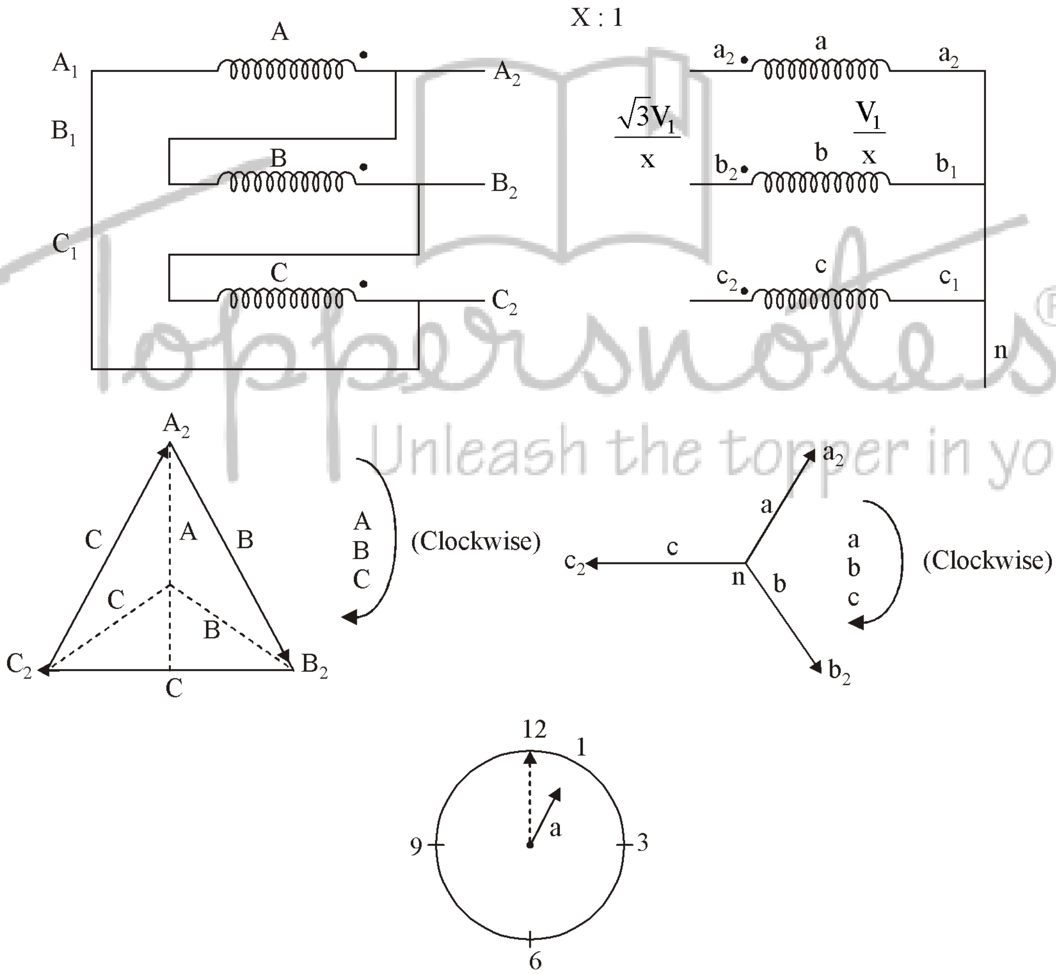

This connection offers +30° phase displacement between primary and secondary and this connection is also called Dy11 connection.








This connection offers 210° phase displacement between primary and secondary and this connection is also called Dy7 connection.






This connection offers –30° phase displacement between primary and secondary and this connection is also called Dy1 connection.

**Comparison Between Different Transformer Connections**

If transformation ratio of transformer is x : 1

| Connections | Line Voltages | Secondary Terminal Voltage |
|---|---|---|
| Δ - Δ | $V_1 : \frac{V_1}{x}$ | 100% |
| Y – Y | $V_1 : \frac{V_1}{x}$ | 100% |
| Δ - Y | $V_1 : \sqrt{3}\frac{V_1}{x}$ | 173% |
| Y - Δ | $V_1 : \frac{V_1}{\sqrt{3}x}$ | 57% |

## 1.20 Features of Δ/Y Connection

1. In this transformer if phase turns ratio is x : 1 the line turns ratio is x : $\sqrt{3}$ .
2. This Δ/Y transformer connection offers highest secondary terminal voltage (73% more) among all the Transformer connections for the same applied voltage and turns ratio.
3. For same voltage rating, Δ-Y requires less no of turns per phase among all the transformer connections, so that this transformer is most economical when compared to other transformer connections.
4. This connection is more economical for step up application.
5. This transformer connection is generally employed at beginning of the transmission line as a step up transformer.
6. As primary side Δ provide closed path for $3^{rd}$ harmonic current, $3^{rd}$ harmonic emf's in individual Y phases on secondary side are absent so that neutral point on secondary side is maintained stable and the transformer can able to supply 1-ϕ and 3-ϕ loads perfectly.
7. Primary side Delta also provides path for Zero Sequence currents under unbalanced load condition, thereby the Transformer can supply unbalanced loads satisfactorily.
8. That's why Δ/Y transformer connection is most ideal choice for distribution applications.

**Example 9 :** A 6.6 kV/415V delta/ star connected distribution transformer has % resistance of 2% and % reactance of 6% :

(i) The voltage regulation of transformer at full load 0.8pf lead is :

(ii) The voltage to be applied on primary side of transformer to maintain the secondary voltage at 415 V with the above load condition is :

***Solution:*** 6.6KV/415V    % R = 2%    % x = 6%    cosϕ = 0.8    and    sinϕ = 0.6

(i) Regulation = % R cosϕ + % x sin ϕ

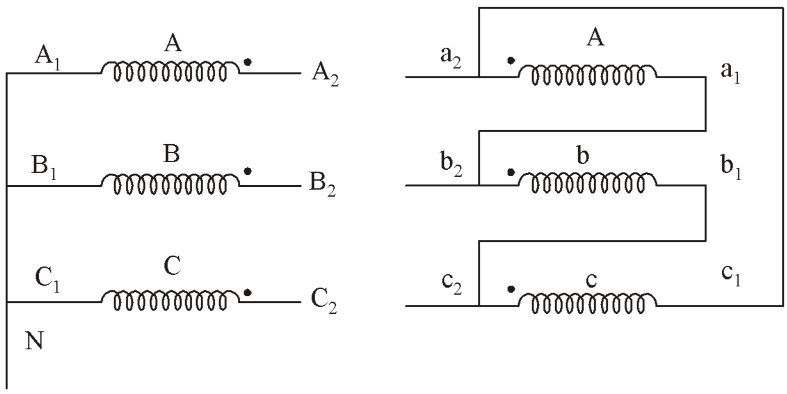
= 2 × 0.8 + 6 × 0.6 = 5.2%.

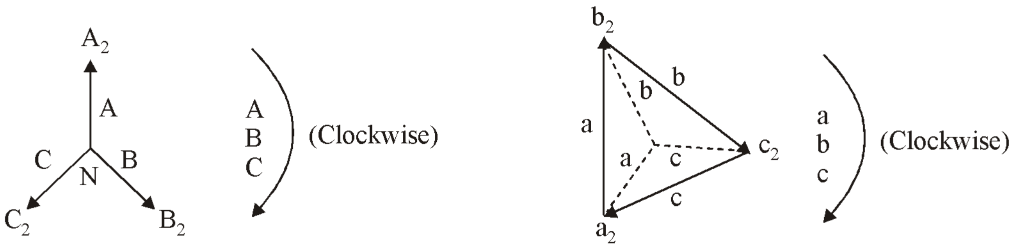
(ii) Voltage $d_{NP}$ = 5.2% of 4.5 = 21.58 V

21.58V + 415 V = 436.58 V

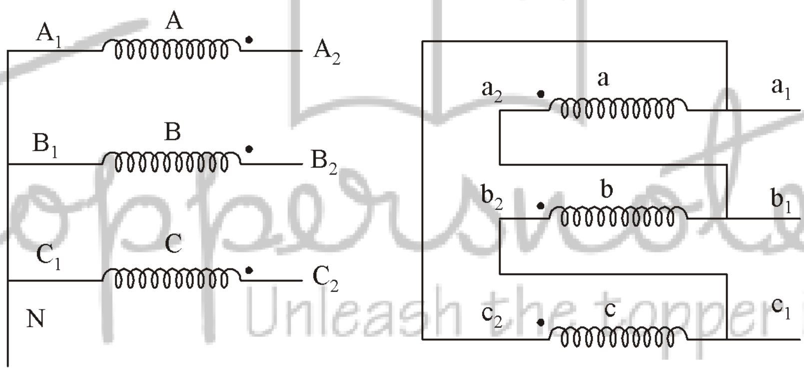
The voltage to be applied is $= \frac{436.5 \times 6.6 \times 1000}{415} = 6943.2V$

**Y-Δ Transformer Connection**

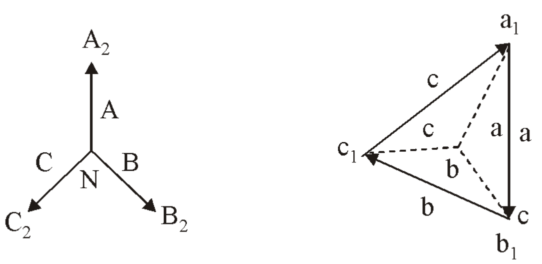


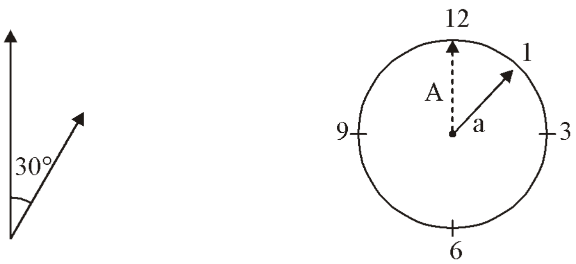




This connection offers 210° phase displacement between primary and secondary and this connection is also called Yd7 connection

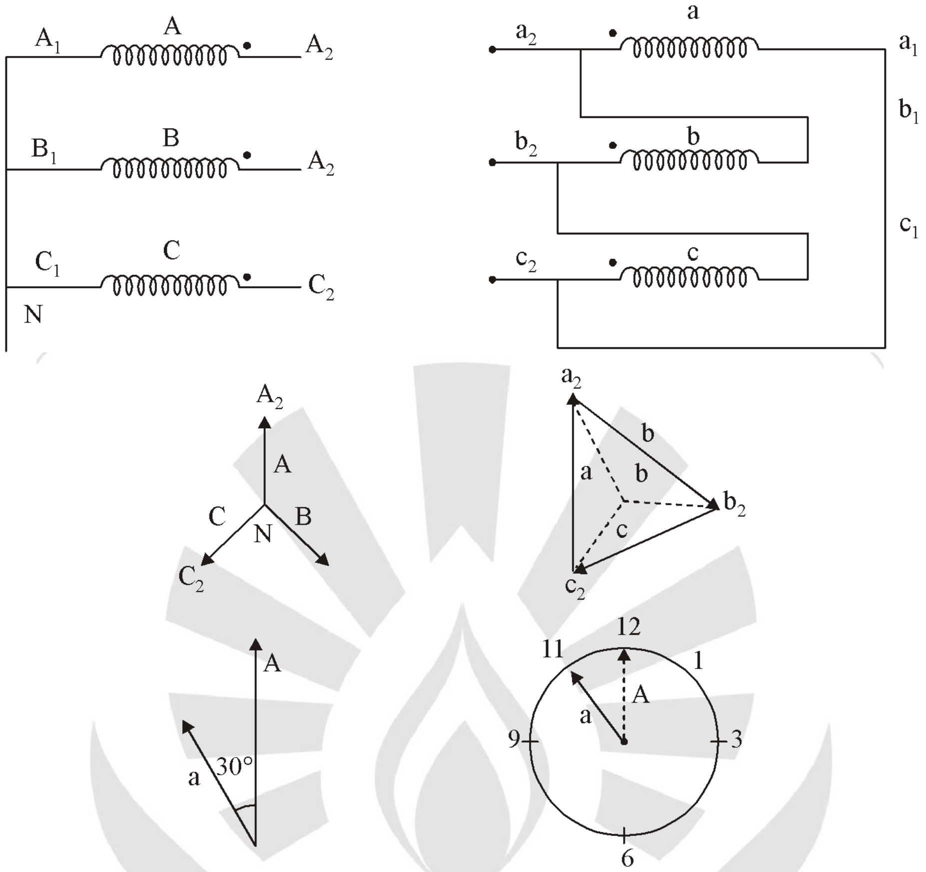




This connection offers –30° phase displacement between primary and secondary and this connection is also called Yd1 connection.




This connection offers +30° phase displacement between primary and secondary and this connection is also called Yd11 connection.

## 1.21 Features of Δ/D Connection

1. In this transformer connection if the phase turns ratio is x : 1, line turns ratio is $x:\frac{1}{\sqrt{3}}$.
2. This transformer connection produce least secondary terminal voltage among all the transformer connections for same applied voltage and turns ratio.
3. This transformer connection produces 42.3% less terminal voltage when compared to Δ/Δ (or) Y/Y connections.
4. This transformer connection requires highest no of turns per phase on both among all transformer connections for same voltage rating. Hence, it is most uneconomical transformer connection among all transformer connections.
5. This transformer connection is economical for Y winding on HV side and Δ winding on LV side. That means it is economical to use this transformer connection for step down applications.
6. This transformer is not suitable for distribution application since neutral is not available on secondary side.
7. In this transformer, secondary side Δ provides closed path for $3^{rd}$ harmonic currents. Therefore the shape of emf is sinusoidal. Y/Δ transformer with switches $S_1$ and $S_2$.

**If switches $S_1$ and $S_2$ are open :**

1. Shape of $I_\mu$ on primary side is sinusoidal.
2. Secondary side $I_\mu$ is zero.
3. The shape flux in transformer core is Trapezoidal.
4. The shape of emf is peak wave.

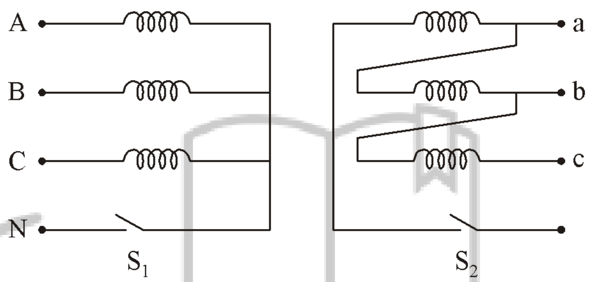
**If switch $S_1$ is open and $S_2$ is closed :**

1. Shape of $I_\mu$ on secondary side is sinewave with frequency 3f.
2. The shape flux in transformer core is sinusoidal. Therefore emf is also sinusoidal.

**If switches $S_1$ and $S_2$ are closed :**

1. Shape of $I_\mu$ on primary side is peak (Fundamental + $3^{rd}$ harmonic).
2. Secondary side $I_\mu$ is only sinusoidal third harmonic.

**Common Data for Examples 10 & 11.**




The star-delta transformer shown above is excited on the star side with balanced, 4-wire, 3-phase, sinusoidal voltage supply of rated magnitude. The transformer is under no load condition.

**Example 10 :** With both $S_1$ and $S_2$ open, the core flux waveform will be :

(a) A sinusoid at fundamental Frequency

(b) Flat-topped with third harmonic

(c) Peaky with third-harmonic

(d) None of these

***Solution***

For sinusoidal excitation, the flux is a flat topped wave with $3^{rd}$ harmonic. As $S_1$ & $S_2$ both are opened there is no closed path for the circulation of $3^{rd}$ harmonic currents. So no compensating flux is produced for $3^{rd}$ harmonic flux. Hence flux remains as flat topped wave.

**Example 11 :** With $S_2$ closed and $S_1$ open, the current waveform in the delta winding will be :

(a) A sinusoid at fundamental frequency

(b) Flat-topped with third harmonic

(c) Only third-harmonic

(d) None of these

***Solution:*** With $S_2$ closed, there is a closed path available for the $3^{rd}$ harmonic currents with in the phases. As the transformer is under no load condition only $3^{rd}$ harmonic currents will be flowing in the delta connected secondary.

## 1.22 Special 3-$\phi$ Transformer Connections

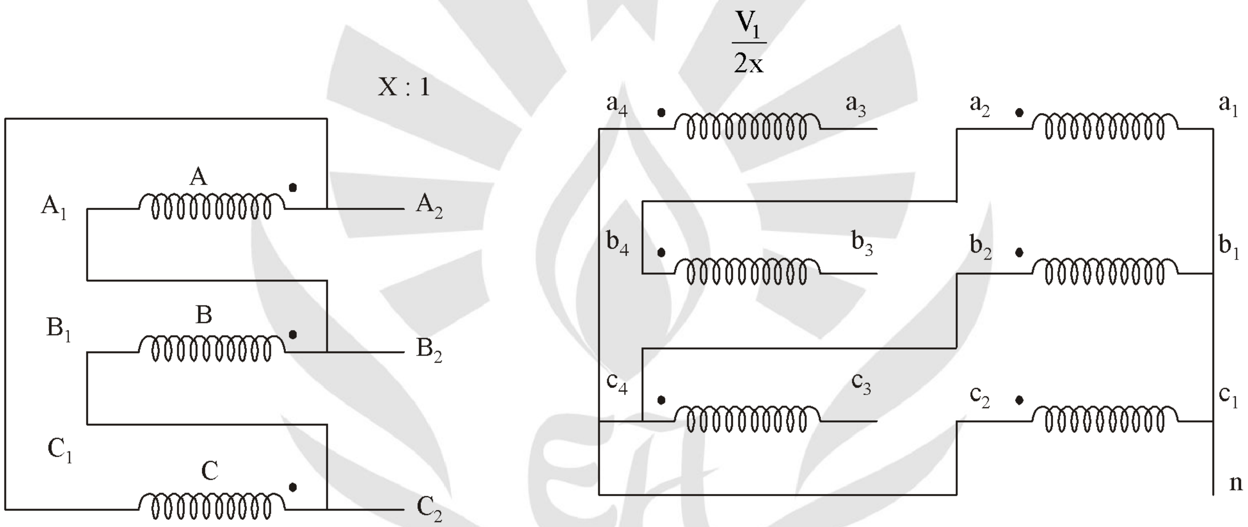
1. $\Delta$ - Zig zag Y
2. Y - Zig zag Y
3. V - V connection (open delta)
4. Scott connection or T - T connection.

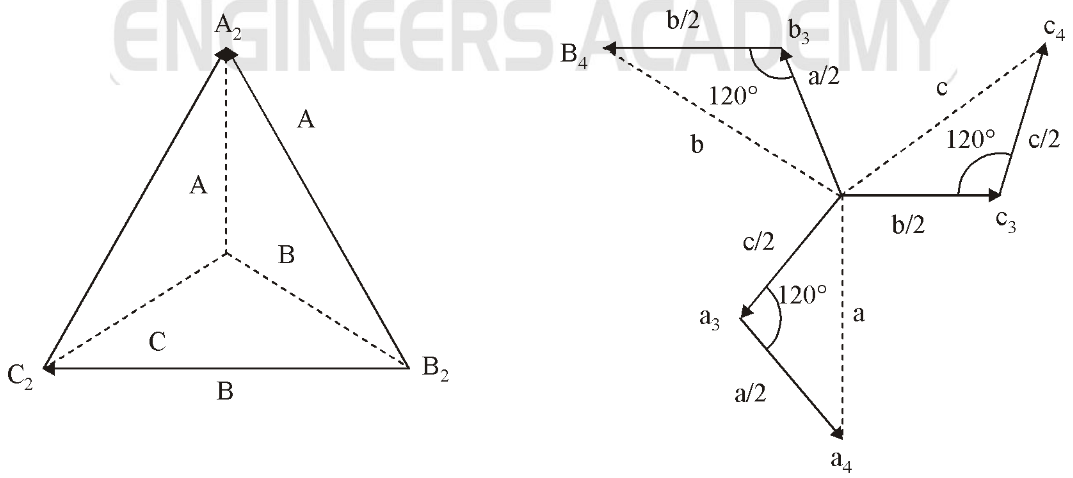
### *1.22.1 Δ - Zig Zag Y*

It can be used in place of D/Y transformer to get exactly stable neutral point.

The purpose of zig zag star is to nullify the 3$^{rd}$ harmonic emf in individual phases so that the phase voltage is sinusoidal wave.

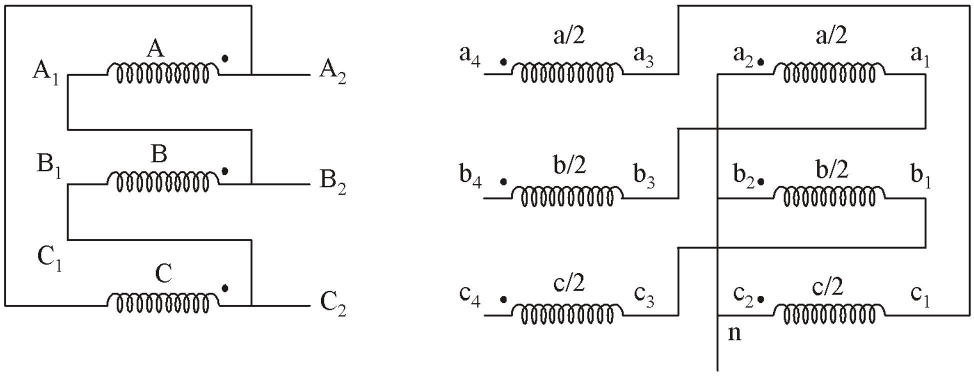
In zig zag Y, each phase is divided into two equal parts. The 3$^{rd}$ harmonic emf's are nullified due to series subtractive polarity connection of different half phases.

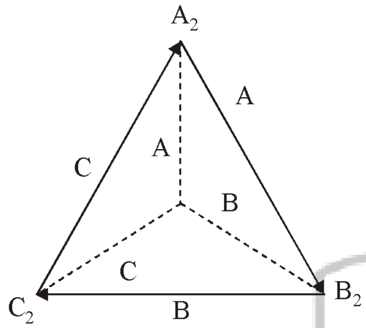


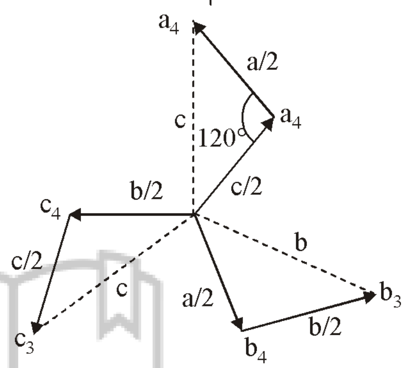



This connection offers 180 degree phase displacement between primary and secondary and this connection is also called Dz6.

For different end connections of zig zag star:




This connection offers zero degree phase displacement between primary and secondary and this connection is also called Dz12.

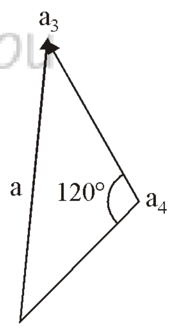
If x : 1 is phase turns ratio of transformer :

If line to line voltage applied on primary is $V_1$, then the line to line voltage on secondary side is

The phase voltage across secondary half winding (say a/2) is $\frac{V_1}{2x}$,

$$V_{a3n}^2 = \left(\frac{V_1}{2x}\right)^2 + \left(\frac{V_1}{2x}\right)^2 + 2\frac{V_1}{2x}\frac{V_1}{2x}\cos 60°$$

$$= \left(\frac{V_1}{2x}\right)^2 + \left(\frac{V_1}{2x}\right)^2 + 2\left(\frac{V_1}{2x}\right)^2 \frac{1}{2}$$




$$V_{a3n} = \frac{\sqrt{3}V_1}{2x} \cdot \text{(Phase Voltage)}$$

$$V_{a3b3} = \sqrt{3}\left(\frac{\sqrt{3}V_1}{2x}\right) \text{ (line to line voltage)}$$

$$= \frac{3V_1}{2x}$$

23.2% less terminal voltage will be obtained when compared to Δ/Y transformer connection.

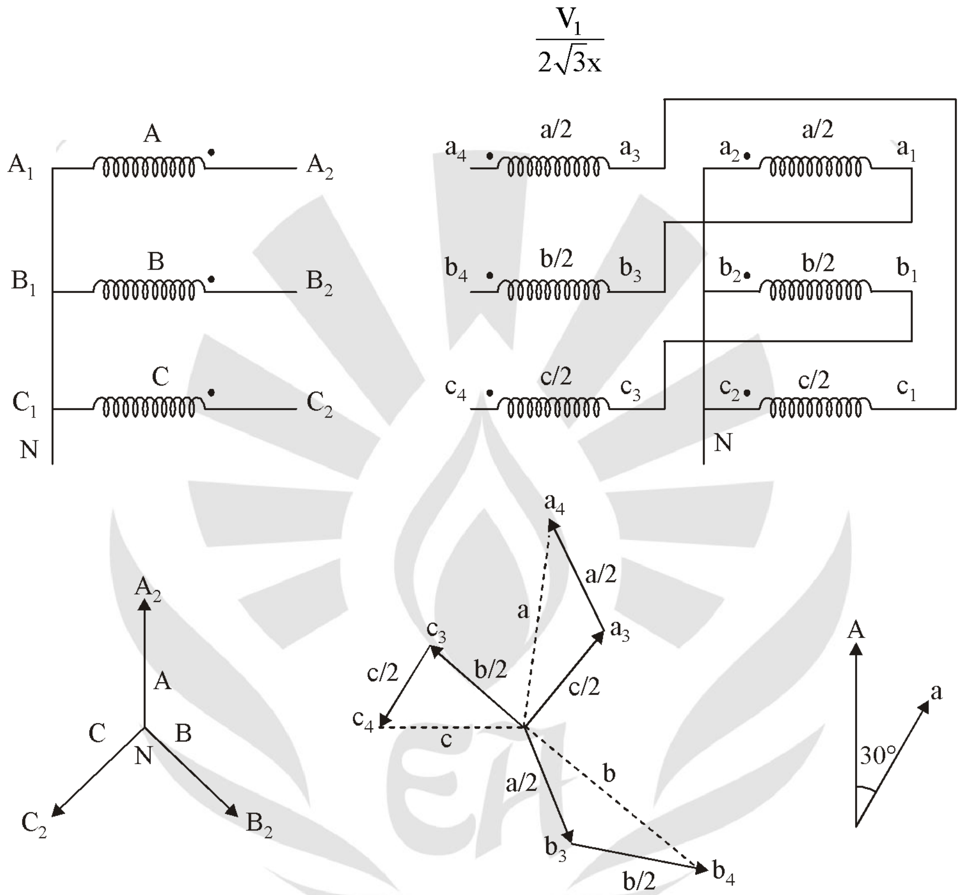
The line turns ratio is $\quad V_1 : \frac{3V_1}{2x} = x : \frac{3}{2} = x : 1.5$

In Δ-zig zag Y, if phase turns ratio is x : 1, the line turns ratio is $x : \frac{3}{2}$.

This Δ-zig zag Y connection produces 23.2% less line voltage when compare to Δ-Y connection.

In zig zag Y-connection as different half phases are connected in series subtractive polarity, the 3rd harmonic emf's in individual phases are nullified and neutral point of the zig zag Y exactly maintained stable.

### *1.22.1 Y-Zig Zag Y Connection*




This connection offers –30° phase displacement between primary and secondary and this connection is also called Yz1.

It indicates that Y-zig zag Y produces displacement of ± 30° and 180° ± 30°.



In Y-zig zag Y, if the phase turns ratio x : 1, the line turns ratio is $x : \frac{\sqrt{3}}{2}$.

This Y-zig zag Y connection produces 13.4% less secondary terminal voltage when compare to Y-Y.

For same voltage rating Y-zig zag requires 9% additional turns per phase when compared to Y/Y.

In zig zag Y as windings 2 different half sections are connected in series subtractive polarity, the resultant 3rd harmonic emf between any terminals to neutral point becomes zero there by 3rd harmonic emf's from individual phases can be eliminated.

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