



AFCAT

**Air Force Common Admission
Test**

Indian Air Force

Volume - 4

Maths & Reasoning



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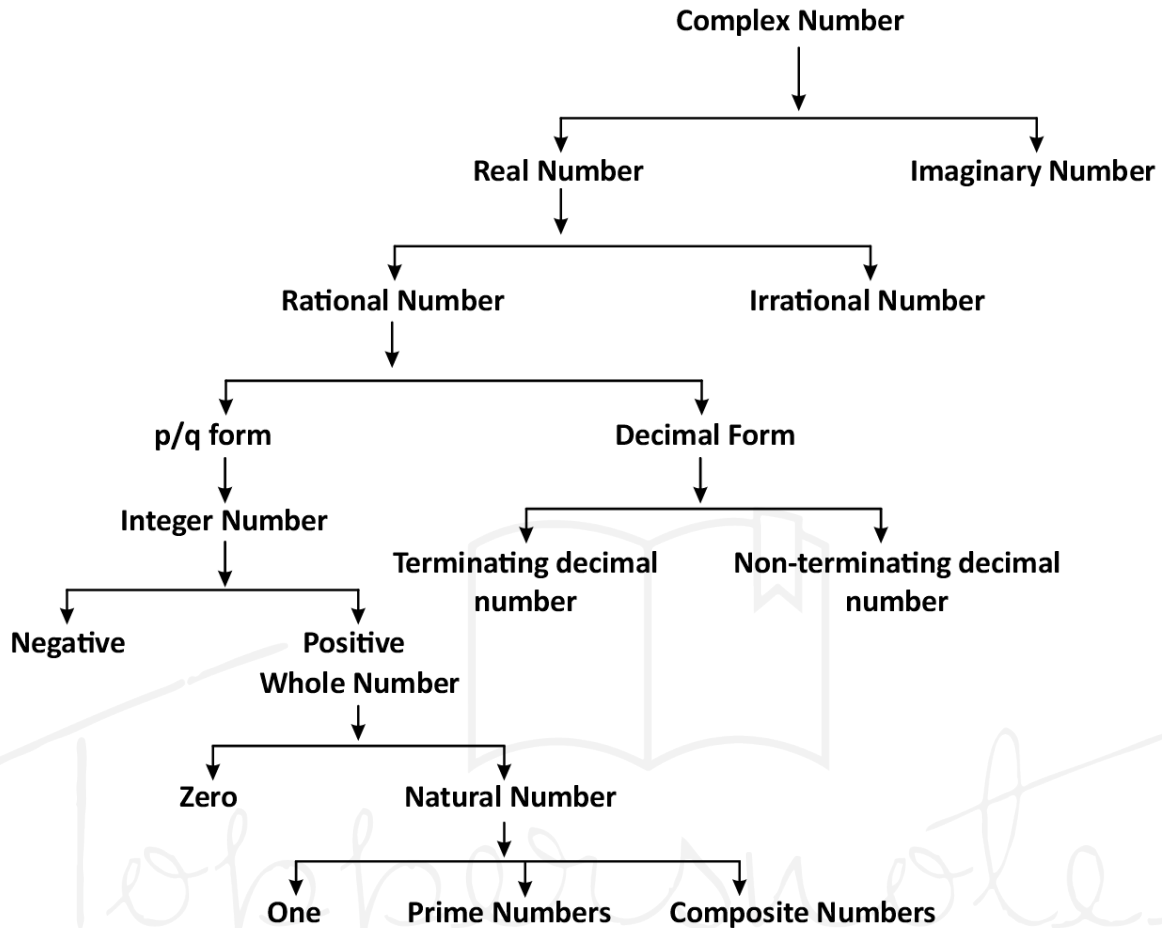
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1 CHAPTER

Number System



Complex Number (Z)

$Z = \text{Real numbers} + \text{Imaginary numbers}$

$$Z = a + ib$$

Where, $a = \text{Real numbers}$.
 $b = \text{Imaginary numbers}$.

Real Numbers

Rational and irrational numbers together are called real numbers. These can be represented on the number line.

Imaginary Numbers

Numbers that can not be represented on the number line.

Integer Numbers

A set of numbers which includes whole numbers as well as negative numbers, is called integer numbers, it is denoted by I .

$$I = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

Natural Numbers

The numbers which are used to count things are called natural numbers.

$$N = \{1, 2, 3, 4, 5, \dots\}$$

Whole Numbers

When 0 is also included in the family of natural numbers, then they are called whole numbers.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

The product of four consecutive natural numbers is always exactly divisible by 24.

Even Numbers

Numbers which are completely divisible by 2 are called even numbers.

$$n^{\text{th}} \text{ term} = 2n$$

$$\text{Sum of first } n \text{ even natural numbers} = n(n+1)$$

Sum of square of first n even natural

$$\text{numbers} = \frac{2n(n+1)(2n+1)}{3}$$

$$\left\{ n = \frac{\text{Last term}}{2} \right\}$$

Odd Numbers

The numbers which are not divisible by 2 are odd numbers.

$$\text{Sum of first } n \text{ odd numbers} = n^2$$

$$\left\{ n = \frac{\text{Last term} + 1}{2} \right\}$$

Natural Numbers

$$\text{Sum of first } n \text{ natural numbers} = \frac{n(n+1)}{2}$$

$$\begin{aligned} \text{Sum of square of first } n \text{ natural numbers} \\ = \frac{n(n+1)(2n+1)}{6} \end{aligned}$$

Sum of cube of first n natural numbers =

$$\left[\frac{n(n+1)}{2} \right]^2$$

The difference of the squares of two consecutive natural numbers is equal to their sum.

$$\text{Example - } 11^2 = 121$$

$$12^2 = 144$$

$$11 + 12 \rightarrow 23$$

$$\text{Difference } 144 - 121 = 23$$

Prime Numbers – Which have only two forms - $1 \times$ numbers

$$\text{E.g. - } \{2, 3, 5, 7, 11, 13, 17, 19, \dots\}$$

Where, 1 isn't a Prime Number.

- The digit 2 is only even prime number.
- 3, 5, 7 is the only pair of consecutive odd prime numbers.
- Total prime numbers between 1 to 25 = 9
- Total prime numbers between 25 to 50 = 6
- There are total of 15 prime numbers between 1-50.
- There are total of 10 prime numbers between 51 – 100.
So there are total 25 prime numbers from 1-100.
- Total prime numbers from 1 to 200 = 46
- Total prime numbers from 1 to 300 = 62
- Total prime numbers from 1 to 400 = 78
- Total prime numbers from 1 to 500 = 95

Co-prime Numbers

Numbers whose HCF is only 1.

$$\text{E.g. - } (4,9), (15, 22), (39, 40)$$

$$\text{HCF} = 1$$

Perfect Number

A number whose sum of its factors is equal to that number (except the number itself in the factors)

$$\text{E.g. - } 6 \rightarrow 1, 2, 3 \rightarrow \text{Here } 1 + 2 + 3 \rightarrow 6$$

$$28 \rightarrow 1, 2, 4, 7, 14 \rightarrow 1 + 2 + 4 + 7 + 14 \rightarrow 28$$

Rational Numbers

Numbers that can be written in the form of P/Q , but where Q must not be zero and P and Q must be integers.

$$\text{E.g. - } 2/3, 4/5, \frac{10}{-11}, \frac{7}{8}$$

Irrational Numbers

These cannot be displayed in P/Q form.

$$\text{E.g. - } \sqrt{2}, \sqrt{3}, \sqrt{11}, \sqrt{19}, \sqrt{26} \dots$$

Perfect square numbers



Unit Digit which can be of square

0
1
4
5 or 25
6
9

Which can't be square

2 —
3 —
7 —
8 —

- The last two digits of the square of any number will be the same as the last two digits of the square of numbers 1-24.

Note: Therefore, everyone must remember the squares of 1-25.

Convert to Binary and Decimal –

1. Convert Decimal Number to Binary Number

To find the binary number equivalent to a decimal number, we continuously divide the given decimal number by 2 until we get 1 as the final quotient.

E.g.

2	89	$2 \times 44 = 88 ; 89 - 88 = 1$
2	44	$2 \times 22 = 44 ; 44 - 44 = 0$
2	22	$2 \times 11 = 22 ; 22 - 22 = 0$
2	11	$2 \times 5 = 10 ; 11 - 10 = 1$
2	5	$2 \times 2 = 4 ; 5 - 4 = 1$
2	2	$2 \times 1 = 2 ; 2 - 2 = 0$
	1	Final quotient

Hence, binary number equivalent to 89 = $(1011001)_2$

2. Convert Binary to Decimal Number

In binary system the value of 1 when it moves one place to its left every time it doubles itself and wherever 0 comes its value is 0.

E.g.

1	0	1	1	0	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Now

$$\begin{aligned} (1011001)_2 &= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 64 + 0 + 16 + 8 + 8 + 0 + 1 \quad \{2^0 = 1\} \\ &= 89 \end{aligned}$$

Finding the Number of Divisors or Number of Factors

First we will do the prime factorization of the number and write it as Power and multiply by adding

One to each power, then the number of divisors will be obtained.

Ex: By how many total numbers can 2280 be completely divided?

Sol. $2280 = 2^3 \times 3^1 \times 5^1 \times 19^1$

$$\begin{aligned} \text{Number of divisors} &= (3 + 1) (1 + 1) (1 + 1) (1 + 1) \\ &= 4 \times 2 \times 2 \times 2 = 32 \end{aligned}$$

Find the unit's digit

1. When the number is in the form of power –

When the unit digit of Base is 0, 1, 5 or 6, the unit digit of the result remains the same for any natural power. When the unit digit of base is 2, 3, 4, 7, 8, or 9, divide the power by 4 and put the same power on the unit digit of the base as the remainder. When the power is rounded off to 4, then the 4th power will be placed on the unit digit of the base.

2. In the form of simplification –

Write the unit digit of each number and simplify it according to the symbol, the result that will come will be its unit digit answer.

Divide by Power of Numbers (Finding the Divisor)

- If $a^n + b^n$ is given –
If n is odd, then $(a+b)$ will be its divisor.
- If $a^n - b^n$ is given –
Divisor (when n is odd) $\rightarrow (a-b)$
Divisor (when n is even) $\rightarrow (a - b)$ or $(a + b)$ or both.

- If $a^n \div (a - 1)$ then the remainder always be 1.
- $a^n \div (a + 1)$ $\left\{ \begin{array}{l} \text{If } n \text{ is an even then the remainder always be 1.} \\ \text{If } n \text{ is an odd then the remainder always be } a. \end{array} \right.$
- If $(a^n + a) \div (a - 1)$ then the remainder always be 2 .
- $(a^n + a) \div (a + 1)$ $\left\{ \begin{array}{l} \text{If } n \text{ is an even then the remainder always be zero (0).} \\ \text{If } n \text{ is an odd then the remainder always be } (a - 1) \end{array} \right.$

Terminating Decimal

Those numbers which end after a few digits after the decimal like - 0.25, 0.15, 0.375 can be written in a fraction number.

Non-Terminating Decimal

Those numbers which continue after the decimal and can be of two types.

0.3333, 0.7777, 0.183183183.....

Repeating

Numbers that never end after the decimal, but repeat, till infinity. It can be written in fractions.

Non Repeating Decimal

Numbers that never end after the decimal point, but they do not repeat their numbers.

Recurring Decimal Fraction

That decimal fraction is the repetition of one or more digits after the decimal point, then one or more digits are repeated after the dot.

Eg. $\frac{1}{3} = 0.333\dots$, $\frac{22}{7} = 3.14285714\dots$. To represent such fractions, a line is drawn over the repeating digit.

$$0.\overline{3524} = \frac{3524 - 35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$$

$$\frac{22}{7} = 3.\overline{14285714\dots} = 3.142857$$

It is called bar.

- Convert pure recurring decimal fraction to simple fraction as follows –

$$0.\overline{p} = \frac{p}{9} \quad 0.\overline{pq} = \frac{pq}{99} \quad 0.\overline{pqr} = \frac{pqr}{999}$$

- Convert a mixed recurring decimal fraction to an ordinary fraction as follows –

$$0.p\overline{q} = \frac{pq - p}{90} \quad 0.pq\overline{r} = \frac{pqr - pq}{900}$$

$$0.p\overline{q}r = \frac{pqr - p}{990} \quad 0.pq\overline{rs} = \frac{pqrs - pq}{9900}$$

Example -

$$(i) 0.\overline{39} = \frac{39}{99} = \frac{13}{33}$$

$$(ii) 0.\overline{625} = \frac{625 - 6}{990} = \frac{619}{990}$$

$$(iii) 0.\overline{3524} = \frac{3524 - 35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$$

Symbol of the Roman Method

1	→	I
2	→	II
3	→	III
4	→	IV
5	→	V
6	→	VI
7	→	VII
8	→	VIII
9	→	IX
10	→	X
20	→	XX
30	→	XXX
40	→	XL
50	→	L
100	→	C
500	→	D
1000	→	M

Rule of Divisibility

Rule of 2	The last digit is an even number or zero (0) as - 236, 150, 1000004
Rule of 3	If the sum of the digits of a number is divisible by 3, then the whole number will be divisible by 3. E.g. 729, 12342, 5631
Rule of 4	Last two digits are zero or divisible by 4. E.g. 1024, 58764, 567800
Rule of 5	The last digit is zero or 5. E.g. 3125, 625, 1250
Rule of 6	If a number is divisible by both 2 and 3 then it is also divisible by 6. E.g. 3060, 42462, 10242
Rule of 7	After multiplying the last digit of a number by 2 and

	subtracting it from the remaining number, if the number is a multiple of 0 or 7 or if any digit is repeated in a multiple of 6, then the number will be divisible by 7. E.g. 222222, 444444444444, 7854
Rule of 8	If the last three digits of a number are divisible by 8 or the last three digits are '000' (zero). E.g. 9872, 347000
Rule of 9	If the sum of the digits of a number is divisible by 9, then the whole number will be divisible by 9.
Rule of 10	The last digit should be zero (0).
Rule of 11	If the difference between the sum of digits at odd places and sum of digits at even places is zero (0) or 11 or a multiple of 11. E.g. 1331, 5643, 8172659
Rule of 12	Composite form of divisible by 3 and 4.
Rule of 13	Repeating the digit 6 times, or multiplying the last digit by 4 and adding it to the remaining number, if the number is divisible by 13, then the whole number will be divisible by 13. E.g. 222222, 17784

Practice Questions

- Q.1** If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of that number, then what will be $\frac{5}{3}$ of that number?
 (a) 12 (b) 18
 (c) 15 (d) 20
- Q.2** If the sum of two numbers is a and their product is b then their reciprocals will be –
 (a) $\frac{1}{a} + \frac{1}{b}$ (b) $\frac{b}{a}$
 (c) $\frac{a}{b}$ (d) $\frac{a}{ab}$
- Q.3** The sum of two numbers is 75 and their difference is 25, then what will be the product of those two numbers?
 (a) 1350 (b) 1250
 (c) 1000 (d) 125
- Q.4** Divide 150 into two parts such that the sum of their reciprocal is $\frac{3}{112}$. Calculate both parts.
 (a) 50, 90 (b) 70, 80
 (c) 60, 90 (d) 50, 100
- Q.5** If the sum of any three consecutive odd natural numbers is 147, then the middle number will be –
 (a) 47 (b) 48
 (c) 49 (d) 51

- Q.6** If the product of first three and last three of 4 consecutive prime numbers is 385 and 1001, then find the greatest prime number.
- Q.7** What will be the sum of the even numbers between 50 and 100?
- Q.8** What will be the sum of odd numbers between 50 and 100?
- Q.9** In a division method, the divisor is 12 times the quotient and 5 times the remainder. Accordingly, if the remainder is 36, then what will be the dividend?
 (a) 2706 (b) 2796
 (c) 2736 (d) 2826
- Q.10** What is the unit digits of $(3694)^{1739} \times (615)^{317} \times (841)^{491}$
 (a) 0 (b) 2
 (c) 3 (d) 5
- Q.11** What will be written in the form of $\frac{p}{q}$ of 18.484848....?
 (a) $\frac{462}{25}$ (b) $\frac{610}{33}$
 (c) $\frac{200}{11}$ (d) $\frac{609}{33}$
- Q.12** Put $\frac{0.936 - 0.568}{0.45 + 2.67}$ in the form of rational number.
- Q.13** What will be the common factor of $\{(127)^{127} + (97)^{127}\}$ and $\{(127)^{97} + (97)^{97}\}$?
 (a) 127 (b) 97
 (c) 30 (d) 224

Answer Key

- | | | | |
|----------------|----------------|-----------------|---|
| Q.1 (d) | Q.2 (c) | Q.3 (b) | Q.4 (b) |
| Q.5 (c) | Q.6 13 | Q.7 1800 | Q.8 1875 |
| Q.9 (c) | Q.10(a) | Q.11(b) | Q.12 $\frac{2024}{17205}$ |
| Q.13(d) | | | |

2

CHAPTER

Time & Work

1. Efficiency – The work done by a person in unit time is called efficiency.

2. If a person can complete a work in x days, then he can do a part of the work in $\frac{1}{x}$ day.

$$\text{Efficiency} = \frac{1}{x} \text{ part/day}$$

$$\text{Efficiency} \propto \frac{1}{\text{Number of days}}$$

3. If the efficiency of a person is more then he will complete the work in less days and if the efficiency is less then he will complete in more days.

4. If the time taken to work together is given, then the time taken by one person to complete the work is = $\frac{\text{Multiplication of days}}{\text{Difference of days}}$

5. If a person leaves the work before the completion of the work, then the time taken to complete the work = $\frac{(x+m)y}{x+y}$

Where, x = time of the person who leaves the work before the completion.

m = The number of days before he leaves work.

y = Time of the person who works from beginning to end.

6. If the number of persons engaged in completing a work is changed to $x : y$, then the time taken to complete the work will change in the ratio of $y : x$.

7. A can complete a work in x days and B can complete the same work in y days then both will complete it together.

$$\text{Time} = \frac{xy}{x+y}$$

8. If three persons A, B and C complete it in x , y and z days, then –

$$\text{Time} = \frac{xyz}{xy + yz + xz}$$

Practice Question

Q.1 A can do a piece of work in 6 days and B can do the same work in 9 days. In how many days will they together complete the work?

- (a) 7.5 days (b) 5.4 days
(c) 3.6 days (d) 3 days

Q.2 A can do $\frac{1}{2}$ of a work in 5 days, B can do $\frac{3}{5}$ of the same work in 9 days, C can do $\frac{2}{3}$ of the same work in 8 days, all three together can do that work in how many days will you complete it ?

- (a) 3 days (b) 5 days
(c) $4\frac{1}{2}$ days (d) 4 days

Q.3 A can do a piece of work in 15 days and B in 20 days. If they work together for 4 days, then what part of the work is left now?

- (a) $\frac{8}{15}$ (b) $\frac{7}{15}$
(c) $\frac{1}{4}$ (d) $\frac{1}{10}$

Q.4 A alone can complete a work in 20 days while B alone can do it in 25 days, both work together for 5 days, then B leaves the work, in how many days A alone can complete the remaining work?

Q.5 A, B and C can do a piece of work in 16, $12\frac{4}{5}$ and 32 days respectively.

They start work together. But after 4 days A left the work. B left the work 3 days before the completion of the work. Then tell in how many days the work is completed?

Q.6 A, B, C can complete a work in 24 days, 32 days and 64 days respectively, they start the work together, A leaves the work after 6 days and B leaves the work after 6 days. left the work the day before, then in how many days the work was completed ?

Q.7 A and B can complete a work in 7 days and 8 days respectively, if both work for one day except one day and A starts the work, then in how many days will the whole work be finished?

Q.8 A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if

he is assisted by B and C on every third day?

- (a) 12 days (b) 15 days
(c) 16 days (d) 18 days

Q.9 If 40 men working 13 hours a day can complete a piece of work in 20 days, then in how many hours a day can 26 men complete the work in 25 days?

- (a) 12 hrs (b) 14 hrs
(c) 16 hrs (d) 18 hrs

Q.10 Some carpenters promised to do a job in 9 days but 5 of them were absent and remaining men did the job in 12 days. The original number of carpenters was.

- (a) 24 (b) 20
(c) 16 (d) 18

Q.11 A is 25% more efficient than B, and B takes 6 days more than A to complete a piece of work. How many days will B take to complete the same work?

- (a) 30 days (b) 20 days
(c) 24 days (d) 28 days

Q.12 A and B together can do a piece of work in 5 days, if A works 2 times its efficiency and B works $\frac{1}{3}$ times its

efficiency, then the work is completed in 3 days, then in how many days A alone will do that work ?

Answer Key

Q.1 (c) **Q.2** (d) **Q.3** (a) **Q.4** 11 days

Q.5 9 days **Q.6** 20 days **Q.7** $7\frac{3}{7}$ days **Q.8** (b)

Q.9 (c) **Q.10**(b) **Q.11**(a) **Q.12** $6\frac{1}{4}$ days

3

CHAPTER

Average

Average = Sum of Trials / Number of Trials

Number Based Average (Formula)

1. Average of first n natural numbers

$$= \frac{n+1}{2}$$
2. Average of first n consecutive even numbers = $(n + 1)$
3. Average of first n consecutive odd numbers = n
4. Average of squares of first n natural numbers = $\frac{n+1}{6} \cdot \frac{2n+1}{2}$
5. Average of cubes of first n natural numbers = $\frac{n}{4} \cdot \frac{n+1}{2}^2$
6. Average of odd numbers from 1 to $n = \frac{n+1}{2}$, (where n = last odd number)
7. Average of even numbers from 1 to $n = \frac{n+2}{2}$, (where n = last even number)
8. If the speed in covering the same distance is a km/hr and b km/hr respectively, then the average speed will be

$$= \frac{2ab}{a+b}$$
9. If the speed for the same distance is a km/hr, b km/hr and c km/hr, then The average speed will be

$$= \frac{3abc}{ad+bc+ca}$$
 km/hr
10. One of the P persons, whose average weight is x kg, leaves, a new person comes in his place, due to which the average weight of the persons is increased by y kg, then the weight of the new person = $(x + P \times y)$ kg

11. The average age of P persons is X years. When Q persons join, the average age becomes y years, then the average age of the new persons = $x + y - x \times \frac{P+Q}{Q}$ years
12. The average age of P persons is x years. When Q persons leaves the average age of the person decreased by y years, then the average age of the persons who went out = $x - \left[y - x \times \frac{P-Q}{Q} \right]$ years
13. The average age of x children is y years. If the age of the father is added to the ages of the children, then their average age becomes z years. Father's age = $z \times (x + 1) - y \times x$ years
14. The average age of P students is x years. The average age of the students when one student left out becomes y years, then the average age of the outgoing student becomes = $P \times x - (P - 1)y$ years
15. If the average value of salary of total P employees and officers in an institution is ₹ x per month and the average value of salary of officers is ₹ y per month and the average value of salary of employees is ₹ z per month, then the total number of employees in the organization

$$= \frac{x - y \times P}{z - y}$$
16. If each amount is multiplied by x , then the average also becomes x times.
17. Finding bowler's average
 Bowler's average = Total runs / Numbers of wickets
 Total runs = $A \times x$, x = Numbers of wicket

Practice Question

- Q.1** The average of 18 numbers is 42. If each is multiplied by $1\frac{1}{2}$ what will be the average of the resulting number?
- Q.2** What is the average of the first 50 natural numbers ?
- Q.3** What is the average of the squares of the first 10 natural numbers ?
(a) 35.5 (b) 36
(c) 37.5 (d) 38.5
- Q.4** The sum of 5 numbers is 240. Out of these, the average of the first two numbers is 30 and that of the last two numbers is 70, what is the third number ?
(a) 40
(b) 60
(c) 75
(d) Cannot be determined
- Q.5** The average of two numbers A and B is 20, that of B and C is 19 and that of C and A is 21, what is the value of A ?
(a) 24 (b) 22
(c) 20 (d) 18
- Q.6** A batsman scored 63 runs in the twelfth innings which increases his average of runs by 2 runs, what is the average of his runs after the twelfth innings ?
(a) 41 (b) 42
(c) 34 (d) 35
- Q.7** In a cricket match, the run rate in the first 10 overs was only 3.2. What should be the run rate for the remaining 40 overs to complete the target of 282 runs ?
(a) 6.25 (b) 6.50
(c) 6.75 (d) 7.0
- Q.8** The average speed of a motorcycle from a place A to B was 65 km/h and from B to A its average speed was 60 km/h. What was the average speed of the motorcycle for the whole journey?
(a) 62.5 km/h (b) 72.2 km/h
(c) 62.4 km/h (d) 60.8 km/h
- Q.9** A car travels at V_1 km/hr for t_1 hour and travels at V_2 km/hr for t_2 hour. What is the average speed of the car during the whole journey ?
(a) $\frac{t_1 + t_2}{v_1 t_1 + v_2 t_2}$ km/h
(b) $\frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$ km/h
(c) $\frac{v_1 t_2 + v_2 t_1}{v + v_2}$ km/h
(d) $\frac{v_1 + v_2}{v_1 t_1 + v_2 t_2}$ km/h
- Q.10** An airplane travels at a speed of 200, 400, 600 and 800 km/hr over the four sides of a square field. Then find the average speed of the airplane for the whole journey ? (in km/h)
(a) 600 km/h (b) 400 km/h
(c) 500 km/h (d) 384 km/h
- Q.11** The average height of 25 boys is 1.4 m. After 5 boys from this group leave the camp, the average height of the remaining boys increases by 0.15 m. What is the average height of the 5 boys who left ?
(a) 0.8 m (b) 0.9 m
(c) 0.95 m (d) 1.05 m
- Q.12** Out of 9 persons, 8 persons spent ₹ 30 each for their meals. The ninth one spent ₹ 20 more than the average expenditure of all the nine. The total money spent by all of them was ?
(a) ₹ 260 (b) ₹ 290
(c) ₹ 292.50 (d) ₹ 400.50

- Q.13** Three years ago, the average age of a family of 5 members was 17 years. A baby having been born the present average age of the family is the same as the average age 3 years ago. Find the present age of the baby ?
(a) 2 years (b) 2.4 years
(c) 3 years (d) 1.5 years
- Q.14** The present age of Ram is thrice the present age of his son and that of his father is $\frac{2}{5}$. The average age of all three is 46 years. What is the difference between the present ages of Ram and his father ?
(a) 68 years
(b) 58 years
(c) 54 years
(d) Cannot be determined
- Q.15** In a primary school, the average age of the students is 8 years and the average age of 12 teachers is 45 years. If the average age of all is 9 years, then what are the number of students?
(a) 432 (b) 540
(c) 408 (d) 416
- Q.16** The average number of printing errors in a book of 512 pages is 4 per page. If the number of these errors in the first 302 pages is 998, then what is the average number of errors per page in the remaining pages ?
(a) 4 (b) 5
(c) 5.5 (d) 6.5
- Q.17** The arithmetic mean of the scores of a group of students in an examination was 52. Among them, the best 20% students got an average of 80 marks and the worst 25% students got an average of 31 marks, then what was the average of the marks of the remaining 55% students?
(a) 45 (b) 50
(c) 51.4 approx. (d) 54.6 approx.
- Q.18** The average marks of the students in a group is 63. Out of these the marks of 3 are 78, 69 and 48, what is the average marks of the remaining 6 students?
(a) 63.5 (b) 64
(c) 63 (d) 62

Answer Key

Q.1 $42 \times \frac{3}{2} = 63$

Q.2 We know that, $1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$

$$\therefore 1 + 2 + 3 + \dots + 50 = \left(\frac{1}{2} \times 50 \times 51 \right) = 1275$$

So, the required average $= \frac{1275}{50} = \frac{51}{2} = 25.5$

Q.3 We know that, $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1)$

$$\therefore 1^2 + 2^2 + 3^2 + \dots + 10^2 = \left(\frac{1}{6} \times 10 \times 11 \times 21 \right) = 385$$

So, the required average $= \frac{385}{10} = 38.5$

Q.4 Sum of 5 numbers = 240

Sum of first 2 numbers = $(30 \times 2) = 60$

Sum of last two numbers = $(70 \times 2) = 140$

\therefore Third number = $240 - (60 + 140) = 40$

Q.5 $(A + B) = (20 \times 2) = 40$

$(B + C) = (19 \times 2) = 38$ and $(C + A) = (21 \times 2) = 42$

On adding: $2(A + B + C) = (40 + 38 + 42) = 120$

$\Rightarrow (A + B + C) = 60$

$\therefore A = (A + B + C) - (B + C) = (60 - 38) = 22$

Q.6 Let the average number of runs after 11 innings = x

Then, average number of runs after 12 innings = $(x + 2)$

$\therefore 12(x + 2) - 11x = 63$

$\Rightarrow 12x - 11x = 63 - 24 \Rightarrow x = 39$

Average run count after 12 innings = $(x + 2) = (39 + 2) = 41$

Short Approach:

Average after twelfth innings = Runs of last innings - (Number of innings - 1) \times Increase in average

$$= 63 - (12 - 1) \times 2$$

$$= 63 - 22 = 41$$

Q.7 Number of runs scored in 10 overs = $(3.2 \times 10) = 32$

Number of runs to be scored in next 40 overs = $282 - 32 = 250$

Run-rate for next 40 overs = $\frac{250}{40} = 6.25$

Q.8 Here, $x = 65$ and $y = 60$

$$\therefore \text{Average speed} = \frac{2xy}{x+y} \text{ km/h}$$

$$= \frac{2 \times 65 \times 60}{65 + 60} \text{ km/h}$$

$$= \frac{2 \times 65 \times 60}{125} \text{ km/h}$$

$$= \frac{312}{5} \text{ km/h} = 62.4 \text{ km/h}$$

Q.9 Distance covered in t_1 hours = t_1v_1 km

Distance covered in t_2 hours = t_2v_2 km

Total Distance = $t_1v_1 + t_2v_2$

Total Time = $t_1 + t_2$

$$\therefore \text{Average Speed} = \frac{v_1t_1 + v_2t_2}{t_1 + t_2} \text{ km/h}$$

Q.10 Let one side of the square be x km long.

\therefore Total distance = $4x$ km

Total time

$$= \frac{x}{200} + \frac{x}{400} + \frac{x}{600} + \frac{x}{800} = \frac{x}{96} \text{ hr}$$

$$\therefore \text{Average Speed} = \frac{4x \times 96}{x} = 384 \text{ km/h}$$

Short Approach:

A — 200					12
B — 400					—6
C — 600		2400			—4
D — 800					3

$$\frac{2400}{\frac{25}{4}} = \frac{2400 \times 4}{25}$$
$$= 96 \times 4 = 384 \text{ km/hr}$$

Q.11 Sum of height of 25 boys = (1.4×25) m = 35 m

Average height of 20 boys = $(1.4 + 0.15)$ m = 1.55 m

Sum of height of 20 boys = (1.55×20) m = 31 m

Sum of height of 5 boys = $(35 - 31)$ meters = 4 meters

Average height of 5 boys = $\frac{4}{5}$ meter = 0.8 meter

Q.12 Let the average expenditure of 9 persons be = x

According to the question, $\frac{30 \times 8 + x + 20}{9} = x$

$$240 + 20 + x = 9x$$

$$\Rightarrow 260 = 8x$$

$$x = \frac{260}{8} = x = 32.5$$

$$\text{Total expenditure} = 32.5 \times 9 = ₹ 292.50$$

Q.13 Let the age of the child be = x years.

Total age of family 3 years ago = 85 years

Presently the total age of the family = 85 + 15 = 100 years

According to question,

$$\frac{100+x}{6} = 17$$

$$\Rightarrow 100 + x = 102$$

$$\Rightarrow x = 2 \text{ years}$$

Short Approach:

Family	3 years ago	Present
	85	100
Family + Child	–	102
		Present age of child = 2 years

Q.14 Let Ram's son age be = x years

Then, Age of Ram = 3x years

$$\text{Age of Ram} = \frac{2}{5} \times (\text{Age of Ram's father})$$

$$\therefore \text{Age of Ram's father} = \frac{5}{2} \times (\text{Age of Ram}) = \left(\frac{5}{2} \times 3x\right) \text{ yrs} = \frac{15x}{2} \text{ yrs}$$

Sum of ages of three = (46 × 3) years = 138 years

$$\therefore x + 3x + \frac{15x}{2} = 138 \Rightarrow 2x + 6x + 15x = 276$$

$$\Rightarrow 23x = 276 \Rightarrow x = \frac{276}{23} = 12$$

The difference between the present ages of Ram and his father = $\left(\frac{15x}{2} - 3x\right) \text{ yrs} = \frac{9x}{2} \text{ yrs}$

$$= \left(\frac{9}{2} \times 12\right) \text{ yrs} = 54 \text{ yrs}$$

Short Approach:

	Ram's father	:	Ram	:	Ram's son
Ratio	15	:	6	:	2
	Sum of ratios = 15 + 6 + 2 = 23				
	Sum of ages of three = 46 × 3 yrs				

<p>The difference between the present ages of Ram and Ram's father</p> $= \frac{46 \times 3}{23} \times (15 - 6)$ $= 6 \times 9$ $= 54 \text{ yrs}$

Q.15 Let the number of students be 'x'.

\Rightarrow The sum of the average ages of all the students / x = 8

\Rightarrow Sum of ages of all students = 8x

Now, sum of age of teachers / 12 = 45

\Rightarrow Sum of ages of teachers = 540

Average of all

\Rightarrow (Sum of ages of students + Sum of ages of teachers) / (x + 12) = 9

$\Rightarrow 8x + 540 = 9x + 108$

$\Rightarrow 9x - 8x = 540 - 108$

$\Rightarrow x = 432$

\therefore Total number of Students = x = 432

Q.16 Total number of errors in the whole book = (512 × 4) = 2048

Number of errors in 302 pages = 998

Number of errors in the remaining 210 pages = (2048 – 998) = 1050

Average number of these errors per page = $\frac{1050}{210} = 5$

Q.17 Let the total number of students = 100

And average marks of remaining 55% students = x

$100 \times 52 = 20 \times 80 + 25 \times 31 + 55 \times x \quad \Rightarrow 5200 = 1600 + 775 + 55x$

$\Rightarrow 5200 = 2375 + 55x \quad \Rightarrow 55x = 2825$

$x = 51.3636 \quad \Rightarrow x = 51.4 \text{ approx.}$

Q.18 Total number of students in the group = (3 + 6) = 9

Sum of marks of 9 students = (63 × 9) = 567

The sum of the marks of 3 students = (78 + 69 + 48) = 195

The sum of the marks of 6 students = (567 – 195) = 372

Their average marks = $\frac{372}{6} = 62$