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ENGINEERS ACADEMY

RPSC - A.En.

Assistant Engineering

ELECTRICAL

Rajasthan Public Service Commission (RPSC)

Volume - 1

Network Theory

SIERRA INNOVATIONS

BASICS OF CIRCUIT ELEMENT AND CIRCUIT LAW

THEORY

1.1 Circuit Element

Circuit element is a part of an electric circuit or network.

One of the three quantitative altributives (resistance, inductance, capacitance) characteristic of an electric circuit.

1.2 Classification of Elements

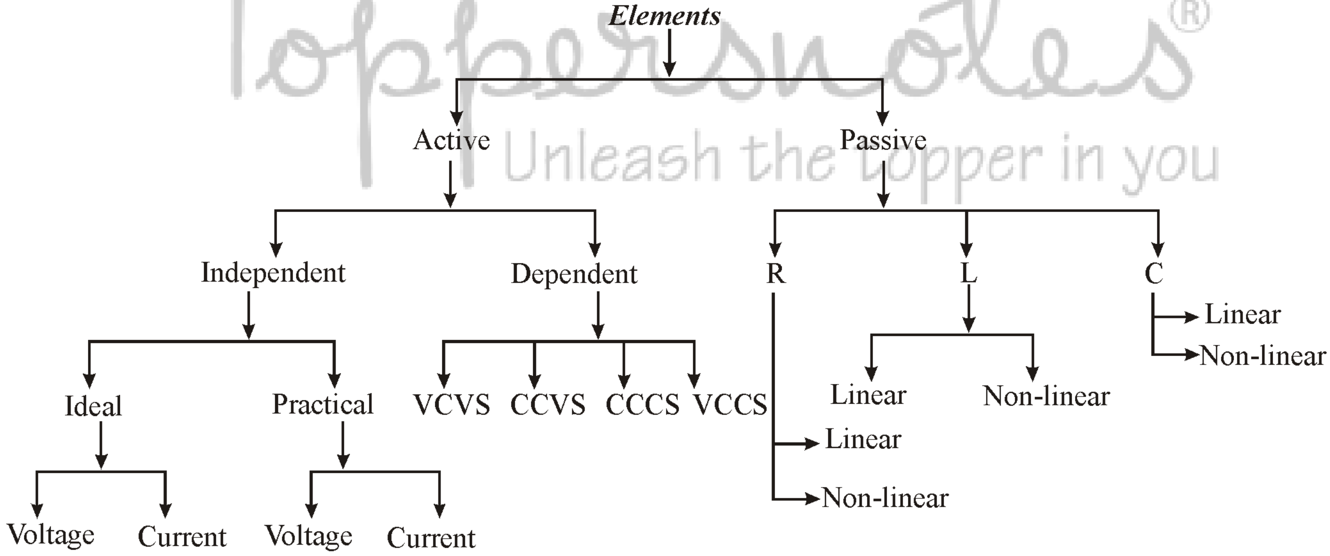
(i) Active and passive element

(ii) Unidirectional and bidirectional

(iii) Linear and non-linear element

(iv) Lumped and distributed element

(v) Time variant and time invariant element



1.2.1 Active Element and Passive Element

Active Element

When the element is capable of delivering energy independently for long time (approximately ∞) or when the element is having property of internal amplification.Then the element is called as active element. It does not require external source of energy.

Example: Voltage and current source are independent source. Transistor and op-amp are dependent source.

PASSIVE ELEMENT

When element can not deliver energy independently, the element is called as passive element.

It requires external source of energy.

(i) Resistance

- It is property of material
- It opposes the flow of charge carrier (e^-)
- It converts electrical energy to heat energy
- It's unit is ohm (Ω) & indicated by R.
- Resistance depends upon temperature

$$R_t = R_0 (1 + \alpha_0 T) \quad \text{...(1)}$$

Where, R_0 = Material Resistance at 0°C

α_0 = Temperature coefficient

T = Temperature

The symbolic representation is

Ohm's law : It states that current density (J) is directly proportional to electric field intensity (E).

$$J \propto E$$

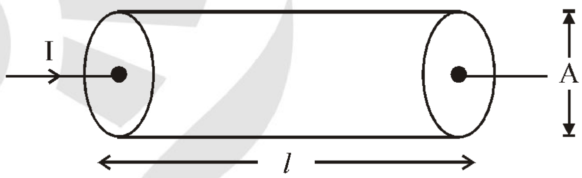
$$J = \sigma E$$

$$J = \frac{I}{A} = \frac{1}{\rho}\frac{V}{l}$$

$$\frac{V}{I} = \frac{\rho l}{A}$$

$$\frac{V}{I} = \frac{\rho l}{A} = R = \text{Constant}$$

$$V = RI$$



- It state that potential difference across the element is directly proportional to current flowing through element.
- It is valid when temperature and conductivity of material are constant.

Ohm's law may be written in several ways

$J = \sigma E$ ***(First form)***

Where σ = Conductivity and its unit is $\Omega^{-1}\ m^{-1}$.

$V = RI$ ***(Second form)***

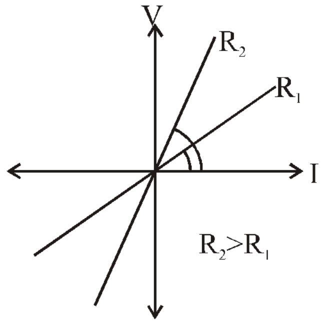
$I = GV$ ***(Third form)***

Where G = Conductance and its unit is siemens Ω^{-1}.

$$V = R\frac{dQ}{dt} \quad \textbf{\textit{(Fourth form)}}$$

Where Q = Charge and its unit is Coulomb (C).

- Ohm's law is applicable for any "bilateral conductor".
- If value of R is negative, then this will act as an active element but practically, this is not possible.



(ii) ***Inductance :***

It opposes rate of change of magnitude and direction of electric current passing through it.

It is indicated by L and unit is Henry.

The symbolic representation is

Faraday first law states that when conductor cuts a magnetic lines of force, an emf is induced in conductor.

Faraday second law states that emf induced in conductor is directly proportional to rate of change of flux (ϕ).

Note : *Flux is the presence of a force field in a specified physical medium.*

$$e \propto \frac{d\phi}{dt}$$

Where, e = Dynamic induced emf

ϕ = Flux

$$e = -N\frac{d\phi}{dt}$$

This is Lenz's Law.

An inductor is a passive electronic component that store energy in the form of magnetic field.

Simplest form of inductor is wire loop or coil.

The inductance is directly proportional to the number of turns in the coil.

Flux linkage $(\Psi) = N\phi$

$$e = \frac{d\Psi}{dt}$$

i.e. $\Psi \propto \phi \propto B \propto i$

$$\Psi \propto i$$

$$\Psi = Li$$

$$e = \frac{d\Psi}{dt}$$

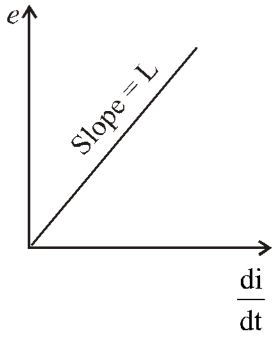
$$e = L\frac{di}{dt}$$

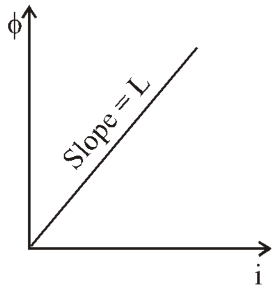
$$\psi = N\phi$$

$$\psi = Li$$

So, $N\phi = Li$

$$L = \frac{N\phi}{i}$$





$$e = L\frac{di}{dt}$$

If current through inductor remains constant, the voltage drop will be zero. It means short circuited coil. This is the steady state condition.

If current changes within zero time (dt = 0). It will give infinite voltage drop across inductor which is not feasible practically. Thus inductor always oppose change in current abruptly.

- When inductance does not depend upon the current, inductor is called as linear inductor.

Example: Air core inductor.

- When inductance depend on the current, inductor is called as non-linear inductor.

Example: Iron core inductor

- Inductor store energy in the form of magnetic field (kinetic energy).

$$L = \frac{N\phi}{i}$$

Where, $\phi = \frac{\text{Magneto motive force (MMF)}}{\text{Reluctance(s)}}$

$$\phi = \frac{NI}{l / A\mu_0\mu_r} = \frac{\mu_0\mu_r NIA}{l}$$

Where, A = Cross sectional area of core

l = Length of core

$\mu_0 = 4\pi \times 10^{-7}$ H/m

μ_r = Relative permeability

Then, $L = \frac{\mu_0\mu_r N^2 A}{l} = \frac{N^2}{l / A\mu_0\mu_r}$

$$L = \frac{N^2}{s}$$

$$L \propto N^2$$

Inductance of an inductor in directly proportional to square of number of turns.

Energy stored in inductor

Voltage across inductor is given by

$$V = L\frac{di}{dt}$$

and power $P = VI$

$$P = Li\frac{di}{dt}$$

$$W = \int Pdt = \int Li\frac{di}{dt}.dt$$

$$= L\int idi$$

$$W = \frac{1}{2}Li^2$$

In ideal inductor, the average power dissipation is equal to zero because it does not have internal resistance.

(iii) Capacitor

It is a device used to store an electric charge consisting of one or more pairs of conductors separated by an insulator.

It is a passive electronic component that store energy in the form of an electrostatic field (potential energy).

It is indicated by (C) and unit is farad (F).

Stored charge is directly proportional to applied voltage.

i.e., $$Q \propto V$$

$$Q = CV$$

$$C = \frac{Q}{V} = \frac{\text{Coulomb}}{\text{Volt}} = \text{Farad}$$

Since the unit of capacitance is very large so we measure capacitance in smaller unit like μF.

Formulae related to capacitor

$$I = C\frac{dV}{dt}$$

$$V = \frac{1}{C}\int_{-\infty}^{t} Idt$$

$$P = CV\frac{dV}{dt}$$

Energy stored in capacitor

$$W = \int Pdt = \int VC\frac{dV}{dt}dt = C\int VdV$$

$$W = \frac{1}{2}CV^2$$

- In ideal capacitor, power dissipation is equal to zero, because it does not have internal resistance.
- When capacitance of capacitor does not depend on the voltage, it is called as linear capacitor.
- When capacitance of capacitor depends on the voltage, it is called as non-linear capacitor.

Example: Varactor diode.

- If voltage is under steady state like DC voltage.

i.e., $$dV = 0$$

$$i = C\frac{dV}{dt} = 0$$

$$i = 0$$

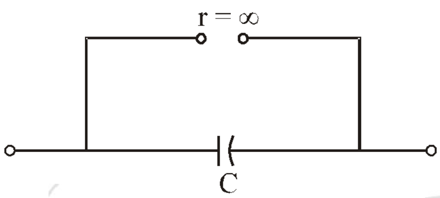
It means, there is no current in the circuit.

- Capacitor does not allow sudden change of voltage because current through capacitor will be infinite which is practically not possible.

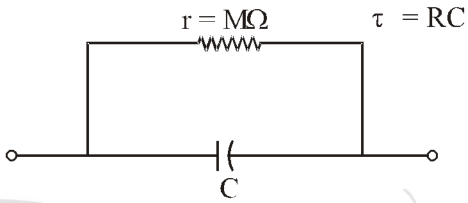
$$i = C\frac{dV}{dt}$$

if $dt = 0,\ i = \infty$

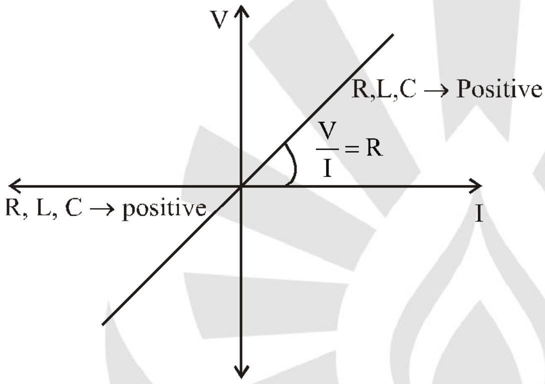
- Ideal capacitor has infinite value of time constant and for practical capacitor, the value of time constant is less



Ideal Capacitor

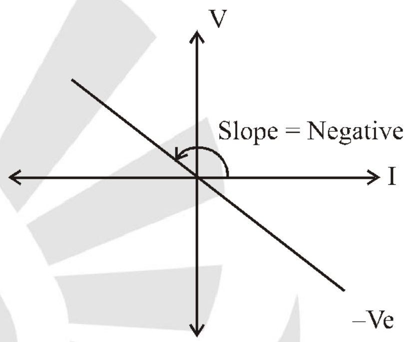


Practical Capacitor



(In I & III quadrant) (Passive)

(consumption of energy)



(II & IV quadrant) (Active)

(Generation of energy)

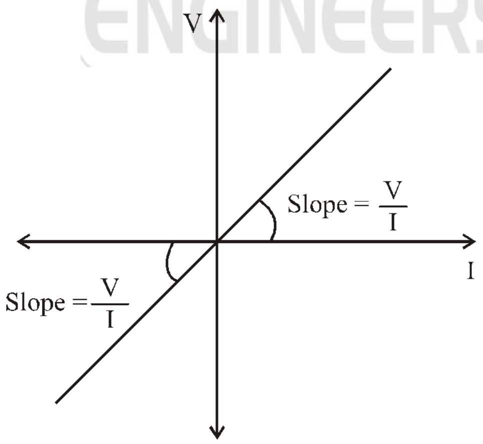
1.2.2 Bidirectional and Unidirectional Element

When element have properties which does not depend on the flow of direction of current, is called as bidirectional element.

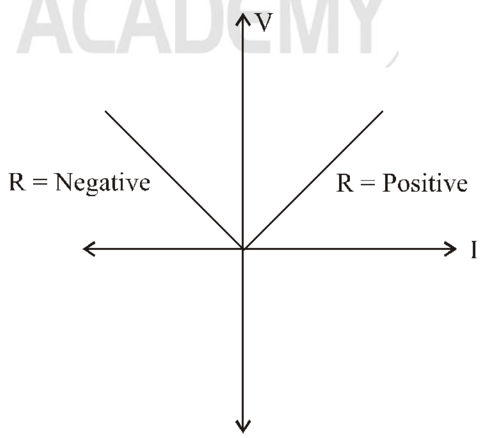
Example: Inductance, Resistor, Capacitor.

When element have properties that they depend on the flow of direction of current is called as unidirectional element.

Example: Diode.



(Slope is same ⇒ R is same)
(Bilateral)



(Unilateral)

1.2.3 Linear and Non-Linear Element

When element follow ohm's law then it is known as linear element.

Note: *(a) Network is valid if frequency < 1 MHz.*

(b) Electromagnetic theory is valid for high frequency (about 1 MHz)

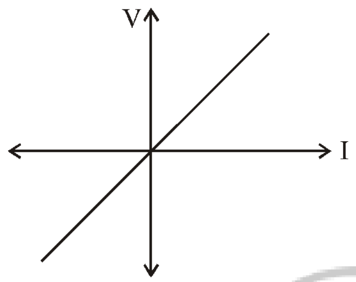
$$\left.\begin{array}{l}\sigma = 0 \;\; R = \infty \\ \sigma = \infty \;\; R = 0\end{array}\right\}\text{No Network analysis.}$$

Active : No need of external source.

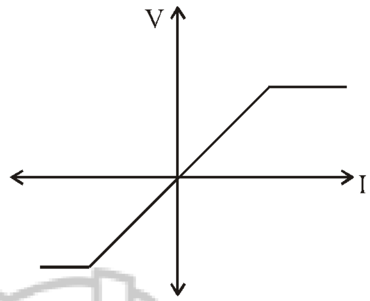
Passive : Requires external source of energy.

R, L, C $\geq$ 0 Passive element (consuming energy)

R, L, C < 0 Active element (negative)



(Linear)

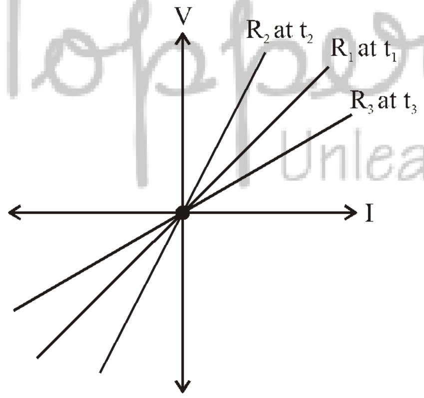


(Non-linear)

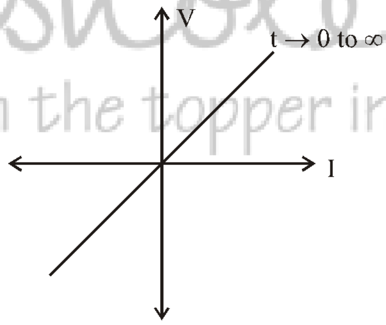
1.2.4 Lumped and Distributed Element

Physically seperate elements such as resistors, capacitors and diodes are lumped materials. On the other hand network elements which are inseparable for analytical purposes are called distributed elements.

1.2.5 Time Variant and Time Invariant



(Time variant)



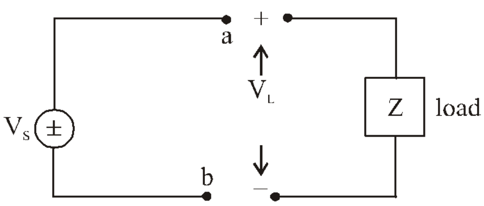
(Time invariant)

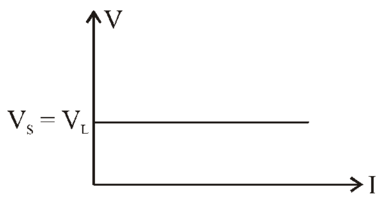
1.3 Independent Source

1.3.1 Voltage Source

(i) Ideal Voltage Source

R_{int} = 0 (internal resistance)

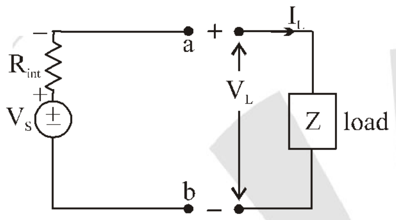


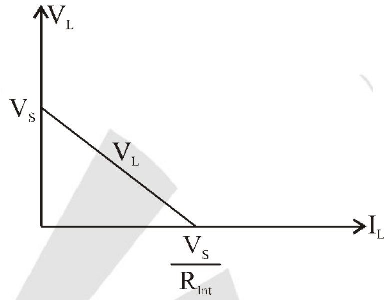


- We can't predict current across ideal voltage source, it can be determine only by other parameter current can be any value (0 to ∞)
- Ohm's law is not applicable here.
- For an ideal voltage source, the terminal voltage is independent of terminal current.
- V-I characteristics of ideal voltage source is non-linear.
- Inherently all the sources are non-linear in nature, since V & I are non-linear, but the non-linearity of independent sources are ignored in network theory, otherwise sources are active and unilateral element.

(ii) Practical Voltage Source

$$R_{int} \neq 0$$



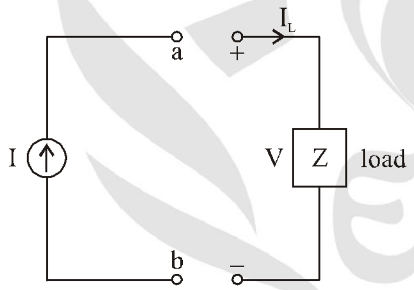


(Current can be predicted)

$$-V_s + IR_{int} + V_L = 0$$
$$V_L = V_s - IR_{int}$$

1.3.2 Current Source

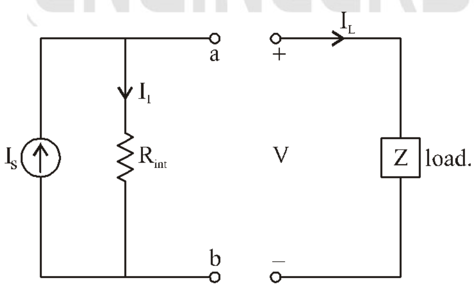
(i) Ideal Current Source

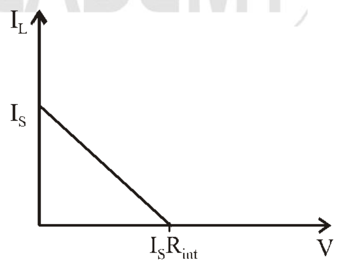


- For ideal current source, current is constant and independent of voltage and load.
- For ideal current source, the terminal current source is independent of terminal voltage.

$$R(\text{internal}) = \infty.$$

(ii) Practical Current Source



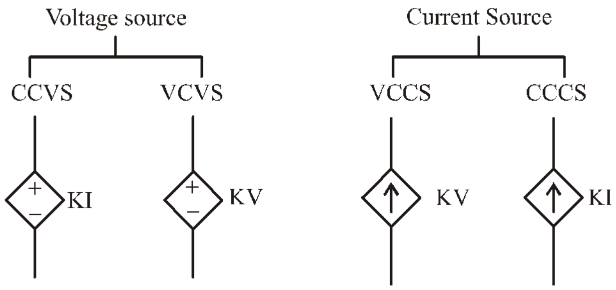


$$I_s - I_1 - I_L = 0$$

$$\Rightarrow \quad I_L = I_s - I_1$$

$$\Rightarrow \quad I_L = I_s - \frac{V}{R_{int}}$$

1.4 DEPENDENT SOURCE



Inherently the controlled sources are also non-linear in nature, since the voltage and current relationship is non-linear. But the linearity of the controlled source is defined w.r.t. controlling independent variable. If they are linear, then the sources are said to be linear, otherwise non-linear.

The presence of controlled source in a Network makes it an active Network.

1.5 KIRCHHOFF'S LAWS

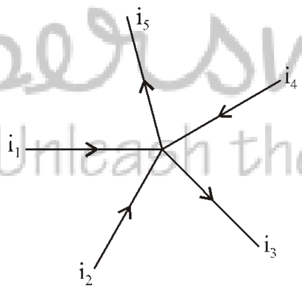
There are simple relationship between currents and voltages of different branches of an electric circuit. These relationships are determined by some basic laws that are known as kirchhoff laws, There are two laws

(i) Current law

(ii) Voltage law

1.5.1 KIRCHOFF'S CURRENT LAW (KCL)

- It is based on law of conservation of charge.
- It states that, algebric sum of current meet at a point is equal to zero.
- All incoming current is positive and outgoing current is negative



$$i_1 + i_2 - i_3 + i_4 - i_5 = 0 \quad ...(i)$$

We know that $i = \frac{dQ}{dt}$

$$\frac{dQ_1}{dt} + \frac{dQ_2}{dt} - \frac{dQ_3}{dt} + \frac{dQ_4}{dt} - \frac{dQ_5}{dt} = 0$$

$$Q_1 + Q_2 - Q_3 + Q_4 - Q_5 = 0 \quad ...(ii)$$

KCL is based on law of conservation of charge.

as, $Q = ne$

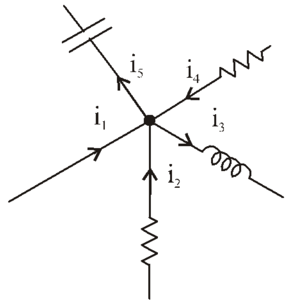
$$\therefore \quad n_1e^- + n_2e^- - n_3e^- + n_4e^- - n_5e^- = 0$$

$$\Rightarrow \quad n_1 + n_2 - n_3 + n_4 - n_5 = 0 \quad ...(iii)$$

where n is number of electron entering or leaving.

Note: *KCL is valid only for lumped electrical circuit not for distributed circuit.*

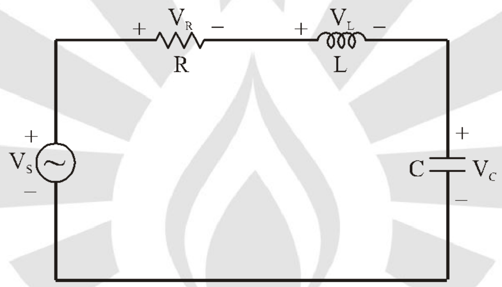
KCL doesn't depends on the type of elements used in the circuit. So KCL is valid for linear, non-linear, active, passive, time-variant & time invariant and other circuit element.



$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

1.5.2 Kirchoff's Voltage Law (KVL)

- It is based on law of conservation of energy.
- It state that algebric sum of the voltage in closed loop is equal to zero.



$$-V_S + V_R + V_L + V_C = 0$$

$\because$ $$V = \frac{W}{Q} \text{ and Q is same in series.}$$

So, $$-\frac{W_S}{Q} + \frac{W_R}{Q} + \frac{W_L}{Q} + \frac{W_C}{Q} = 0$$

$$-W_S + W_R + W_L + W_C = 0$$

- KVL is based on law of conservation of energy.
- KVL is valid for lumped electrical circuit not for distributed circuit.
- KVL is independent of nature of circuit elements.

KVL + Ohm's law $\rightarrow$ Mesh analysis

KCL + Ohm's law $\rightarrow$ Nodal analysis

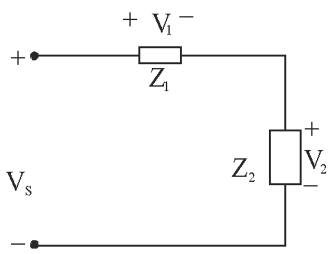
1.6 Ohm's Law

- $V = IR$
- $J = \sigma E$
- $V_C = \frac{1}{C}\int_0^t dt$
- $I_C = C\frac{dV_C}{dt}$
- $I_L = \frac{1}{L}\int_{-\infty}^t V_L dt$
- $V_L = L\frac{di}{dt}$

- Ohm's law is valid only at constant temperature.
- Ohm's law is not valid for ideal source. (due to non-linearity).
- But in Nodal and mesh, ideal sources can be used and their non-linearities can be neglected.

1.7 Equivalent Circuit

1.7.1 Series



$$Z_{eq} = Z_1 + Z_2$$

Resistance $Z = R$

$$R_{eq} = R_1 + R_2 \quad \textit{(Series)}$$

Inductance $Z = j\omega L$

$$j\omega L_{eq} = j\omega L_1 + j\omega L_2$$

$$L_{eq} = L_1 + L_2 \quad \textit{(Series)}$$

Capacitance $Z = \frac{1}{j\omega C}$

$$\frac{1}{j\omega C_{eq}} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \quad \textit{(Series)}$$

Voltage Division Rule

$$Z_{eq} = Z_1 + Z_2$$

$$I = \frac{V_s}{Z_{eq}} = \frac{V_s}{Z_1 + Z_2}$$

$$V_1 = IZ_1$$

$$V_2 = IZ_2$$

$$V_1 = \frac{Z_1}{Z_1 + Z_2} V_S$$

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_S$$

Resistance
$$V_1 = \frac{R_1}{R_1 + R_2} V_S$$

$$V_2 = \frac{R_2}{R_1 + R_2} V_S$$

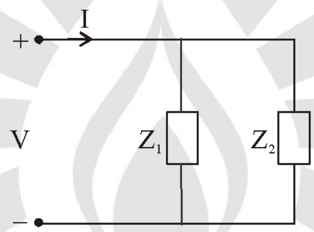
Inductance
$$V_1 = \frac{L_1}{L_1 + L_2} V_S$$

$$V_2 = \frac{L_2}{L_1 + L_2} V_S$$

Capacitance
$$V_1 = \frac{C_2}{C_1 + C_2} V_S$$

$$V_2 = \frac{C_1}{C_1 + C_2} V_S$$

1.7.2 Parallel



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

Resistance
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$\Rightarrow$
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Inductance
$$\frac{1}{j\omega L_{eq}} = \frac{1}{j\omega L_1} + \frac{1}{j\omega L_2}$$

$\Rightarrow$
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Capacitance
$$j\omega C_{eq} = j\omega C_1 + j\omega C_2$$

$\Rightarrow$
$$C_{eq} = C_1 + C_2$$

Current Division Rule

$$V = IZ_{eq}$$

$$I_1 = \frac{V}{Z_1} = \frac{Z_1 Z_2}{Z_1(Z_1 + Z_2)} I = \frac{Z_2}{(Z_1 + Z_2)} I$$

$$I_2 = \frac{V}{Z_2} = \frac{Z_1}{Z_1 + Z_2} I$$

Resistance

$$I_1 = \frac{Z_2}{Z_1 + Z_2} I$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

Inductance

$$I_1 = \frac{L_2}{L_1 + L_2} I$$

$$I_2 = \frac{L_1}{L_1 + L_2} I$$

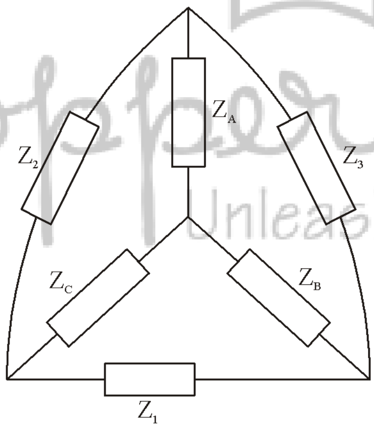
Capacitance

$$I_1 = \frac{C_1}{C_1 + C_2} I$$

$$I_2 = \frac{C_2}{C_1 + C_2} I$$

1.8 Delta and Star Connection

When elements are connected neither in series nor in parallel. To reduce this kind of network star-delta transform is used.



$Z_A, Z_B, Z_C \rightarrow$ Star Network
$Z_1, Z_2, Z_3 \rightarrow$ Delta Network

Delta to Star Conversion	*Star to Delta Conversion*
• $Z_A = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$	• $Z_1 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$
• $Z_B = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$	• $Z_2 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B}$
• $Z_C = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$	• $Z_3 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$

Star connection is also known as T-connection.

Note : *If Delta* $\Rightarrow 3\Omega$ *(each resistance)*

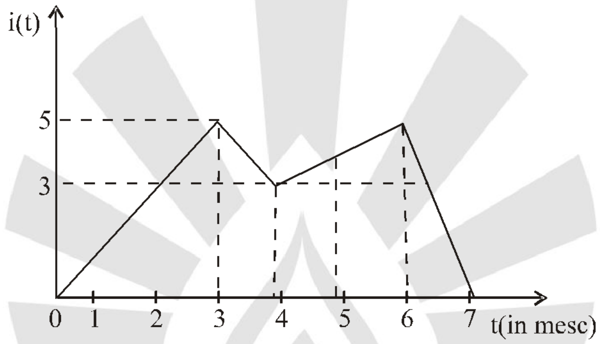
then Star $\Rightarrow 1\Omega$ *(each resistance)*

for R & L; *Delta = 3 × Star*

for C; *Delta =* $\frac{1}{3}$ *× Star*

Star to Delta, impedance $\uparrow \Rightarrow C \downarrow \Rightarrow R\ \&\ L \uparrow$

Example 1: The current flowing through the capacitor is shown in figure, then determine the charge stored by the capacitor upto the 7 msec?



Solution : The area of under curve i(t) and t gives stored charge

$$Q = \int i(t)dt$$

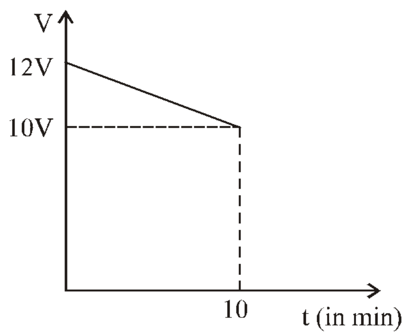
$$= \int_0^3 i(t)dt + \int_3^4 i(t)dt + \int_4^6 i(t)dt + \int_6^7 i(t)dt$$

$$= \left(\frac{1}{2}\times 3\times 5\right) + \frac{1}{2}\times 1\times(3+5) + \frac{1}{2}\times 2\times(3+5) + \frac{1}{2}\times 1\times 5$$

$$= \frac{15}{2} + 4 + 8 + \frac{5}{2} = 22 \text{ milli coulomb}$$

Example 2: A fully charged mobile phone with 12 V is good for 10 min talk time Assume that during the talk time the battery delivers a constant current of 2 Amp and its voltage drops linearly from 12 Volts to 10 Volts as shown in the figure.

How much energy does the battery delievers during talk time.



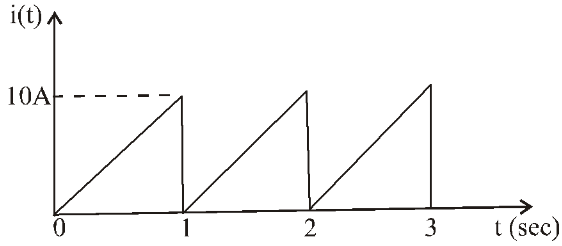
Solution :

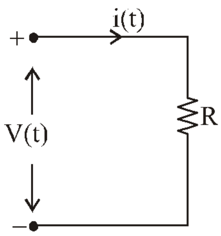
$$E = \int P\,dt = \int VI\,dt = 2\int_0^{t_1} V\,dt$$

$$t_1 = 10 \text{ min} = 600 \text{ sec}$$

$$E = 2\int_0^{600} V\,dt = 2\times\left[600\left(\frac{10+12}{2}\right)\right] = 13.2 \text{ kJ}$$

Example 3 : The current is flowing through 10 Ω resistor. Find the average power dissipated by the resistor.





Solution :

$$P = I^2R$$

$$P_{avg} = I_{rms}^2 R$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)\,dt$$

$$m = \frac{10-0}{1-0} = 10$$

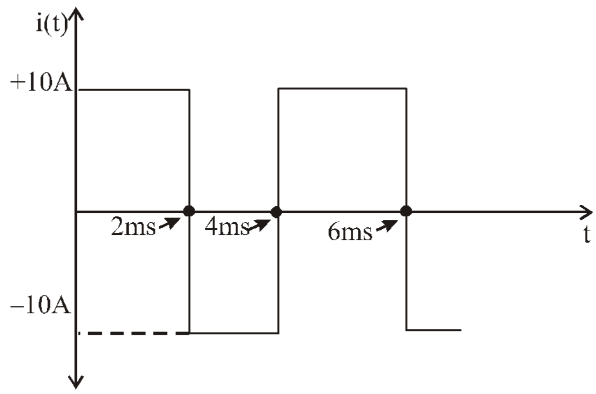
So, $y = mx + C$

$$i(t) = 10t$$

So, $$I_{rms}^2 = \frac{1}{T}\int_0^1 100t^2\,dt = 100\left(\frac{t^3}{3}\right)_0^1 = \frac{100}{3}$$

$$P_{avg} = I_{rms}^2 R = \frac{100}{3}\times 10 = 333.33 \text{ W}$$

Example 4 : Find the average power dissipated by 1Ω resistor.



Solution :

$$I_{rms}^2 = \frac{1}{4\times10^{-3}}\left[\int_0^2 (10)^2 dt + \int_2^4 (-10)^2 dt\right]$$

$$= \frac{1}{4\times10^{-3}}[100(2-0)+100(4-2)]\times10^{-3}$$

$$= \frac{1}{4\times10^{-3}}\times400\times10^{-3} = 100 \text{ A}$$

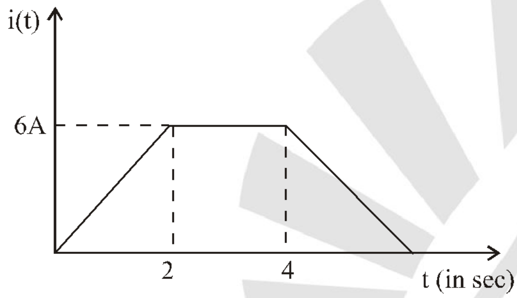
$$P_{rms} = I_{rms}^2 \ R = 100 \times 1 = 100 \text{ W}$$

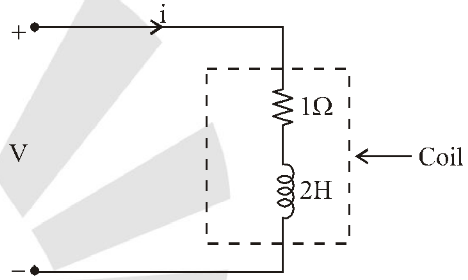
$$P_{max} = |I^2|R \ (10)^2 \times 1$$

$$= 100 \text{ W}$$

In general, $P_{avg} \leq P_{max}$

Example 5: The figure shows the current flowing through the coil of resistance 1Ω and inductance 2H.





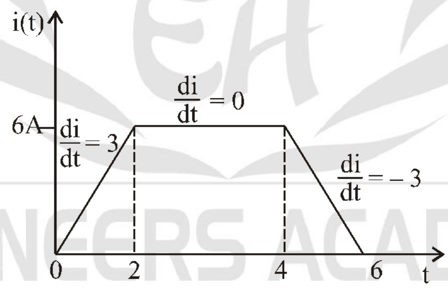
Determine

(i) What would be the energy stored across inductor for first 2 sec.

(ii) The energy absorbed by the inductor upto first 4 sec.

(iii) Find the energy absorbed across coil for first 6 sec?

Solution : (i)



$$V = \frac{Ldi}{dt}$$

$$E = \int Pdt$$

$$E = \int_{t_1}^{t_2}\left(L\frac{di}{dt}\right) \ i(t)dt \ = 2\left[\int_0^2 i\frac{di}{dt}dt\right]$$

$$= 2\left[\int_0^2 3t(3)dt\right] = 36 \text{ Joules}$$

(ii)
$$E = 2\left[\int_0^2 i\frac{di}{dt}dt + \int_2^4 i\frac{di}{dt}dt\right]$$

$$= 2\left[\int_0^2 3t(3)dt + 0\right] = 36 \text{ Joules}$$

(iii)
$$E_L = 2\left[\int_0^2 i\frac{di}{dt}dt + \int_2^4 i\frac{di}{dt}dt + \int_4^6 i\frac{di}{dt}dt\right]$$
$$= 36 + 0 - 36 = 0$$

Energy absorbed by resistance $E_R = \int P\, dt$

$$E_R = \left[\int_0^2 9t^2 dt + \int_2^4 36\, dt + \int_4^6 9(t-6)^2 dt\right]$$

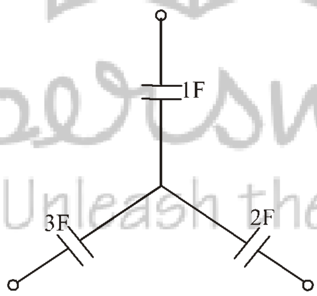
$$E_R = [24 + 72 + 24] = 120 \text{ J}$$

Energy across coil $E = E_L + E_R$
$$= 0 + 120 = 120 \text{ J}$$

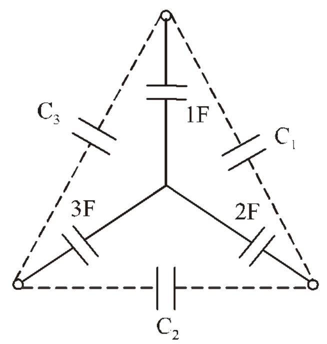
Note: ***(i)*** *Whenever current is changing w.r.t. time, then only inductor will store the energy, but for constant current inductor will not store the energy (constant energy).*

(ii) *Same in case of capacitor w.r.t. voltage.*

Example 6 : Find the delta equivalent for the given circuit :



Solution :



$$\frac{1}{C_1} = \frac{\frac{1}{3}\times\frac{1}{1}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{1}\times\frac{1}{2}}{\frac{1}{3}} = 3 \Rightarrow C_1 = \frac{1}{3}\text{F}$$

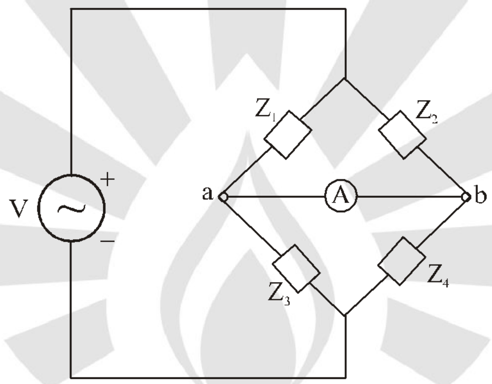
$$\frac{1}{C_2} = \frac{\frac{1}{3}\times\frac{1}{2}+\frac{1}{1}\times\frac{1}{2}+\frac{1}{1}\times\frac{1}{3}}{\frac{1}{1}} = 1$$

$$\Rightarrow \quad C_2 = 1\ F$$

$$\frac{1}{C_3} = \frac{\frac{1}{1}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{1}\times\frac{1}{2}}{\frac{1}{2}} = 2$$

$$\Rightarrow \quad C_3 = \frac{1}{2} F$$

Wheat Stone Bridge :



Condition for balanced bridge

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

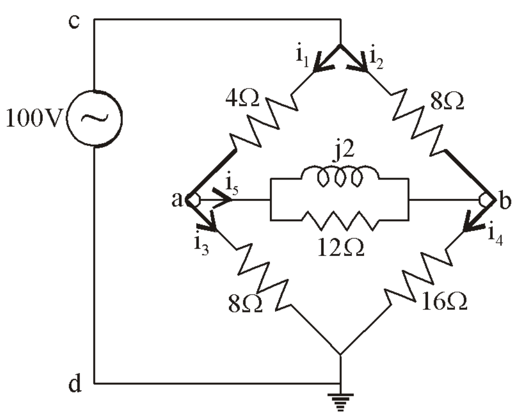
When bridge is balanced then no current will flow through ammeter.

Example 7 : Find out the following

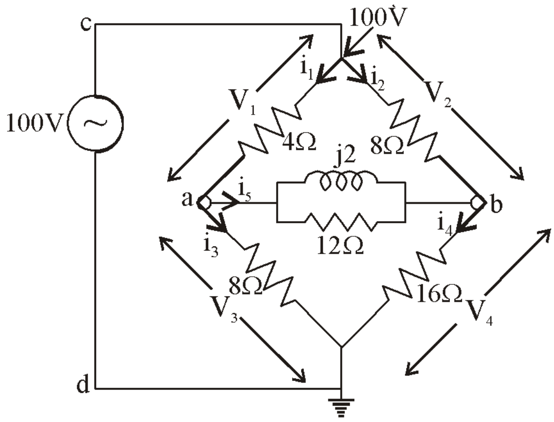
(i) Z_{cd} = ?

(ii) i_1, i_2, i_3, i_4, i_5 = ?

(iii) V_a, V_b, V_c, V_d = ?



Solution :



Since, bridge is balanced.

So, open → ab

then

$$R_{eq} = 12 \parallel 24$$

$$\frac{1}{R_{eq}} = \frac{36}{12\times 24} = \frac{1}{8}$$

$$R_{eq} = 8\Omega$$

$$Z_{cd} = 8\Omega$$

$$i_5 = 0$$

$$i_1 = i_3 = \frac{100}{12}\text{A}$$

$$i_2 = i_4 = \frac{100}{24}\text{A}$$

$$V_C = 100\text{ V}$$

$$V_D = 0\text{ V}$$

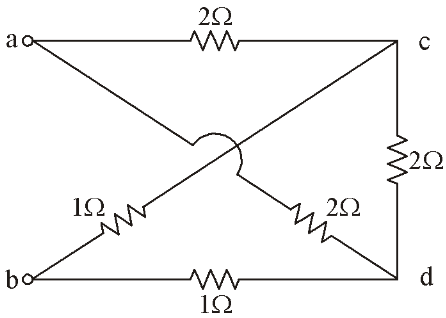
$$V_3 = V_a = i_3R_3 = \frac{100}{12}\times 8 = \frac{200}{3}\text{V}$$

$$V_4 = V_b = i_4R_4 = \frac{100}{24}\times 16 = \frac{200}{3}\text{V}$$

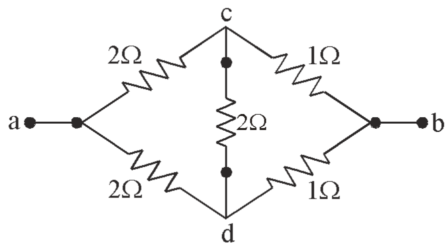
$$V_1 = i_1R_1 = \frac{100}{12}\times 4 = \frac{100}{3}\text{V}$$

$$V_2 = i_2R_2 = \frac{100}{24}\times 8 = \frac{100}{3}\text{V}$$

Example 8 : Determine the equivalent resistance across 'ab' terminal of above circuit?



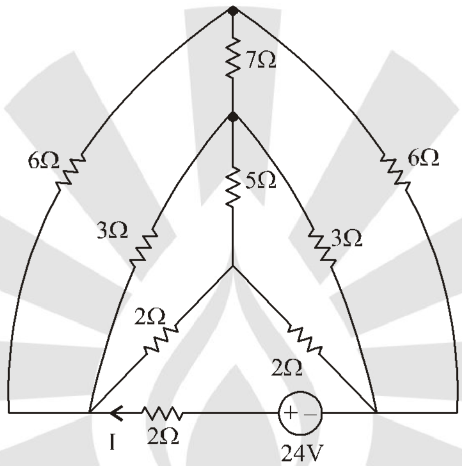
Solution :



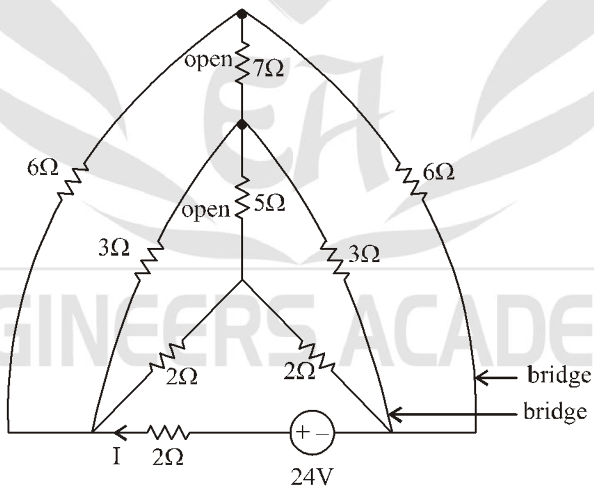
Wheatstone bridge is balanced

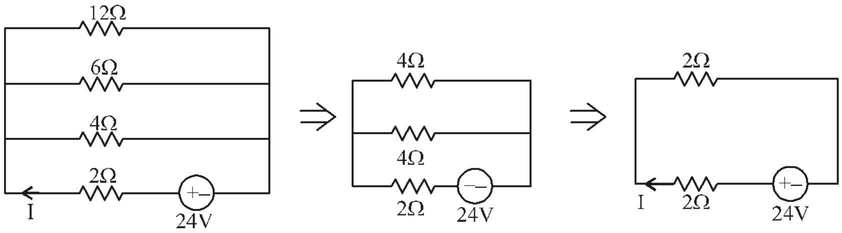
$$R_{ab} = 3 \parallel 3 = 1.5\ \Omega$$

Example 9 : Find the current I for circuit shown below.



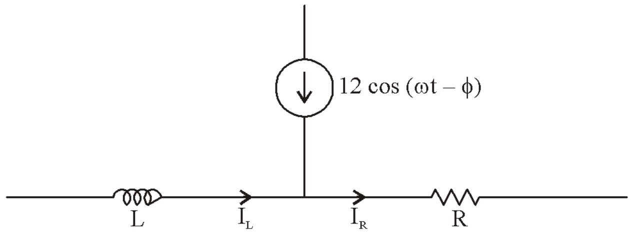
Solution : 5 Ω and 7 Ω will act as an open circuit because it satisfy wheat stone bridge condition.





$$I = \frac{24}{4} = 6 \text{ Amp}$$

Example 10: Find ϕ for the circuit shown below.



Where, $i_L(0) = 1$ A

$i_R(t) = 4\ e^{-3t} + 3e^{-4t}$ A

Solution : $I_L + 12\cos(\omega t - \phi) = I_R$

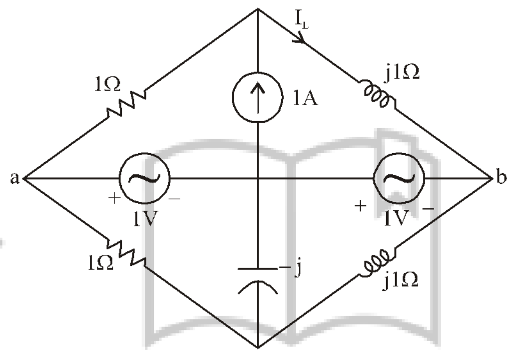
at $t = 0$

$\Rightarrow$ $1 + 12\cos(0 - \phi) = 7$

$\Rightarrow$ $12\cos\phi = 6$

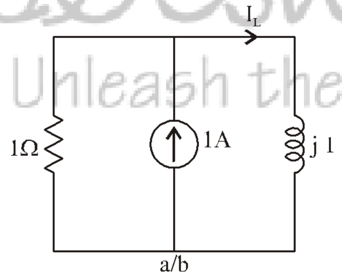
$\Rightarrow$ $\phi = 60°$

Example 11 : Find the current I_L for circuit shown below.



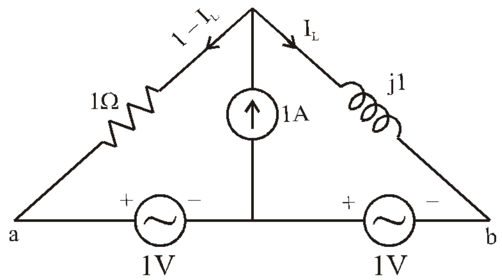
Solution : Approach-I $V_{ab} = 0,$

Because above bridge network is in balanced condition



$$I_L = 1\times\frac{1}{1+j} = \frac{1}{1+j}$$

Approach-II



$$I_R + I_L - 1 = 0$$

$$I_R = 1 - I_L$$

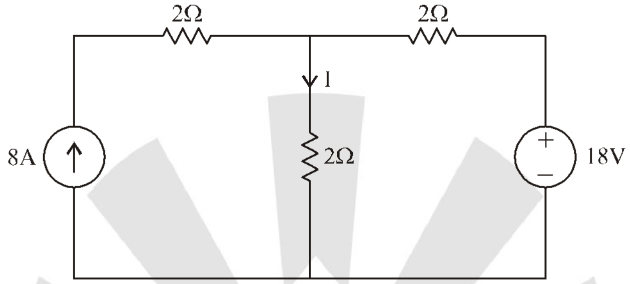
Applying KVL

$$\Rightarrow +1 - 1 + (1 - I_L)\,1 - I_L(j1) = 0$$

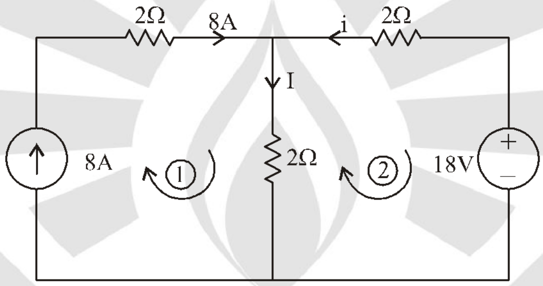
$$\Rightarrow \quad 1 - I_L - I_L\,(j) = 0$$

$$\Rightarrow \quad I_L = \frac{1}{1+j}$$

Example 12 : Find current I for the circuit shown below.



Solution :



By Apply KCL, $\quad i + 8 = I$

$$i = I - 8 \qquad \text{...(i)}$$

Now, by applying KVL in loop 2 ...(ii)

$$2I + 2i - 18 = 0$$

Put the value of i in equation (i)

$$2I + 2(I - 8) - 18 = 0$$

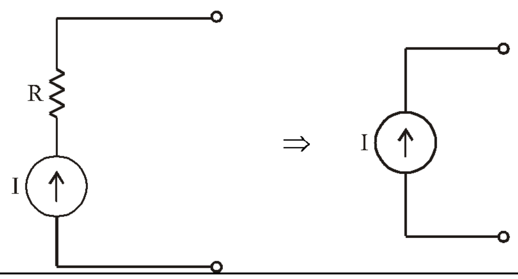
$$2I + 2I - 16 - 18 = 0$$

$$4I = 34$$

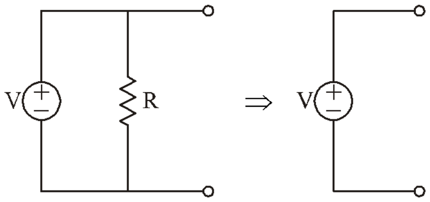
$$I = \frac{34}{4}$$

$$I = 8.5 \text{ A}$$

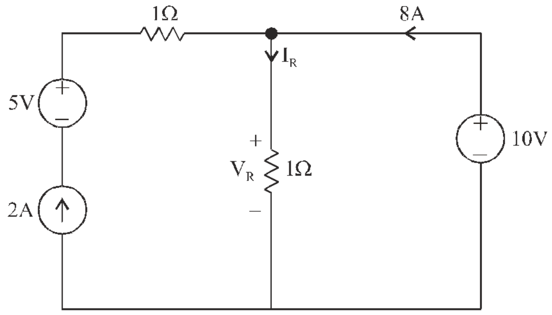
Note: ***(i)*** *A resistor in series with ideal current source can be neglected from the nodal analysis for voltage and current calculation, but not for power calculation.*



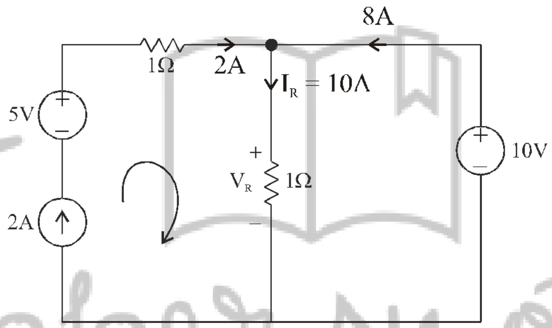
(ii) *A resistor is parallel with ideal voltage source can be neglected for nodal analysis for V & I calculations, but not for power calculations.*



Example 13 : Find the value of V_R, I_R and voltage drop across current source of 2A?



Solution :



By appling KCL at Node

$$2 + 8 = I_R$$

$$I_R = 10 \text{ A}$$

$$V_R = 1 \times I_R$$

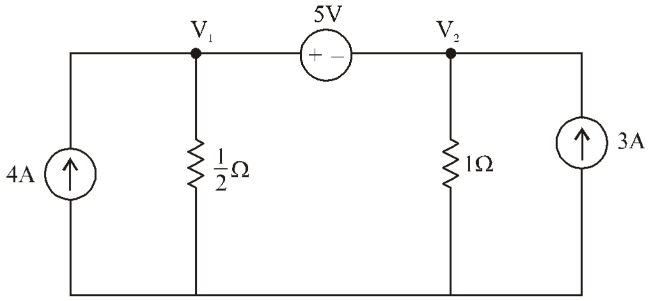
$$= 1 \times 10 = 10 \text{ Volt}$$

$$V_{2A} + 5 - 2 - 10 = 0$$

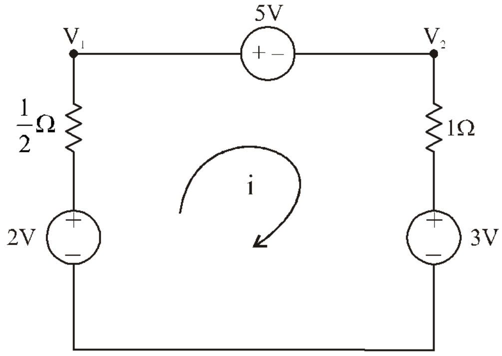
$$V_{2A} = 7 \text{ Volt}$$

1.8 Super Node

Example 14 : Find the value of V_1 and V_2.



Solution : Approach-I: Using source conversion



$$2-\frac{i}{2}-5-i-3 = 0$$

$$-\frac{3i}{2} = 6$$

$$i = -4\text{ A}$$

$$V_1 = 2-\frac{1}{2}\times(-4) = 2+2 = 4\text{ V}$$

$$V_2 = 3-1\times(4) = -1\text{ V}$$

Approach-II Super node :

$$-4+\frac{V_1}{\frac{1}{2}}+\frac{V_2}{1}-3 = 0$$

$$-4+2V_1+V_2-3 = 0$$

$$2V_1+V_2 = 7 \quad \text{....(i)}$$

$$V_1-V_2 = 5 \quad \text{....(ii)}$$

From equations (i) & (ii)

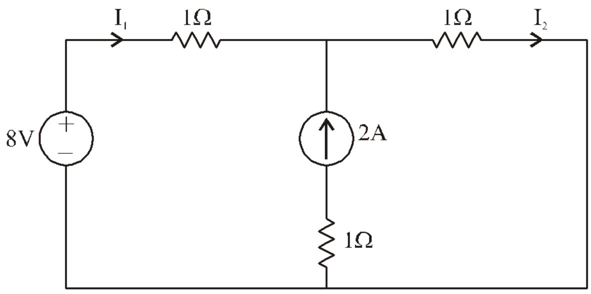
$$V_1 = 4\text{ V}$$

$$V_2 = -1\text{ V}$$

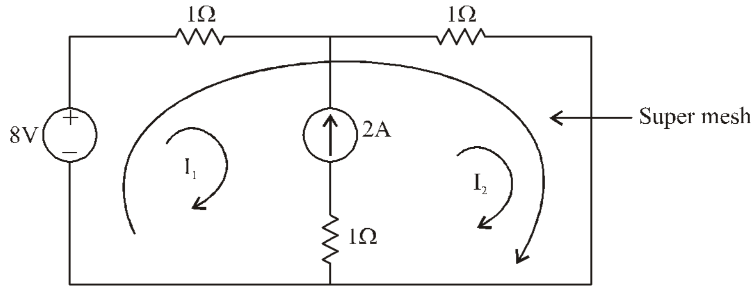
Note : *Whenever there is an ideal voltage source between two nodes then it is not possible to write the independent nodal equation at two nodes, hence,* ***supernode*** *is used.*

1.9 Super–Mesh

Example 15 : Find the value of I_1 and I_2.



Solution :



$$8 - i_1 - i_2 = 0$$

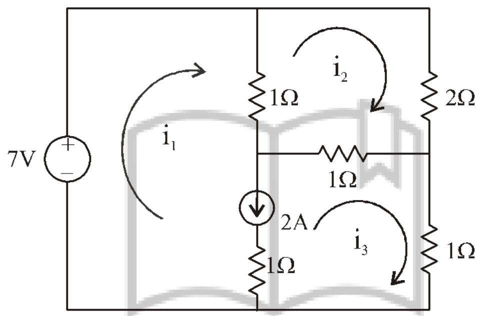
$$i_1 + i_2 = 8 \quad ...(i)$$

$$-i_1 + i_2 = 2 \quad ...(ii)$$

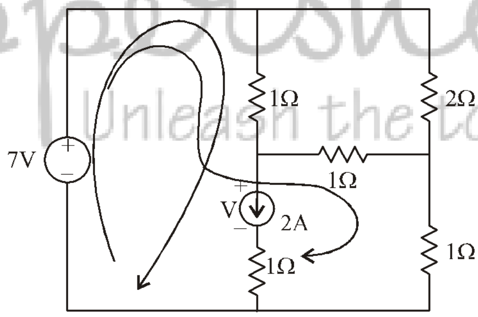
$$i_1 = 3 \text{ A}$$

$$i_2 = 5 \text{ A}$$

Example 16 : Find the value of i_1, i_2 and i_3. Also find voltage across current source.



Solution :



Applying KVL as shown above

$$7 - 1\,(i_1 - i_2) + (i_2 - i_3) - i_3 = 0$$

$$7 - i_1 + 2i_2 - 2i_3 = 0 \quad ...(i)$$

Applying KCL

$$i_1 - i_3 = 2$$

$$\Rightarrow \quad i_1 = 2 + i_3 \quad ...(ii)$$

Applying KCL

$$-(i_2 - i_1) - 2i_2 - (i_2 - i_3) = 0$$

$$i_1 - 4i_2 + i_3 = 0 \quad ...(iii)$$

Now equations (i) & (iii)

$$5 + 2i_2 - 3i_3 = 0 \qquad ...(iv)$$

$$2 - 4i_2 + 2i_3 = 0 \qquad ...(v)$$

$$i_3 = 3\text{ A}$$

$$i_2 = 2\text{ A}$$

$$i_1 = 5\text{ A}$$

Voltage across current source,

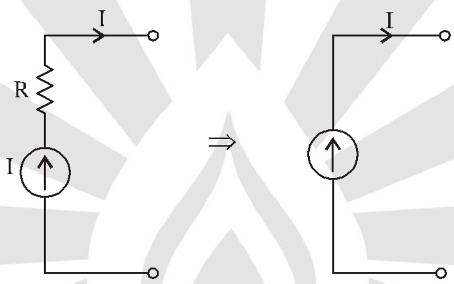
$$V - (i_3 - i_2) - i_3 - (i_3 - i_1) = 0$$

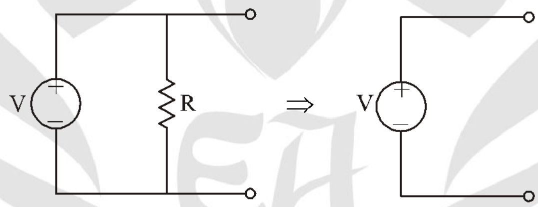
$$V + i_1 + i_2 - 3i_3 = 0$$

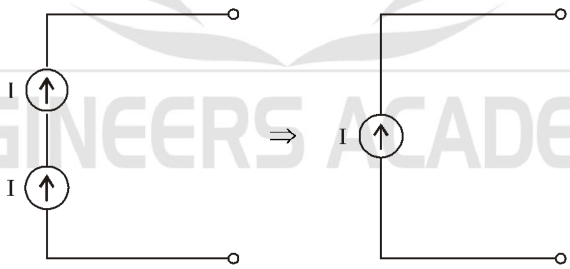
$$V = -5 - 2 + 9$$

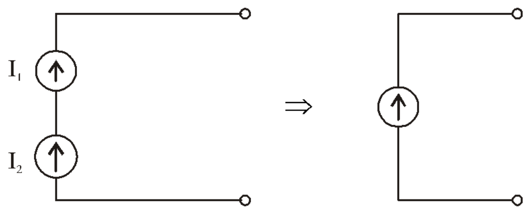
$$V = 2\text{ Volt}$$

1.10 Equivalent Circuit W.R.T. Source Point of View



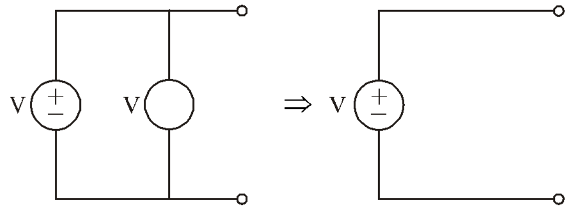


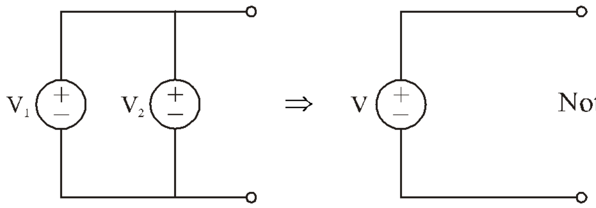




(Not possible)

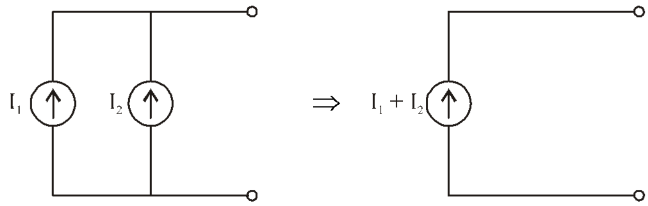
***Note :** The above circuit does not satisfy KCL.*

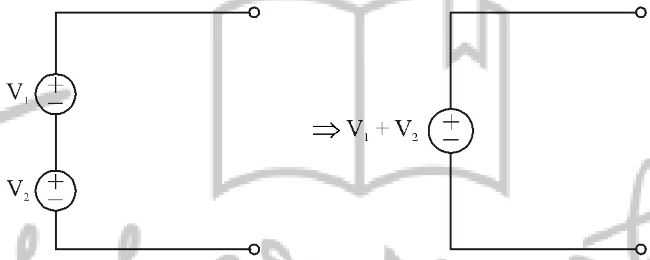




Not possible

Note : *The above circuit does not satisfy KVL.*



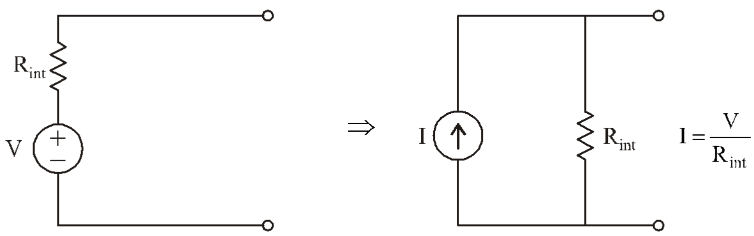


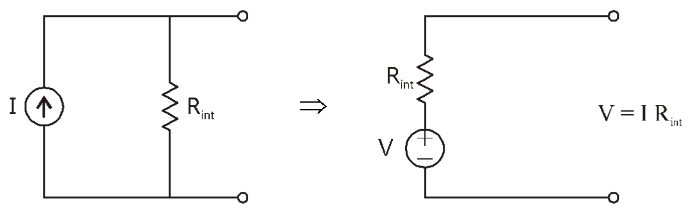
Note :

(i) *Two ideal current source can be connected in series if their magnitude are equal. Otherwise there would be oscillations in circuit and results in unstability.*

(ii) *Two ideal voltage source can be connected in parallel if their magnitude are equal. Otherwise their would be oscillations in the circuit and results in unstability.*

1.11 Source Transformation



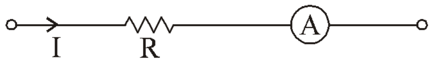


(i) Source transformation is valid only for practical sources not for ideal source.

(ii) Source transformation is a network simplification technique which is applicable only for practical source.

1.12 AMMETER

(1) Ammeter must be connected in series.



(2) Internal resistance

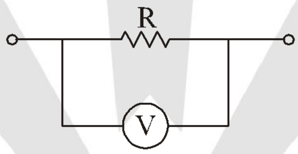
For ideal ammeter $R_{int} = 0$

For practical ammeter R_{int} = finite (low)

So, there would be voltage drop across R_{int}.

(3) Practical ammeter shows less reading then ideal ammeter.

1.13 VOLTMETER



(1) Voltmeter must be connected in parallel.

(2) Internal resistance

For ideal voltmeter $R_{int} = \infty$

For practical voltmeter R_{int} = finite (high)

(3) Practical voltmeter shows less reading then ideal voltmeter

Note : Values of R_{int} for ideal

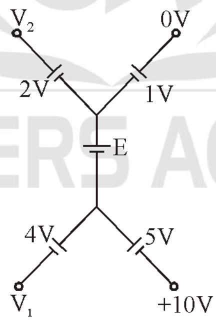
(i) Ammeter : 0

(ii) Voltmeter : ∞

(iii) Current source : ∞

(iv) Voltage source : 0

Example 17 : Find E, V_2 and V_1.



Solution :

$$0 - 1 - E - 5 - 10 = 0$$

$$E = -16 \text{ V}$$

$$0 - 1 - E - 4 - V_1 = 0$$

$$V_1 = 11 \text{ V}$$

$$V_2 - 2 - E - 5 - 10 = 0$$

$$V_2 = 1 \text{ V}$$

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