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MPPSC

State Civil Services

Prelims

Madhya Pradesh Public Service Commission

Volume - 7

Mathematics and Data Interpretation

SIERRA INNOVATIONS

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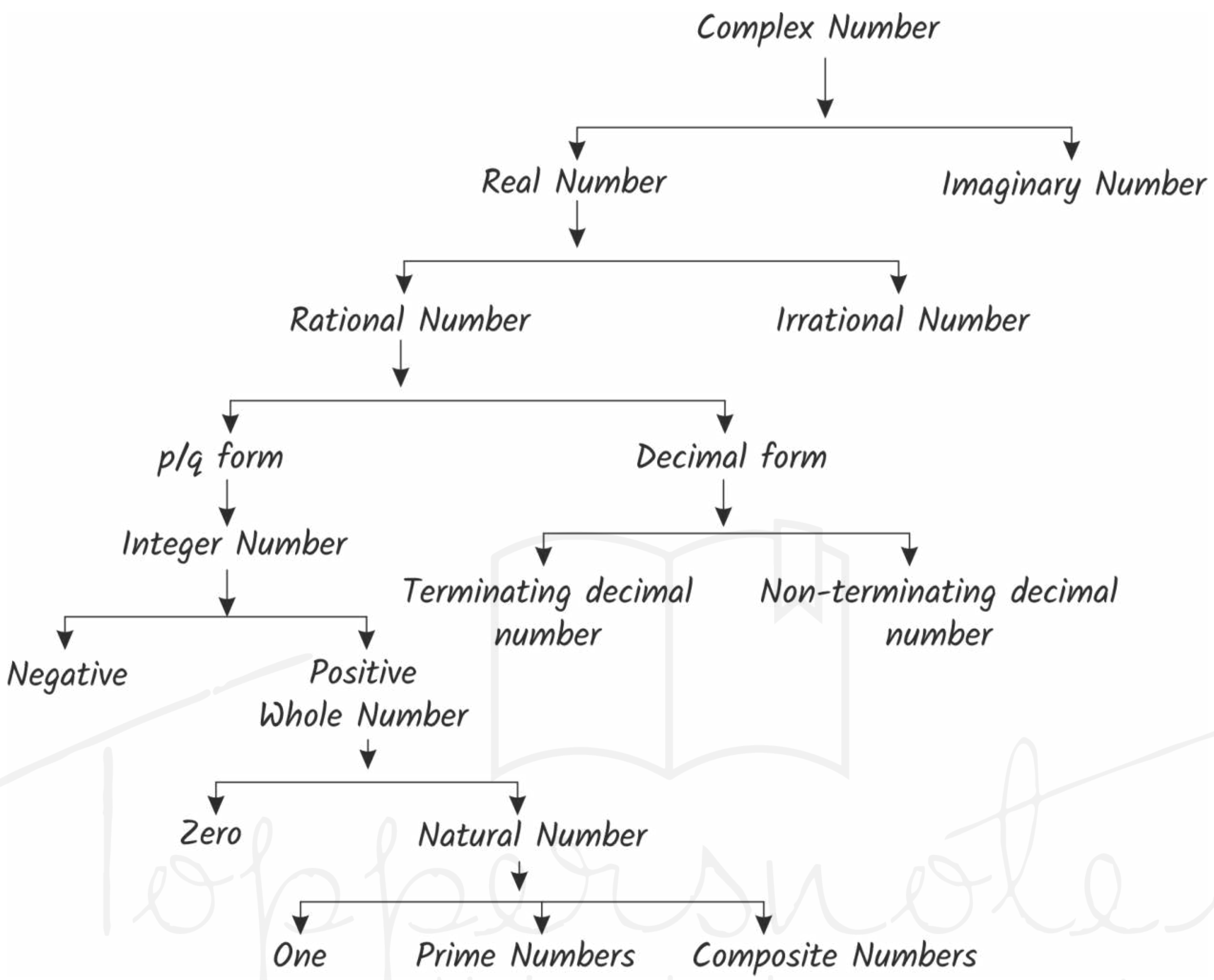
1
CHAPTER

Number System



Real Numbers

Rational and irrational numbers together are called real numbers. These can be represented on the number line.

Integer Numbers

A set of numbers which includes whole numbers as well as negative numbers, is called integer numbers, it is denoted by I.

$I = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\ldots\}$

Natural Numbers

The numbers which are used to count things are called natural numbers.

$N = \{1, 2, 3, 4, 5, \ldots\ldots\}$

Whole Numbers

When 0 is also included in the family of natural numbers, then they are called whole numbers.

W = {0, 1, 2, 3, 4, 5, ...}

The product of four consecutive natural numbers is always exactly divisible by 24.

Even Numbers

Numbers which are completely divisible by 2 are called even numbers.

n^{th} term = 2n

Sum of first n even natural numbers = n(n+1)

Sum of square of first n even natural numbers = $\frac{2n(n+1)(2n+1)}{3}$

$$\left\{n = \frac{\text{Last term}}{2}\right\}$$

Odd Numbers

The numbers which are not divisible by 2 are odd numbers.

Sum of first n odd numbers = n^2

$$\left\{n = \frac{\text{Last term} + 1}{2}\right\}$$

Natural Numbers

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

Sum of square of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

Sum of cube of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

Prime Numbers – Which have only two factors: 1and the number itself.

E.g. - {2, 3, 5, 7, 11, 13, 17, 19........}

- 1 isn't a Prime Number.
- The digit 2 is only even prime number.
- 3, 5, 7 is the only pair of consecutive odd prime numbers.

Co-prime Numbers

Numbers whose HCF is only 1.

E.g. - (4,9), (15, 22), (39, 40)

HCF = 1

Rational Numbers

Numbers that can be written in the form of P/Q, but where Q must not be zero and P and Q must be integers.

E.g. - 2/3, 4/5, $\frac{10}{-11}, \frac{7}{8}$

Irrational Numbers

These cannot be displayed in P/Q form.

E.g. - $\sqrt{2}, \sqrt{3}, \sqrt{11}, \sqrt{19}, \sqrt{26}...$

Finding the Number of Divisors or Number of Factors

First we will do the prime factorization of the number and write it as Power and multiply by adding One to each power, then the number of divisors will be obtained.

E.g. *By how many total numbers can 2280 be completely divided?*

Sol. $2280 = 2^3 \times 3^1 \times 5^1 \times 19^1$

Number of divisors $= (3 + 1)(1 + 1)(1 + 1)(1 + 1)$

$= 4 \times 2 \times 2 \times 2 = 32$

Find the unit's digit

1. **When the number is in the form of power –**

   When the unit digit of Base is 0, 1, 5 or 6, the unit digit of the result remains the same for any natural power. When the unit digit of base is 2, 3, 4, 7, 8, or 9, divide the power by 4 and put the same power on the unit digit of the base as the remainder. When the power is rounded off to 4, then the 4^{th} *power will be placed on the unit digit of the base.*

2. **In the form of simplification –**

   Write the unit digit of each number and simplify it according to the symbol, the result that will come will be its unit digit answer.

Divide by Power of Numbers (Finding the Divisor)

1. *If* $a^n + b^n$ *is given –*

   If n is odd, then (a+b) will be its divisor.

2. *If* $a^n - b^n$ *is given –*

   Divisor (when n is odd) $\rightarrow$ *(a–b)*

   Divisor (when n is even) $\rightarrow$ *(a – b) or (a + b) or both.*

1. If $a^n \div (a-1)$ then the remainder always be 1.
2. $a^n \div (a+1)$ $\begin{cases} \text{If } n \text{ is an even then the remainder always be 1.} \\ \text{If } n \text{ is an odd then the remainder always be } a. \end{cases}$
3. If $(a^n + a) \div (a-1)$ then the remainder always be 2 .
4. $(a^n + a) \div (a+1)$ $\begin{cases} \text{If } n \text{ is an even then the remainder always be zero (0).} \\ \text{If } n \text{ is an odd then the remainder always be } (a-1) \end{cases}$

Rule of Divisibility

Rule of 2	The last digit is an even number or zero (0) as - 236, 150, 1000004
Rule of 3	If the sum of the digits of a number is divisible by 3, then the whole number will be divisible by 3. E.g. 729, 12342, 5631
Rule of 4	Last two digits are zero or divisible by 4. E.g. 1024, 58764, 567800
Rule of 5	The last digit is zero or 5. E.g. 3125, 625, 1250
Rule of 6	If a number is divisible by both 2 and 3 then it is also divisible by 6. E.g. 3060, 42462, 10242
Rule of 7	After multiplying the last digit of a number by 2 and subtracting it from the remaining number, if the number is a multiple of 0 or 7 or if any digit is repeated in a multiple of 6, then the number will be divisible by 7. E.g. 222222, 444444444444, 7854
Rule of 8	If the last three digits of a number are divisible by 8 or the last three digits are '000' (zero). E.g. 9872, 347000
Rule of 9	If the sum of the digits of a number is divisible by 9, then the whole number will be divisible by 9.
Rule of 10	The last digit should be zero (0).
Rule of 11	If the difference between the sum of digits at odd places and sum of digits at even places is zero (0) or 11 or a multiple of 11. E.g. 1331, 5643, 8172659
Rule of 12	Composite form of divisible by 3 and 4.

Rule of 13	Repeating the digit 6 times, or multiplying the last digit by 4 and adding it to the remaining number, if the number is divisible by 13, then the whole number will be divisible by 13. E.g. 222222, 17784

Examples

Q.1 If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of that number, then what will be $\frac{5}{3}$ of that number?

(a) 12 (b) 18 (c) 15 (d) 20

Sol. **(d)**

Let the number = x

According to the question,

$\Rightarrow \frac{9x-2x}{12} = 7$

$\Rightarrow 7x = 7 \times 12$

$\Rightarrow x = 12$

Hence, $\frac{5}{3}$ part of the number

$= \frac{x-5}{3} \Rightarrow \frac{12 \times 5}{3} = 20$

Q.2 If the sum of two numbers is a and their product is a then their reciprocals will be –

(a) $\frac{1}{a}+\frac{1}{b}$ (b) $\frac{b}{a}$ (c) $\frac{a}{b}$ (d) $\frac{a}{ab}$

Sol. **(c)**

Let the two numbers be P and Q respectively.

$P + Q = a$

$PQ = b$

$\frac{1}{P}+\frac{1}{Q} \Rightarrow \frac{Q+P}{PQ} = \frac{a}{b}$

Q.3 The sum of two numbers is 75 and their difference is 25, then what will be the product of those two numbers?

(a) 1350 (b) 1250 (c) 1000 (d) 125

Sol. **(b)**

Let the greater number is x and smaller number is y.

$\therefore x + y = 75$ ....(i)

and, $x - y = 25$ ....(ii)

2x = 100 (By adding the equation i and ii)

x = 50

Putting the value of x in eqn. (i),

50 + y = 75

y = 75 – 50 = 25

Hence, the product of both the numbers = xy = 50 × 25 = 1250

Q.4 Divide 150 into two parts such that the sum of their reciprocal is $\frac{3}{112}$. Calculate both parts.

(a) 50, 90 (b) 70, 80 (c) 60, 90 (d) 50, 100

Sol. **(b)**

Let the first part is x then its second part be (150 – x).

According to the question,

$$\Rightarrow \frac{1}{x}+\frac{1}{(150-x)}=\frac{3}{112}$$

$$\Rightarrow \frac{150-x+x}{x(150-x)}=\frac{3}{112}$$

$\Rightarrow$ 3x(150 – x) = 150 × 112

$$\Rightarrow 150x-x^2=\frac{150\times 112}{3}$$

$\Rightarrow x^2 - 150x + 5600 = 0$

$\Rightarrow x^2 - 70x - 80x + 5600 = 0$

$\Rightarrow x(x - 70) - 80(x - 70) = 0$

$\Rightarrow (x - 80)(x - 70) = 0$

$\therefore$ x = 80 or 70

If the first part = 80 then the second part = 150 – 80 $\Rightarrow$ 70

If the first part = 70 then the second part = 150 – 70 $\Rightarrow$ 80

Q.5 If the sum of any three consecutive odd natural numbers is 147, then the middle number will be –

(a) 47 (b) 48 (c) 49 (d) 51

Sol. **(c)**

x = Suppose an odd number.

According to the question,

(x) + (x + 2) + (x + 4) = 147

3x +6 = 147

$$x=\frac{141}{3}=47$$

Hence, the middle number = (x + 2) = 47 + 2 = 49

Q.6 If the product of first three and last three of 4 consecutive prime numbers is 385 and 1001, then find the greatest prime number.

Sol. Let a, b, c & d are four prime numbers.

$abc = 385$ (i)

$bcd = 1001$ (ii)

$$\frac{abc}{bcd} = \frac{385}{1001} = \frac{5}{13}$$

Greatest prime number = 13

> **Trick:**
>
> Sum of first n odd numbers = n^2
>
> $1 + 3 + 5 + \ldots\ldots + 99 = ?$
>
> $? = \left(\frac{99+1}{2}\right)^2 = 2500$ Ans.

Q.7 What will be the sum of the even numbers between 50 and 100?

Sol. $52 + 54 + 56 + \ldots\ldots\ldots + 98$

$= (2 + 4 + 6 + \ldots + 98) - (2 + 4 + 6 + \ldots + 50)$

$n = \frac{98}{2} = 49,\ n = \frac{50}{2} = 25$

$= 49 \times 50 = 2450,\ 25 \times 26 = 650$

$\therefore\ ? = 2450 - 650 = 1800$ Ans.

Q.8 What will be the sum of odd numbers between 50 and 100?

Sol. $51 + 53 + \ldots\ldots\ldots + 99$

$= (1 + 3 + 5 + \ldots + 99) - (1 + 3 + 5 + \ldots + 49)$

$= \frac{99+1}{2} = \frac{100}{2} = 50,\ \frac{49+1}{2} = \frac{50}{2} = 25$

$\therefore\ ? = (50)^2 - (25)^2$

$= 2500 - 625 = 1875$ Ans.

Q.9 In a division method, the divisor is 12 times the quotient and 5 times the remainder. Accordingly, if the remainder is 36, then what will be the dividend?

(a) 2706 (b) 2796 (c) 2736 (d) 2826

Sol. **(c)**

Remainder = 36

$\therefore$ Divisor $= 5 \times 36 = 180$

$\therefore$ Quotient $= \frac{180}{12} = 15$

$\therefore$ Dividend = Divisor × Quotient + Remainder

= 180 × 15 + 36

= 2700 + 36

= 2736

Q.10 What is the unit digits of $(3694)^{1739} \times (615)^{317} \times (841)^{491}$

(a) 0 (b) 2 (c) 3 (d) 5

Sol. Unit digit in $(3694)^{1793}$ = 4 ; Unit digit in 4 = Unit didits in $\left\{\left(4^2\right)^{896} \times 4\right\}$

= Unit digit in (6 × 4) = 4

Unit digit in $(615)^{317}$ = Unit digit in $(5)^{317}$ = 5

Unit digit in $(841)^{491}$ = Unit digit in $(1)^{491}$ = 1

5 × 4 × 1 = 20, Unit digit = 0

Q.11 What will be written in the form of $\frac{p}{q}$ of 18.484848....?

(a) $\frac{462}{25}$ (b) $\frac{610}{33}$ (c) $\frac{200}{11}$ (d) $\frac{609}{33}$

Sol. Let x = 18.484848......................... then,

100x = 1848. 484848........

On subtracting, $99x = 1830 \Rightarrow x = \frac{1830}{99} = \frac{610}{33}$

Hence, the required form as $\frac{p}{q}$ of 18.484848...... $= \frac{610}{33}$

Q.12 Put $\frac{0.\overline{936} - 0.\overline{568}}{0.\overline{45} + 2.\overline{67}}$ in the form of rational number.

Sol. $0.\overline{936} = \frac{936}{999}, 0.\overline{568} = \frac{568}{999}$

$$\therefore \left(0.\overline{936} - 0.\overline{568}\right) = \left(\frac{936}{999} - \frac{568}{999}\right) = \frac{(936 - 568)}{999} = \frac{368}{999}$$

$$0.\overline{45} = \frac{45}{99}, 2.\overline{67} = 2 + 0.\overline{67} = 2 + \frac{67}{99} = \frac{198 + 67}{99} = \frac{265}{99}$$

$$\therefore \left(0.\overline{45} + 2.\overline{67}\right) = \left(\frac{45}{99} + \frac{265}{99}\right) = \frac{(45 + 265)}{99} = \frac{310}{99}$$

$$\text{Given expression} = \left(\frac{\overset{184}{\cancel{368}}}{\underset{111}{\cancel{999}}} \times \frac{\overset{11}{\cancel{99}}}{\underset{155}{\cancel{310}}}\right) = \frac{2024}{17205}$$

Q.13 What will be the common factor of $\left\{(127)^{127}+(97)^{127}\right\}$ and $\left\{(127)^{97}+(97)^{97}\right\}$?

(a) 127 (b) 97 (c) 30 (d) 224

Sol. $(x + y)$ is one of the factor of $(x^m + y^m)$ If m is an odd.

$\therefore$ The factor of $\left\{(127)^{127}+(97)^{127}\right\}=(127+97)=224$

Similarly, the factor of $\left\{(127)^{97}+(97)^{97}\right\}=(127+97)=224$

Hence, the common factor of both is 224.

2 CHAPTER Simplification

- In simplification, we represent the given data in a simple form, such as the data is done in fraction, in decimal, in division, in power and by solving or changing the mathematical operation.
- If different types of operations are given on some number, then how can we solve it so that the answer to the question is correct, for that there is a rule which we call the rule of VBODMAS.
- Which operation we should do first, it decides the rule of VBODMAS.

V → Vinculum
B → Bracket [{()}]
O → OF
D → Divide (/)
M → Multiplication (×)
A → (+) Addition
S → Subtraction (–)

- The first of all these mathematical operations is V which means Vinculum (line bracket). If there is a line bracket in the question, then first we will solve it and then (BODMAS) Rule will work in it.
- B (Bracket) in the second place means brackets which can be –
  1. Small bracket ( )
  2. Middle/curly bracket { }
  3. Big bracket [ ]
- First the small brackets, then the curly bracket, and then the big brackets are solved.
- In the third place is "O" which is formed from "of" or "order", which means "multiply" or "of".
- In the fourth place is "D" which means "Division", in the given expression do the first division in different actions if given.
- There is “M” in the fifth place which means “Multiplication”, in the given expression after “Division” we will do “Multiplication”.
- Sixth position is held by “A” which is related to “Addition”. Addition action takes place after division and multiplication.
- There is "S" in the seventh place which is made of "Subtraction".

Q. Simplify –

$$\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{ of }4\frac{1}{3}\right)$$

Sol: Step 1 – Convert the mixed fraction into simple fraction

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{ of }\frac{13}{3}\right)$$

Now, according to VBODMAS –

Step 2 – $\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{3-2}{12}\right)\right\}\right]\div\left(\frac{1}{2}\text{ of }\frac{13}{3}\right)$

Step 3 – $\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{1}{12}\right)\right\}\right]\div\frac{13}{6}$

Step 4 – $\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\left(\frac{30-1}{12}\right)\right\}\right]\div\frac{13}{6}$

Step 5 – $\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\frac{29}{12}\right\}\right]\div\frac{13}{6}$

Step 6 – $\left[\frac{13}{4}\div\left\{\frac{30-29}{24}\right\}\right]\div\frac{13}{6}$

Step 7 – $\left[\frac{13}{4}\div\frac{1}{24}\right]\div\frac{13}{6}$

Step 8 – $\left[\frac{13}{4}\times 24\right]\div\frac{13}{6}$

Step 9 – $13\times 6\times\frac{6}{13}$

$= 36$ Ans.

Algebraic Formulas –

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
4. $(a^2 - b^2) = (a + b)(a - b)$
5. $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$
6. $a^2+\frac{1}{a^2}=\left(a+\frac{1}{a}\right)^2-2$
7. $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}\left[(a-b)^2+(b+c)^2+(c-a)^2\right]$

8. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 - ab + b^2)$

9. $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + ab + b^2)$

10. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a+b+c)\left\{(a-b)^2 + (b-c)^2 + (c-a)^2\right\}$$

If $a + b + c = 0$, then

$a^3 + b^3 + c^3 = 3abc$

11. $a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$

12. $a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$

Square and Square Root Table

Square	Square Root	Square	Square Root
$1^2 = 1$	$\sqrt{1} = 1$	$16^2 = 256$	$\sqrt{256} = 16$
$2^2 = 4$	$\sqrt{4} = 2$	$17^2 = 289$	$\sqrt{289} = 17$
$3^2 = 9$	$\sqrt{9} = 3$	$18^2 = 324$	$\sqrt{324} = 18$
$4^2 = 16$	$\sqrt{16} = 4$	$19^2 = 361$	$\sqrt{361} = 19$
$5^2 = 25$	$\sqrt{25} = 5$	$20^2 = 400$	$\sqrt{400} = 20$
$6^2 = 36$	$\sqrt{36} = 6$	$21^2 = 441$	$\sqrt{441} = 21$
$7^2 = 49$	$\sqrt{49} = 7$	$22^2 = 484$	$\sqrt{484} = 22$
$8^2 = 64$	$\sqrt{64} = 8$	$23^2 = 529$	$\sqrt{529} = 23$
$9^2 = 81$	$\sqrt{81} = 9$	$24^2 = 576$	$\sqrt{576} = 24$
$10^2 = 100$	$\sqrt{100} = 10$	$25^2 = 625$	$\sqrt{625} = 25$
$11^2 = 121$	$\sqrt{121} = 11$	$26^2 = 676$	$\sqrt{676} = 26$
$12^2 = 144$	$\sqrt{144} = 12$	$27^2 = 729$	$\sqrt{729} = 27$
$13^2 = 169$	$\sqrt{169} = 13$	$28^2 = 784$	$\sqrt{784} = 28$
$14^2 = 196$	$\sqrt{196} = 14$	$29^2 = 841$	$\sqrt{841} = 29$
$15^2 = 225$	$\sqrt{225} = 15$	$30^2 = 900$	$\sqrt{900} = 30$

Cube and Cube Root Table

Cube	Cube Root	Cube	Cube Root
$1^3 = 1$	$\sqrt[3]{1} = 1$	$16^3 = 4096$	$\sqrt[3]{4096} = 16$
$2^3 = 8$	$\sqrt[3]{8} = 2$	$17^3 = 4913$	$\sqrt[3]{4913} = 17$

$3^3 = 27$	$\sqrt[3]{27} = 3$	$18^3 = 5832$	$\sqrt[3]{5832} = 18$
$4^3 = 64$	$\sqrt[3]{64} = 4$	$19^3 = 6859$	$\sqrt[3]{6859} = 19$
$5^3 = 125$	$\sqrt[3]{125} = 5$	$20^3 = 8000$	$\sqrt[3]{8000} = 20$
$6^3 = 216$	$\sqrt[3]{216} = 6$	$21^3 = 9261$	$\sqrt[3]{9261} = 21$
$7^3 = 343$	$\sqrt[3]{343} = 7$	$22^3 = 10648$	$\sqrt[3]{10648} = 22$
$8^3 = 512$	$\sqrt[3]{512} = 8$	$23^3 = 12167$	$\sqrt[3]{12167} = 23$
$9^3 = 729$	$\sqrt[3]{729} = 9$	$24^3 = 13824$	$\sqrt[3]{13824} = 24$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$	$25^3 = 15625$	$\sqrt[3]{15625} = 25$
$11^3 = 1331$	$\sqrt[3]{1331} = 11$	$26^3 = 17576$	$\sqrt[3]{17576} = 26$
$12^3 = 1728$	$\sqrt[3]{1728} = 12$	$27^3 = 19683$	$\sqrt[3]{19683} = 27$
$13^3 = 2197$	$\sqrt[3]{2197} = 13$	$28^3 = 21952$	$\sqrt[3]{21952} = 28$
$14^3 = 2744$	$\sqrt[3]{2744} = 14$	$29^3 = 24389$	$\sqrt[3]{24389} = 29$
$15^3 = 3375$	$\sqrt[3]{3375} = 15$	$30^3 = 27000$	$\sqrt[3]{27000} = 30$

Arithmetic Progression

The series in which each term can be found by adding or subtracting with its preceding term is called the arithmetic progression.

E.g. 2, 5, 8, 11, ...

n^{th} term of an Arithmetic Progression

$T_n = a + (n - 1)\, d$

Where, a = First term

d = Common difference (2^{nd} term – 1^{st} term)

n = Number of all terms.

Addition of n^{th} terms of an Arithmetic Progression –

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

If the first and last term is known –

$$S_n = \frac{n}{2}[a + \ell]$$

Where, $\ell =$ Last term

Arithmetic progression between the two variables

$A = \dfrac{a+b}{2}$ [The arithmetic progression of a & b is A]

Geometric Progression

If the ratio of each term of the series to its preceding term is a certain variable, then it is called a geometric series. This fixed variable is called the common ratio.

n^{th} term of Geometric Series –

$T_n = a.r^{n-1}$

Where, a = First term r = Common ratio n = Number of terms

Addition of n^{th} terms of Geometric Series –

$$S_n = a\left(\frac{1-r^n}{1-r}\right); \text{ When } r<1$$

$$S_n = a\left(\frac{r^n-1}{r-1}\right); \text{ when } r>1$$

1. Geometric series between two variables $G=\sqrt{ab}$
2. If the arithmetic mean and geometric mean between two positive quantities a and b are A and G, then A > G, $\frac{a+b}{2} > \sqrt{ab}$

Relation between Arithmetic Mean, Geometric Mean and Harmonic Mean

Let A, G and H be the arithmetic mean, geometric mean and harmonic mean between two quantities a and b respectively, then

$\boxed{G^2 = AH}$ and $\boxed{A>G>H}$

Examples

Ex.1 The value of 24 × 2 ÷ 12 + 12 ÷ 6 of 2 ÷ (15 ÷ 8 × 4) of (28 ÷ 7 of 5) is –

(a) $4\frac{32}{75}$ (b) $4\frac{8}{75}$ (c) $4\frac{2}{3}$ (d) $4\frac{1}{6}$

Sol: (d)

24 × 2 ÷ 12 + 12 ÷ 6 of 2 ÷ (15 ÷ 8 × 4) of (28 ÷ 7 of 5)

= 24 × (2/12) + 12 ÷ 12 ÷ [(15/8) × 4] of (28 ÷ 35)

= 4 +1 ÷ (15/2) of 4/5

= 4 +1 ÷ 6

= 4 + 1/6

= $4\frac{1}{6}$ Ans.

Ex.3 Evaluate –

$$2\frac{3}{4} \div 1\frac{5}{6} \div \frac{7}{8} \times \left(\frac{1}{3}+\frac{1}{4}\right) + \frac{5}{7} \div \frac{3}{4} \text{ of } \frac{3}{7}$$

(a) $\frac{56}{77}$ (b) $\frac{49}{80}$ (c) $\frac{2}{3}$ (d) $3\frac{2}{9}$

Sol: According to question –

$$\left(\frac{2\frac{3}{4}}{1\frac{5}{6}}\right) \div \frac{7}{8} \times \left(\frac{1}{3}+\frac{1}{4}\right) + \frac{5}{7} + \frac{3}{4} \text{ of } \frac{3}{7}$$

$$= \frac{\frac{11}{4}}{\frac{11}{6}} \div \frac{7}{8} \times \frac{7}{12} + \frac{5}{7} \div \left(\frac{3}{4} \times \frac{3}{7}\right)$$

$$= \frac{3}{2} \times \frac{8}{7} \times \frac{7}{12} + \frac{5}{7} \times \frac{28}{9}$$

$$= 1 + \frac{20}{9}$$

$$= \frac{29}{9} = 3\frac{2}{9} \text{ Ans.}$$

Ex.4 If $(102)^2 = 10404$ then the value of $\sqrt{104.04} + \sqrt{1.0404} + \sqrt{0.010404}$ is equals to?

(a) 0.306 (b) 0.0306 (c) 11.122 (d) 11.322

Sol: (d)

According to question –

$$= \sqrt{104.04} + \sqrt{1.0404} + \sqrt{0.010404}$$

$$= \sqrt{\frac{10404}{100}} + \sqrt{\frac{10404}{10000}} + \sqrt{\frac{10404}{1000000}}$$

$$= \frac{102}{10} + \frac{102}{100} + \frac{102}{1000}$$

$= 10.2 + 1.02 + 0.102 = 11.322$

Ex.7 What is the smallest number to be added to 710 so that the sum becomes a perfect cube?

(a) 29 (b) 19 (c) 11 (d) 21

Sol: (b)

Clearly, $\sqrt[3]{729} = 9$

$\therefore$ 19 must be added to 710 to get a perfect cube.