Toppersnotes
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RRB - JE

ELECTRICAL

Railway Recruitment Board

Volume - 1

Network Theory

SIERRA INNOVATIONS

CONTENTS

BASICS OF CIRCUITS & CIRCUITAL LAWS

THEORY

1.1. *INTRODUCTION*

The phenomenon of transferring charge from one point to another is termed as electric current. An electric current may be defined as the time rate of electric charge q across-sectional boundary. Random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge.

Conductivity is the ability of the path to transfer electrons. The resistivity is the resistance offered to passage of electron. Resistance is the inverse of conductance.

Charge : Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field. The smallest charge that exist is the charge carried by e^- is equal to -1.6×10^{-19} The MKS unit of charge is Coulomb (C). When an electron is removed from a substance, that substance becomes positively charged. A substance with excess of electrons is negatively charged.

Current : The time rate of flow of electric charge across a cross-sectional boundary is termed as current. Mathematically, current is given by

$$i = \frac{dq}{dt}$$

If the charge q is given in coulombs, and the time t in seconds, the current is measured in amperes and denoted by the letter 'A'. or columb/second. The charge transfer between t_0 and t is obtain by $q(t) = \int_{t_0}^{t} i(t)dt$

Voltage : Voltage is defined as work done in moving a unit positive charge once around the closed path. Mathematically, voltage is given by

$$V = \frac{dW}{dq}$$

The unit of potential is Volt (V).

Power : The instantaneous power delivered to a circuit element is the product of the instantaneous value of voltage and current of the element. Mathematically power is given by the relation.

$$p(t) = v(t)\, i(t)$$

The unit of power is Watt (W).

In terms of energy power is defined as, "The time rate of change of energy is called power". Mathematically, power is given by

$$P = \frac{dW}{dt}$$

Energy : The energy delivered to a circuit element over the time interval (t_0, t) is given by

$$E = \int_{t_0}^{t} p(t)dt = \int_{t_0}^{t} v(t)i(t)dt$$

The unit of energy is Watt-sec. or Joules.

1.2. *CIRCUIT COMPONENTS*

(i) Resistance : It is a linear, two terminal component which opposes the flow of current through it. Resistance dissipates energy as heat or some other way. It is measured in Ohms (Ω).

Most materials used to carry current in the form of wires. The resistance R of a conductor varies directly with the length *l* and is inversely proportional to the cross sectional area A.

i.e. $$R = \rho \frac{\ell}{A}$$

where ρ is the constant and is called resistivity. The unit of ρ is ohm-meter.

The ambient temperature of a body is the temperature surrounding it. When the ambient temperature of a resistor is varied, a change in resistance is noted which is related by the following relation

$$R_t = R_0\left[1+\alpha\left(T_t - T_0\right)\right]$$

where, R_0 is the resistance at temperature T_0 = 0°C and R_t is the resistance at temperature T_t°C.

Resistances can be connected either in series or in parallel.

Series connection of resistors : If n resistors are connected in series then these can be replaced by a single resistance R_{eq} such that

$$R_{eq} = R_1 + R_2 + \ldots\ldots + R_n$$

In series combination of resistance, current remains same.

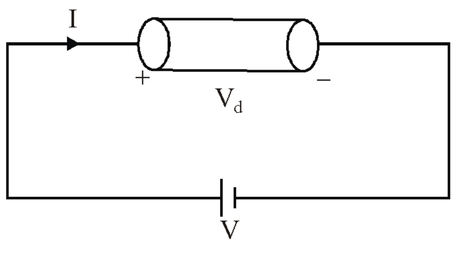
Parallel combination of resistors : In n resistances are connected in parallel then these can be replaced by a single resistance R_{eq} such that.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots + \frac{1}{R_n}$$

In parallel combination of resistances, voltage across parallel branches remains constant.

Ohm's Law : Ohm's law states that the current (I) flowing through a conductor is directly proportional to the voltage (V) across the ends of a conductor, provided physical conditions of a conductor such as pressure, mechanical strain, temperature etc. are kept constant.

$$I \propto V$$
$$I = G \times V$$



Ohm's law also state that at constant temperature potential difference across an element is directly proportional to current flowing through the element.

i.e. $V_d \propto I$

or $V_d = IR$

(ii) Inductance : It is a property of two terminal circuit element by virtue of which it is capable of storing energy in an electromagnetic field with the help of current passing through it. The device is known as inductor.

Inductance can be two types :

(a) Self Inductance : The property of coil due to which it opposes any increase or decrease of current or flux through it is called Self-inductance.

Self-inductance of a coil is given by the relation

$$L = \frac{N\phi}{I} = \frac{N^2\mu A}{\ell}$$

where, N = number of turns in coil

A = area of cross-section

l = length of coil

I = Current in ampere

ϕ = flux in Weber

L = Coefficient of self-inductance in Henry.

As per lenz's law, this self induced e.m.f. opposes the cause (i.e. current in the coil). This property which opposes any change in the current is called self inductance. Current through inductor is given by

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)dt$$

Energy stored by an inductor

$$W_L = \frac{1}{2}Li^2$$

(b) Mutual Inductance (M) : Mutual inductance is the property of two coil because of which each opposes any change in the current flowing through the other by developing an induced emf.

Coefficient of Coupling : Coefficient of coupling (K) between two coils is the measure of coupling of two coils.

$$M = 1$$

Mathematically, $$K = \frac{M}{\sqrt{L_1L_2}}, \text{ value of } K < 1.$$

Coefficient of coupling $$K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

Dot Convention for Coupled Circuits

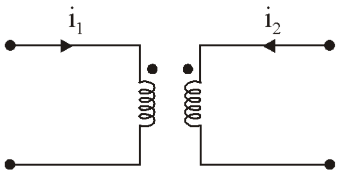


Figure (a)

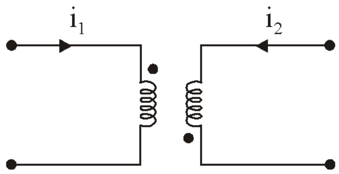


Figure (b)

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A dot placed at any end of the two coils indicates that, these coils are mutually coupled.

Now the mutual inductance M between two coils

(i) Will be positive if the current enters through both the dots or leaves both the dots, (as shown in the above figure (a).

(ii) Will be negative when current enters through a dot in one coil and leaves the dot in another coil as shown in figure (b) above.

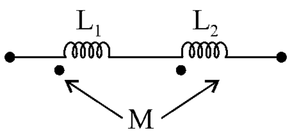
Series combination of inductors : If a number of inductors are connected in series, then they can be replaced by a single inductor of inductance L_{eq} such that

$$L_{eq} = L_1 + L_2 + L_3 + ... + L_n$$

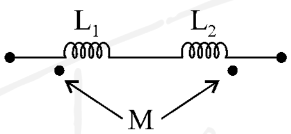
Parallel combination of inductors : If a number of inductors are arranged in a parallel to one another then the equivalent inductance is given by:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ... + \frac{1}{L_n}$$

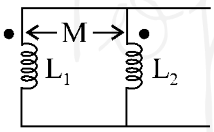
Combination of Inductors with Mutual Inductance

(i) 

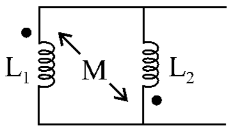
$$L_{eq} = L_1 + L_2 + 2M$$

(ii) 

$$L_{eq} = L_1 + L_2 - 2M$$

(iii) 

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

(iv) 

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

(iii) Capacitance : An electrical element which stores energy due to voltage across it and is independent of the current through it is called capacitor. The unit of capacitance is Farad and denoted by 'C'.

The voltage across the capacitance is given by

$$V = \frac{Q}{C} = \frac{1}{C}\int_{-\infty}^{t} i\,dt \quad (\therefore \text{ where Q is the charge on capacitance in coulombs.})$$

The current through the capacitor is given by

$$i = C\frac{dV}{dt}$$

For a parallel plate capacitor whose plates are having an area of A (in square meter) each the separation between the plates is d (in meters). Then capacitance is given by

$$C = \frac{\varepsilon A}{d} \text{ Farad}$$

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Various types of capacitors are as follows

(a) Ceramic capacitor : They are manufactured by depositing directly on each side of ceramic dielectric, coating of silver that serves as capacitor plates. These capacitors have very high capacitance per unit volume.

(b) Paper capacitor : They are manufactured by winding long narrow sheets of alternate layers of aluminium foil and was impregnated paper, into impact rolls.

(c) Electrolytic capacitors : They can only be used in circuits that maintains the polarity of the voltage in one direction. If the voltage polarity is reversed, the capacitor acts as a short circuit. These capacitors have a very high values of capacitance ranging from 1 to 2μF to the order to several thousand microfarads.

1.3 *CIRCUIT TERMINOLOGY*

(i) **Circuit element :** Any individual circuit component like inductor, resistor, capacitor etc. with two terminals.

(ii) **Branch :** A branch is a group of circuit elements with two terminals. Thus a branch consists of number of elements.

(iii) **Node :** A node is the meeting point of three or more branches at a common point. It is also referred as junction.

(iv) **Mesh and Loop :** A set of branches forming a closed path, with the omission of any branch makes the path open. Mesh must not have any other circuit inside it. Loop may have other loops or meshes inside it.

(v) **Active Network :** A network containing energy sources together with other circuit elements.

(vi) **Passive Network :** A network containing circuit elements without any energy sources.

(vii) **Lumped Network :** Network whose elements are physically separable are called as lumped elements, such as resistors, inductors, capacitors etc.

(viii) **Distributed Network :** A network in which resistors, capacitors, inductors etc. are not electrically separable and isolated as individual elements. A transmission line has distributed resistor, inductor, and capacitor which are not isolated from the network.

(ix) **Bilateral Network :** A bilateral network is defined as those whose elements can transmit well in either direction. For example, elements made of high conductivity material like iron core conductors are bilateral elements.

(x) **Unilateral Network :** A unilateral network is one whose elements follow different laws relating to voltage and current for different direction of current and voltage polarities. For example, vacuum tubes, crystal and metal rectifiers are unilateral elements.

(xi) **Linear and Non Linear Networks :** A linear network is one for which the principle of superposition holds. A circuit element is linear if the relation between current and voltage involves a constant coefficient.

e.g. $$V = Ri, V = L\frac{di}{dt}, \ V = \frac{1}{C}\int i \, dt$$

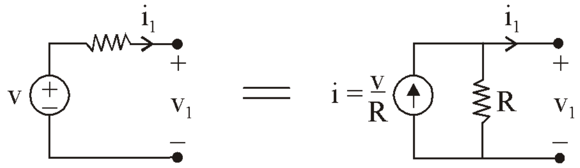
Therefore, it is concluded that a linear network must be bilateral, but a bilateral network is not necessarily linear. For example, an iron-core conductor is bilateral but it is non-linear.

(xii) Current Source : A generator which maintain its output current independent of the voltage across its terminals. It is indicated by a circle enclosing an arrow for reference current direction. An ideal current source has infinite internal resistance.

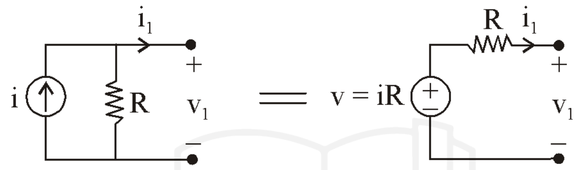
(xiii) Voltage Source : A generator which maintain its value of potential independent of the output current. An AC source is indicated by a circle enclosing a wave line. An ideal voltage source has zero internal resistance.

1.4 *SOURCE TRANSFORMATION*

A voltage source is transformed into its equivalent current source as

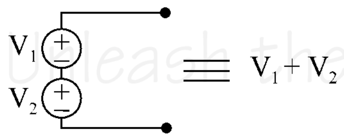


A current source is transformed into its equivalent voltage source as

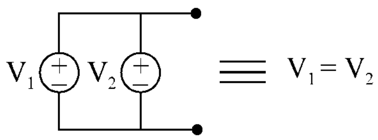


✍ ***Key points :***

- An ideal voltage source has zero internal resistance.
- Ideally the internal resistance of a voltmeter should be infinite.
- An ideal current source has infinite internal resistance.
- Ideally the internal resistance of an ammeter should be zero.
- If two voltage sources are connected in series, they are summed up.



- If two voltage sources are connected in parallel, they are treated as equal.



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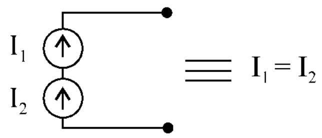
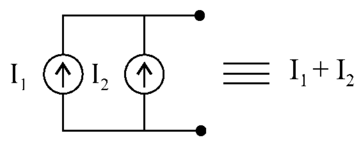


- If two current sources are connected in parallel, they are summed up.



- **Circuit Response of single elements**

Element	Voltage across element	Current in element
Resistance, R	$v(t) = R\,i(t)$	$i(t) = \frac{v(t)}{R}$
Inductance, L	$v(t) = L\frac{di}{dt}$	$i(t) = \frac{1}{L}\int_0^t v\,dt$
Capacitance, C	$v(t) = \frac{1}{C}\int_0^t i\,dt$	$i(t) = C\frac{dv}{dt}$

1.5 *KIRCHOFF'S LAWS*

Kirchoff's basic circuit laws provides two methods for the solution of networks. They are as follows

(i) Kirchoff's Current Law (KCL) : It is based on conservation of charge. It states that, "The algebraic sum of currents at any node of a circuit is zero".

The direction of current coming towards node is taken as positive and outgoing current as negative. The reverse sense of directions can also be taken.

Minimum number of nodal equations to calculate the voltage and current of every port of network is = n – 1 (where, n = number of nodes)

Sign Convention : *Take positive sign for current entering a node and negative sign for leaving a node.*

(ii) Kirchoff's Voltage Law (KVL) : It is based on conservation of energy. It states that, "The algebraic sum of voltages in any closed path of network that is transverse in any single direction is zero".

Sign Convention : *Current i causes a positive drop of voltage when flowing from positive to negative potential and a negative drop when flowing from negative to positive potential.*

Minimum number of loop equations = b – n + 1

where b = number of branches.

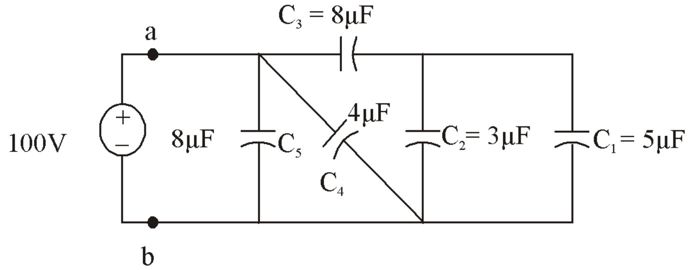
n = number of loops.

Solved Examples

Example : For the circuit shown in figure below

(a) Calculate the equivalent capacitance across the terminals a-b.

(b) Also calculate the charging time to charge these capacitances by a steady direct current of constant magnitude of 10A.



Solution : Equivalent capacitance between terminals a-b is given by-

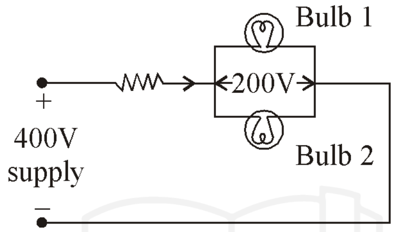
$$C_{eq} = \left[\frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} \| C_4\right] \| C_5 = \left[\frac{(5\mu + 3\mu) \times 8\mu}{5\mu + 3\mu + 8\mu} \| 4\mu\right] \| 8\mu = 4\mu F$$

Then, net charge is given by

$$Q_{net} = C_{eq} \times V = 4 \times 10^{-6} \times 100 = 400\mu F$$

Therefore, charging time is $$t = \frac{Q_{net}}{I} = \frac{400 \times 10^{-6}}{10} = 40\,\mu\text{sec}$$

Example : Two bulbs of rating 100 W and 220 V are required to be connected across a 400 Volts supply. Find the value of the resistance to be inserted in the line so that the voltage across the bulbs does not exceed 220 V



Solution : Total power drawn from the circuit is 2 × 100 = 200 W

Hence supply current I is given by $$I = \frac{200W}{220V} = 0.91A$$

Let R be the resistance connected in series such that the voltage across the bulbs is 220 V

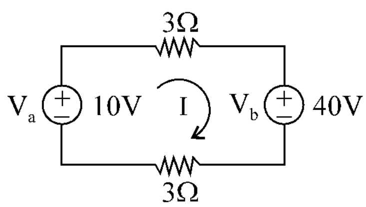
Now Supply voltage = drop in R + voltage across bulbs

$$400 = V_R + 220$$

or $$V_R = IR = 180$$

or $$R = \frac{180}{0.91} = 197.8\Omega$$

Example : The two constant voltage sources V_a and V_b act in the same circuit as shown in figure below. Calculate the power delivered by each voltage source ?



Solution : Applying KVL to the circuit

$$3 \times I + 3 \times I - 10 + 40 = 0$$

or $$I = -5A$$

Power delivered by $V_a = V_a \times I = 10(-5) = -50W$

Power delivered by $V_b = V_b \times I = 40(5) = 200\ W$