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RRB - JE

MECHANICAL

Railway Recruitment Board

Volume - 10

Machine Design

SIERRA INNOVATIONS

CONTENTS

FATIGUE

THEORY

1. VARIABLE LOAD

A material subjected to repetitive or Fluctuating stress will fail at a stress level much lower then required to cause fracture and failure occuring is called fatigue failure limit.

Sometimes this failure occurs without prior indications. The fatigue of material is affected by size, relative magnitude and static and fluctuating load and no. of load reversal.

2. FLUCTUATING STRESSES

In many applications, the components are subjected to forces, which are not static, but vary in magnitude with Respect to time. The stresses induced due to such forces are called Fluctuating stresses.

It is observed that about 80% of failures of mechanical components are due to 'Fatigue Failure' resulting from Fluctuating stresses.

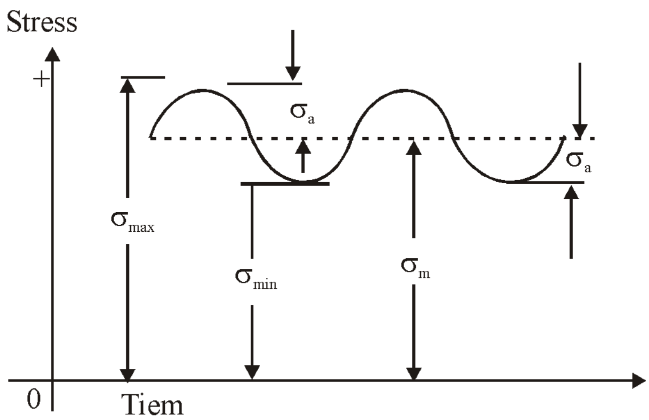
➢ There are three types of Mathematical model for cyclic stresses

(i) Fluctuating or Alternating stresses

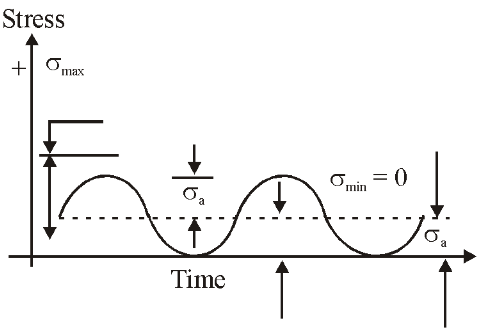
(ii) Repeated stresses

(iii) Reversed stresses

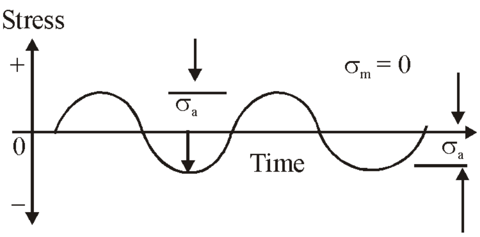
(i) Fluctuating or Alternating Stresses :



(ii) Repeated Stresses:



(iii) Reversed Stresses :



σ_{max} and σ_{min} are maximum and minimum stresses, while σ_m and σ_a are called Mean stress and Amplitude stress.

$$\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min})$$

$$\sigma_v = \sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

The Tensile Stress is Considered as positive While Compressive Stress as Negative.

The Repeated Stress and Revesed stress are Special cases of fluctuating stress with $(\sigma_{min} = 0)$ and $(\sigma_m = 0)$ respectively

3. FATIGUE FAILURE

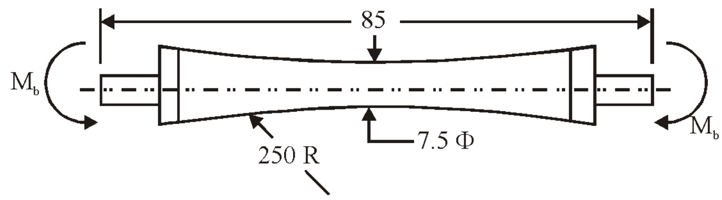
- For example A wire can be cut very easily in few cycles of bending and unbending. This is a fatigue failure and magnitude of stress required to fracture is very Low.
- "Fatigue failure is defined as time delayed fracture under cyclic Loading". examples of parts in which fatigue failure are common are Transmission Shafts, connecting rods, Gears, Suspension springh and boll Bearings.
- Fatigue cracks are not visible till they reach the surface of component and by that time the failure has already taken place. The Fatigue failure is sudden and total.
- The fatigue failure `depends upon number of factors such as the Number of cycles, mean stress, stress amplitude, stress concentration, residual stress, corrosion and creep.

4. ENDURANCE LIMIT

The fatigue or endurance limit of a material is defined as the maximum amplitude of completely reversed stress taht the standard specimen can sustain for an Unlimited numnber of cycles without fatigue failure.

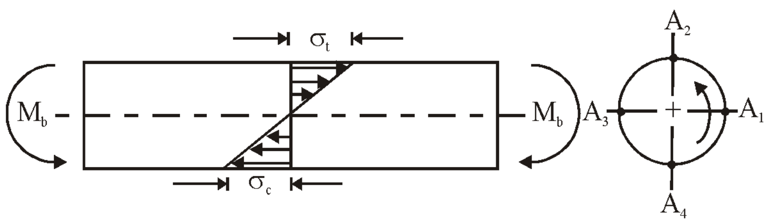
10^6 cycle is considered as a sufficient Number of cycles to define endurance Limit.

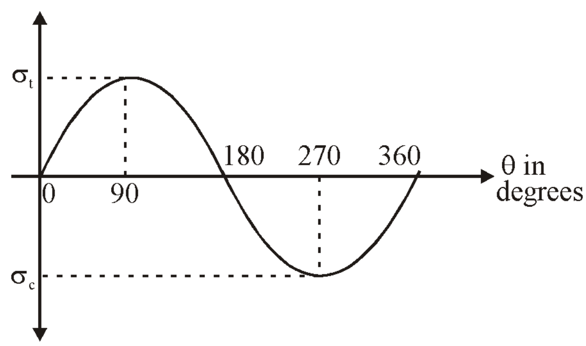
In laboratory the endurance limit is determined by means of a rotating Beam machine developed by **R.R Moore.**



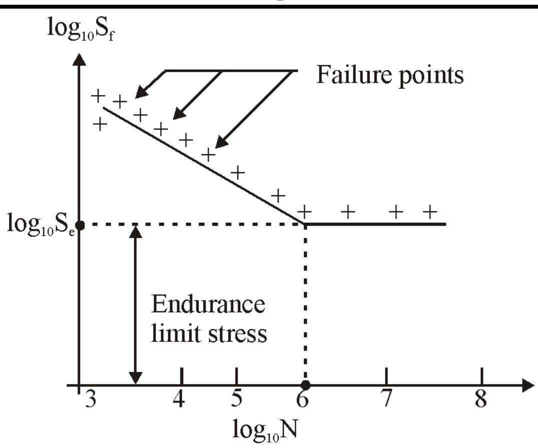
Principle of a rotating beam of circular cross sexrtion is subjected to bending moment (M_b). and beam is subjected to completely reversed stresses with tensile stress in first half and compressive stress in second half.

$$\boxed{\sigma_t \text{ or } \sigma_c = \frac{M_b Y}{I}}$$





- The Results of Rotating beam fatigue testing machine are plotted by means of an S-N curve
- S-N curve is the graphical representation of stress amplitude (S_f) Versus the number of stress cycles (N) before fatigre failure on a Log Log graph Paper.
- S-N Diagram is also called Wohler Diagram.

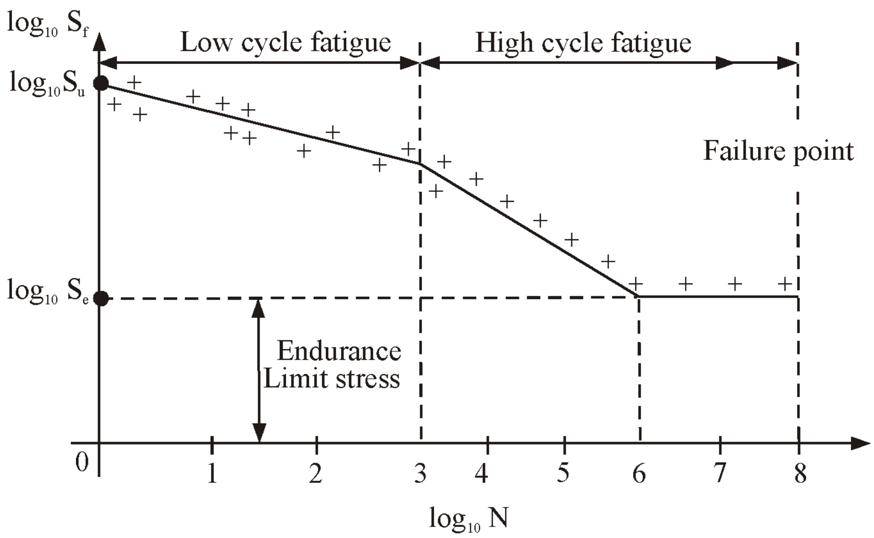


S-N Curve for Steels

Note: *Endurance Limit is not a Property of material like Ultimate tensile strength It is affected by factors such as size of component, shape of component, surface finish, temperature and Notch sensitivity of material.*

5. LOW CYCLE AND HIGH CYCLE FATIGUE

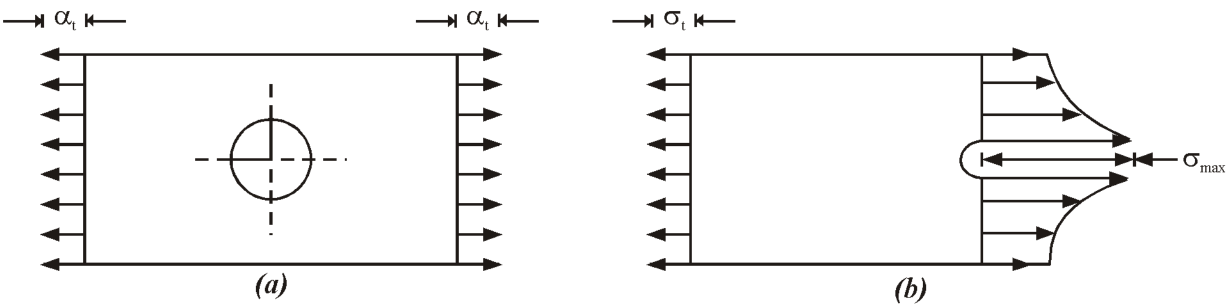
Any fatigue failure when number of stress cycles are less than 1000, is called low-cycle fatigue. Any failure when number of stress cycles are more than 1000 is called high-cycle fatigue.



Low and High Cycle Fatigue

6. STRESS CONCENTRATION

Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt change of the cross-section.



Stress Concentration

It order to consider the effect of stress concentrations, a factor called stress concentration factor (k_t).

$$\boxed{(k_t) = \frac{\text{Highest value of actual stress near discontinuity}}{\text{Nominal stress obtained by elementary equations for minimum cross-section}}}$$

$$\boxed{k_t = \frac{\sigma_{max}}{\sigma_o} = \frac{\tau_{max}}{\tau_o}}$$

σ_o and τ_o are stresses determined by elementary equation. σ_{max} and τ_{max} are localized stresses at Discontinuity. The magnitude of stress concentration factor depends upon the Geometry of Component.

6.1 Causes of Stress Concentration Factor :

(i) Variation in Properties materials

(ii) Load Application

(iii) Abrupt change in section

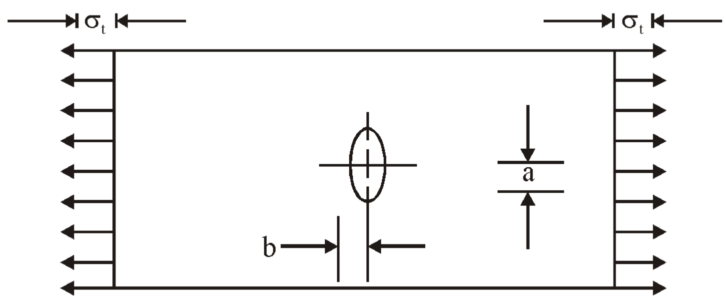
(ii) Discontinuity in Component

(iv) Machining scratches

6.2 Stress Conentration Factor for Elliptical Hole :

According to theory of elasticity

$$k_t = 1 + 2\left(\frac{a}{b}\right)$$



Stress Concentration due to Elliptical Hole

Where a = half width (or semi-major axis) of ellipse perpendicular to direction of Load

b = half width (or semi-minor axis) of ellipse in direction of Load.

if $b = 0$

then $k_t = \infty$

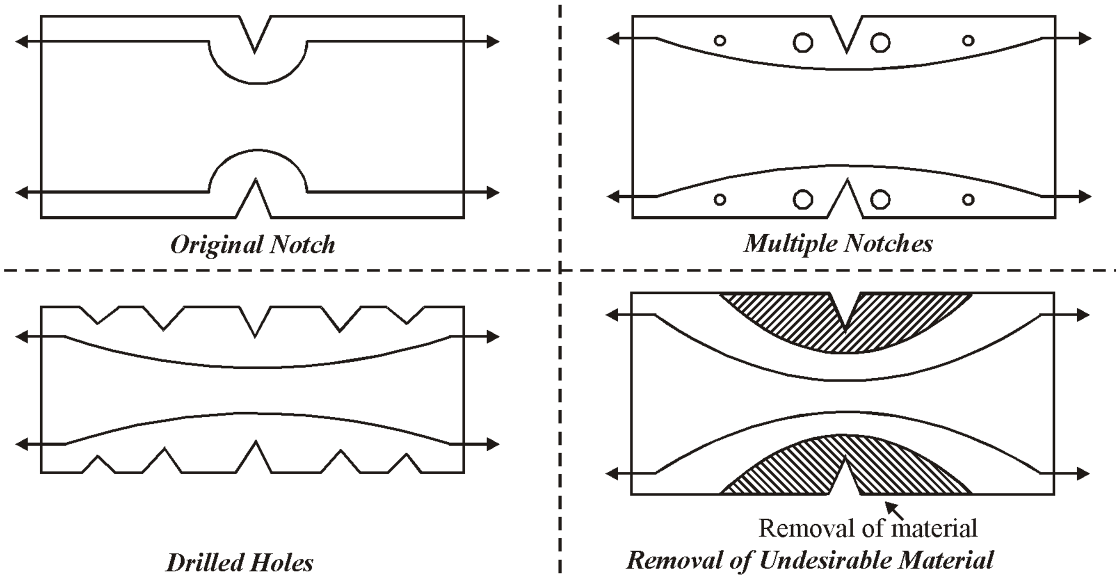
if $a = b$

then $k_t = 1 + 2 = 3$

Therefore the k_t for small circular hole in a flat plate, which is subjected to tensile force is 3.

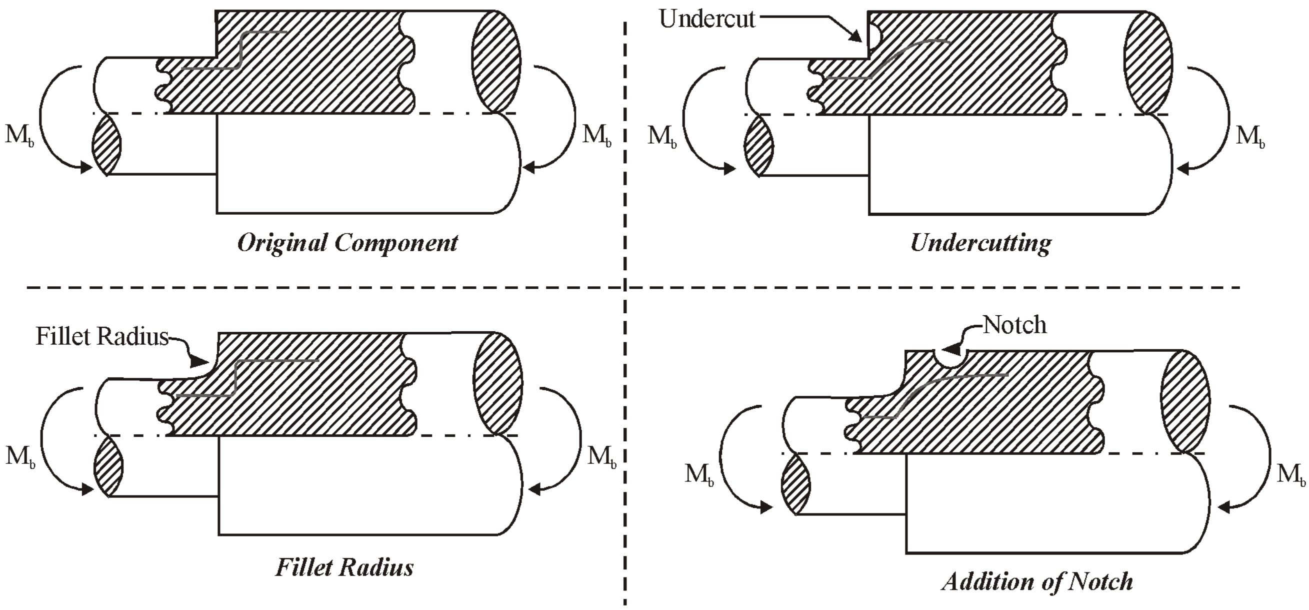
6.3 Reduction of Stress Concentration:

(i) Additional Notches and holes in tension member



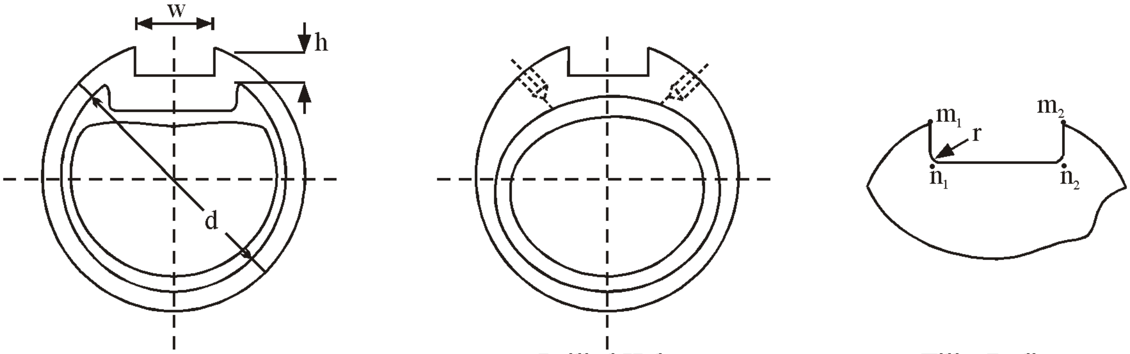
Reduction of Stress Concentration due to V-notch

(ii) Fillet Radius, Undercutting and Notches for Member in Bending



Reduction of Stress Concentration due to Abrupt Change in Cross section

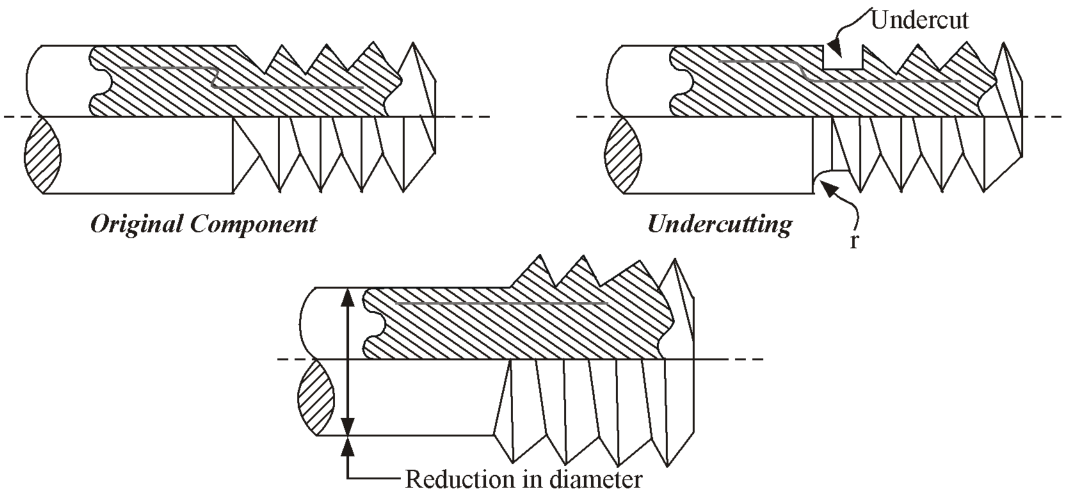
(iii) Drilling Additional holes for shaft



Original Shaft *Drilled Holes* *Fillet Radius*

Reduction of Stress Concentration in Shaft with Keyway:

(iv) Reduction of stress concentration in threaded member



Original Component *Undercutting*

Reduction in Shank Diameter

Reduction of Stress Concentration in Threaded Components

Note: *Stress concentration can be Reduce to some Limit but It can not be eliminate.*

7. FATIGUE STRESS CONCENTRATION FACTOR (k_f)

$$(k_f) = \frac{\text{Endurance Limit of the Notch Free Specimen}}{\text{Endurance Limit of the Notched Specimen}}$$

This factor (k_f) is applicable to actual materials and depends upon the grain size of materials. the greater reduction in endurance limit of fine grained material as compared to coarse-grained materials, due to stress concentration.

8. NOTCH SENSITIVITY

"Noth sensitivity is defined as the susceptibility of a material to succumb to the damaging effect of stress raising notches in fatigue loading."

The Notch Sensitivity factor (q)

$$(q) = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoritical stress over nominal stress}}$$

$$\sigma_0 = \text{Nominal stress}$$
$$(k_f)\,.\,\sigma_0 = \text{Actual Stress}$$
$$(k_t)\,.\,\sigma_0 = \text{Theoritical stress}$$

$$q = \frac{(k_f.\sigma_0 - \sigma_0)}{(k_t.\sigma_0 - \sigma)}$$

$$\boxed{q = \frac{k_f - 1}{k_t - 1}}$$

$$\boxed{k_f = 1 + q(k_t - 1)}$$

Some of the conclvsion are:

(i) When material has no sensitivity to notches

$$q = 0 \text{ and } k_f = 1$$

(ii) When material is fully sensitive to notches

$$q = 1 \text{ and } k_f = k_t$$

9. EFFECTS OF DIFFERENT FACTORS AND ENDURANCE LIMIT

S_e^1 = Endurance limit stress of a rotating beam specimen subjected to reversed bending stress (N/mm^2).

S_e = Endurance limit stress of a praticular mechanical component subjected to reversed bending stress (N/mm^2)

Relationship between

S_e^1 and S_e is

$$S_e = k_a k_b k_c k_d S_e^1$$

where,

k_a = surface finish factor

k_b = size factor

k_c = relaiablity factor

k_d = modifying factor to accunt for stress concentration.

9.1 Surface Finish Factor (k_a) :

For a mirror polished material, the surface finish factor is unity.

9.2 Size Factor (k_b):

Diameter (d) (mm)	**(k_b)**
d = 7.5	1.00
7.5 < d = 50	0.85
d > 50	0.75

9.3 Reliability Factor (k_c):

Reliability R(%)	(k_c)
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

9.4 Modifying Factor to Accoont for Stress Concentration (k_d):

$$\boxed{(k_d) = \frac{1}{(k_f)}}$$

9.5 Relationship Between Enduronce Limit and Ultimate Tensile Strength :

For steel $S_e^1 = 0.5\ S_{ut}$

For cast Iron and cast steel $S_e^1 = 0.4\ S_{ut}$

For wrough aluminum alloys $S_e^1 = 0.4\ S_{ut}$

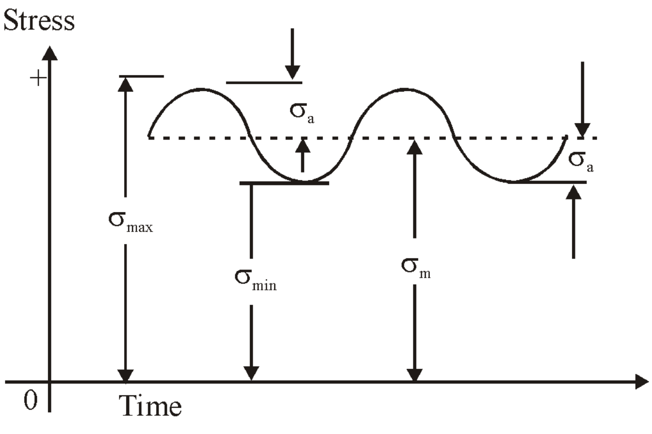
For cast aluminium alloys $S_e^1 = 0.3\ S_{ut}$

10. REVERSED STRESSES-DESIGN FOR FINITE AND INFINITE LIFE

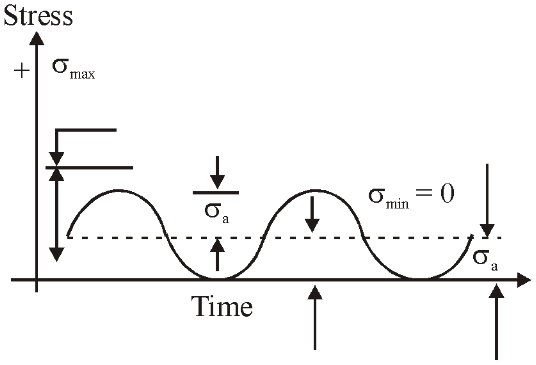
Infinite Life : There are two types of problems in fatigue design :

(i) Components subjected to completely reversed stress.

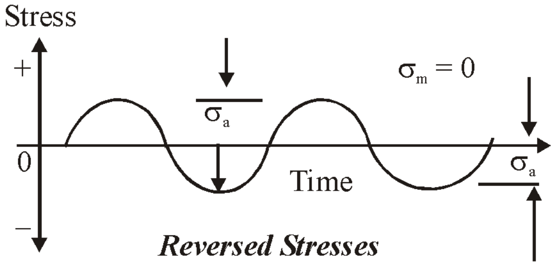
(ii) Components subjected to fluctuating stresses.



Reversed Stresses



Repeated Stresses



Reversed Stresses

The mean stress is zero in case of completely reversed stresses. The stress distribution consists of tensile stresses for the first half cycle and compressive stresses for the remaining half cycle and the stress cycle passes through zero. In case of fluctuating stresses, there is always a mean stress, and the stresses can be purely tensile, purely compressive or mixed depending upon the magnitude of the mean stress. Such problems are solved with the help of the modified.

- The design problems for completely reversed stresses are further divided into two groups:

  (i) Design for infinite life

  (ii) Design for finite life

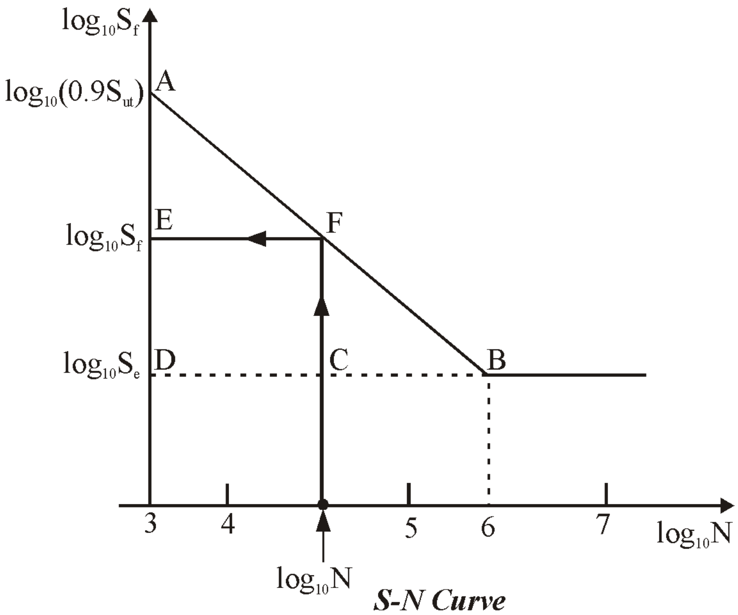
Case-I : When the component is to be designed for infinite life, the endurance limit becomes the criterion of failure. The criterion of failure. the amplitude stress induced in such components should be lower than the endurance limit in order to withstand the infinite number of cycles. Such components are designed with the help of the following equations :

$$\sigma_a = \frac{S_e}{(f_s)}$$

$$\tau_a = \frac{S_{se}}{(f_s)}$$

where, (σ_a) and (τ_a) are stress amplitudes in the component and S_e and S_{se} are corrected endurance limits in reversed bending and torsion respectively.

Case-II : When the component is to be designed for finite life, the S-N curve as shown in figure can be used.



S-N Curve

The curve is valid for steels. ***It consists of a straight line AB drawn from (0.9 S_{ut}) at 10^3 cycles to (S_e) at 10^6 cycles on a long-long paper.***

the design procedure for such problems is as follows :

(i) Locate the point A with co-ordinates

$[3, \log_{10} (0.9\ S_{ut})]$ since $\log_{10}(10^3) = 3$

(ii) Locate the point B with co-ordinates

$[6, \log_{10}(S_e)$ since $\log_{10}(10^6)] = 6$

(iii) Join $\overline{AB}$, which is used as a criterion of failure for finite-life problems.

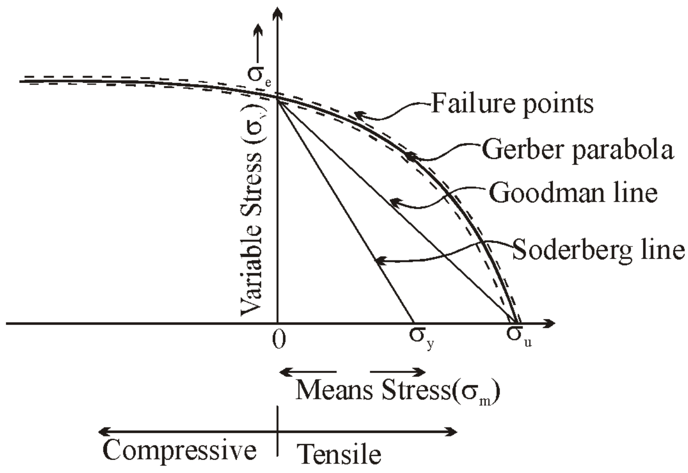
(iv) Depending upon the life N of the component, draw a vertical line passing through $\log_{10}(N)$ on the abscissa. This line intersects $\overline{AB}$ at point F.

(v) Draw a line $\overline{FE}$ parallel to the abscissa. The ordinate at the point E, i.e., $\log_{10}(S_f)$, gives the fatigue strength corresponding to N cycles.

The value of the fatigue strength (S_f) obtained by the above procedure is used for the design calculations.

11. COMBINED MEAN AND VARIABLE STRESSES

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in figure as function of variable stress (σ_v) and mean stress (σ_m). The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failure are rare when the mean stress is compressive or (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.



11.1 Geber Method for Combination of Stress :

The relationship between variable stress (σ_y) and mean stress (σ_m) for axial and bending loading for ductile materials are shown in figure. The point σ_e represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point σ_u represents the static ultimate strength corresponding to $\sigma_v = 0$.

$$\boxed{\sigma_v = \sigma_e\left[\frac{1}{F.S.} - \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S.\right]}$$

or

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)F.S. + \frac{\sigma_v}{\sigma_e}$$

where,

F.S. = Factor fo Safety

σ_m = Mean stress (Tensile or compressive)

σ_u = Ultimate stress (Tensile or Compressive)

σ_e = Endurance limit for Reversed Loading.

Considering the fatigue stress concentration factor (K_f)

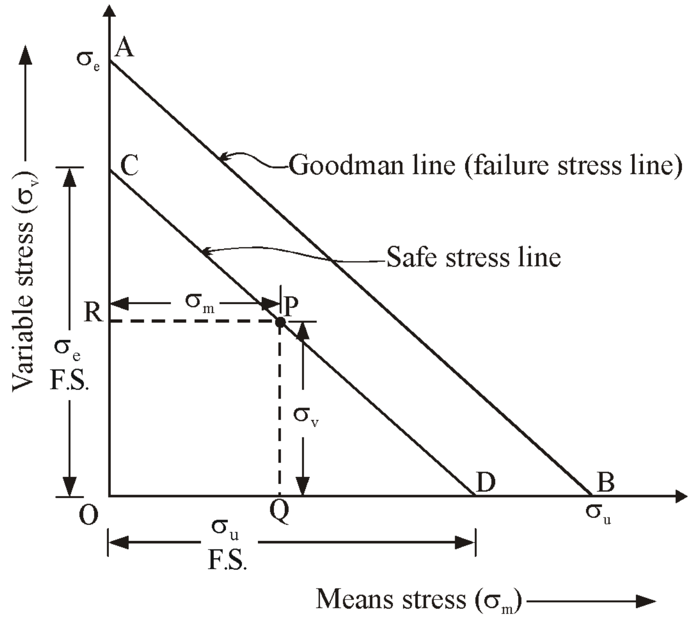
$$\boxed{\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}}$$

11.2 Goodman Method for Combination of Stresses :

A godman line is used when the design is based on ultimate strength. *(Brittle Material)*

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line AB in figure, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

In figure, line AB connecting σ_e and s_u is called Goodman's failure stress line. If a suitable factor of safety (F.S.) is applied to endurance limit and ultimate strength, a safe stress line CD may be drawn parallel to the line AB. Let us consider a design point P on the line CD.



Goodman Method

Now from similar traingles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD}$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration. In such cases the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v).

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e}$$

Where, F.S. = Factor of safety

σ_m = Mean stress

σ_u = Ultimate stress

σ_v = Variable stress

σ_e = Endurance limit for reversed loading

K_f = Fatigue stress concentration factor

Considering the load factor, sufrace finish factor and size factor.

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}}$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad (\because \quad \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

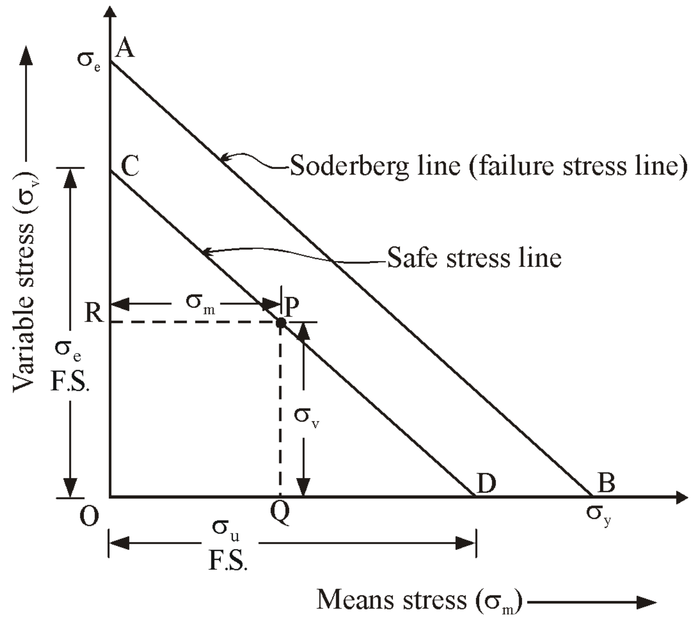
Where, K_b = Load factor for reversed bending load

K_{sur} = Surface finish factor

K_{sz} = Size factor.

11.3 Soderberg Method for Combination of Stresses :

This line is used when design is based on yield strength. *(Ductile Material)*



Soderberg Method

$$\boxed{\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}}$$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (K_f) should be applied to only variable stress (σ_v).

$$\boxed{\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}}$$

Considering the load factor, sufrace finish factor and size factor.

$$\boxed{\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}}$$

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