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BPSC - AE

ASSISTANT ENGINEER

Civil Engineering

Bihar Public Service Commission

Volume - 6

Transportation Engineering

SIERRA INNOVATIONS

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THEORY

1.1 HISTORY OF HIGHWAY PLANNING IN INDIA

1.1.1 Jayakar Committee

In 1927 Jayakar committee for Indian road development was appointed. The major recommendations and the resulting implementations were

- Committee found that the road development of the country has become beyond the capacity of local governments and suggested that Central government should take the proper charge considering it as a matter of national interest.

- One of the recommendations was the holding of periodic road conferences to discuss about road construction and development. This paved the way for the establishment of a semi-official technical body called Indian Road Congress (IRC) in 1934.

- The committee suggested imposition of additional taxation on motor transport which includes duty on motor spirit, vehicle taxation, license fees for vehicles plying for hire. This led to the introduction of a development fund called Central road fund in 1929. This fund was intended for road development.

- A dedicated research organization should be constituted to carry out research and development work. This resulted in the formation of Central Road Research Institute (CRRI) in 1950, New Delhi.

- They gave more stress on long term planning programme, for a period of 20 years (hence called twenty year plan) that is to formulate plans and implement those plans with in the next 20 years.

1.1.2 Nagpur Road Plan 1943 - 1961

The roads were divided into four classes

1. National highways which would pass through states and places having national importance for strategic, administrative and other purposes.

2. State highways which would be the other main roads of a state.

3. District roads which would take traffic from the main roads to the interior of the district. According to the importance, some are considered as major district roads and the remaining as other district roads.

4. Village roads which would link the villages to the road system.

5. They suggested that the length of the road should be increased so as to give a road density of 16kms/100sq.km.

1.1.3 Bombay Road Plan 1961 - 1981

The highlights of the plan were

• It was the second 20 year road plan (1961-1981).

• The total road length targeted to construct was about 10 lakhs.

• Rural roads were given specific attention. Scientific methods of construction was proposed for the rural roads. The necessary technical advice to the Panchayaths should be given by State PWD's.

• They suggested that the length of the road should be increased so as to give a road density of 32kms/100sq.km.

• The construction of 1600 km of expressways was also then included in the plan.

1.1.4 Lucknow Road Plan 1981 - 2001

Some of the salient features of this plan are as given below

• This was the third 20 year road plan (1981-2001). It is also called Lucknow road plan.

• It aimed at constructing a road length of 12 lakh kilometres by the year 2001 resulting in a road density of 82kms/100 sq.km.

• The plan has set the target length of NH to be completed by the end of seventh, eighth and ninth year plan periods.

• It aims at improving the transportation facilities in villages, towns etc. such that no part of country is farther than 50 km from NH.

• One of the goals contained in the plan was that expressways should be constructed on major traffic corridors to provide speedy travel.

• Energy conservation, environmental quality of roads and road safety measures were also given due importance in this plan.

1.2 CROSS SECTIONAL ELEMENTS OF PAVEMENT

The features of the cross-section of the pavement influences the life of the pavement as well as the riding comfort and safety. Camber, kerbs and geometry of various cross-sectional elements are important aspects to be considered in this regard.

1.2.1 Pavement Unevenness

The longitudinal profile of the road pavement has to be 'even' in order to provide good riding comfort to fast moving vehicles and to minimise the vehicle operation cost. Presence of undulations on the pavement surface is called 'pavement unevenness'.

The unevenness of pavement surface is commonly measured by using a simple equipment called 'Bump Integrator' (BI), in terms of 'unevenness index', which is the cumulative measure of vertical undulations of the pavement surface recorded per unit length of the road.

1.2.2 Light Reflecting Characteristics

Night visibility depends upon the colour and light reflecting characteristics of the pavement surface. The glare caused by the reflection of head lights is considerably high on wet pavement surface than on the dry pavement. Light coloured or white pavement surface give good visibility at night particularly during rains; however white or light colour of pavement surface may produces glare and eye strain during bright sunlight. Black top pavement surface on the order hand provides very poor visibility at nights, especially when the surface is wet.

1.2.3 Friction

Friction between the wheel and the pavement surface is a crucial factor in the design of horizontal curves and thus the safe operating speed. Further, it also affects the acceleration and deceleration ability of vehicles. Lack of adequate friction can cause skidding or slipping of vehicles.

* Skidding happens when the path traveled along the road surface is more than the circumferential movement of the wheels due to friction.
* Slip occurs when the wheel revolves more than the corresponding longitudinal movement along the road.
* Coefficient of longitudinal friction = 0.35 to 0.4, Coefficient of lateral friction = 0.15

1.2.4 Camber

Camber or cant is the cross slope provided to raise middle of the road surface in the transverse direction to drain of rain water from road surface. The objectives of providing camber are
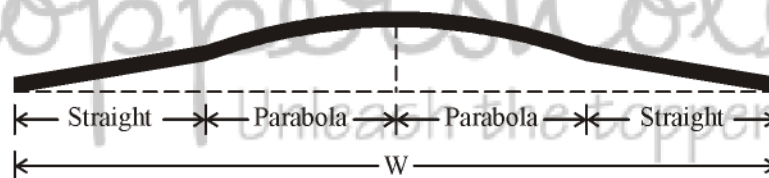
* Surface protection especially for gravel and bituminous roads.
* Sub-grade protection by proper drainage.
* Quick drying of pavement which in turn increases safety.
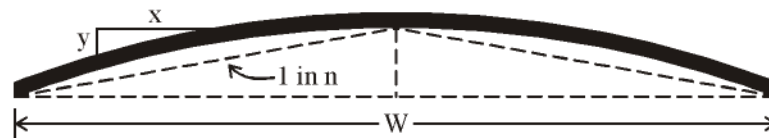
(a) Parabolic camber $y = \dfrac{2x^2}{nW}$
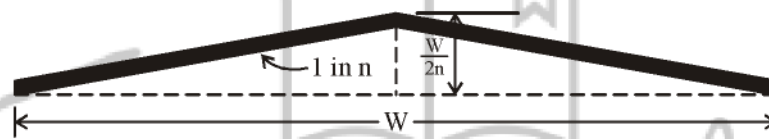
(b) Straight line camber $y = \dfrac{W}{2n}$

(c) Combination of straight and parabolic camber

Fig. : Different types of camber

Table : IRC values for camber

| S. No.	Surface type	Heavy rain	Light rain
1.	Concrete/Bituminous	2 %	1.7 %
2.	Thin bituminous surface	2.5 %	2 %
3.	Gravel/WBM	3 %	2.5 %
4.	Earthen	4 %	3 %

1.2.5 Width of Carriage Way

Width of the carriage way or the width of the pavement depends on the width of the traffic lane and number of lanes. Width of a traffic lane depends on the width of the vehicle and the clearance. Side clearance improves operating speed and safety. The maximum permissible width of a vehicle is 2.44 m.

and the desirable side clearance for single lane traffic is 0.68 m. This require minimum of lane width of 3.75 m for a single lane road. However, the side clearance required is about 0.53 m, on either side and 1.06 m in the center. Therefore, a two lane road require minimum of 3.5 meter for each lane.

Table : IRC Specification for carriage way width

| Traffic Lane	Width
Single lane	3.75 m
Two lane, no kerbs	7.0 m
Two lane, raised kerbs	7.5 m
Intermediate carriage	5.5 m
Multi-lane	3.5 m
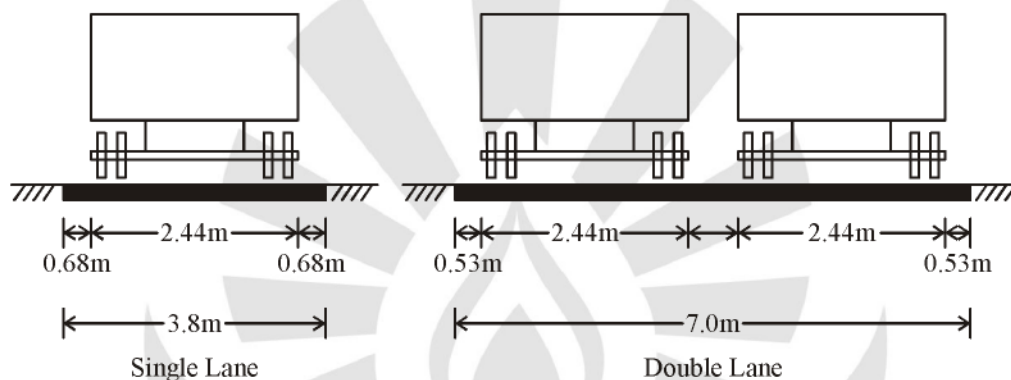
Fig. : Lane width for single and two lane roads

1.2.6 Kerbs

Kerbs indicate the boundary between the carriage way and the shoulder or islands or footpaths. Different types of kerbs are

• Low or mountable kerbs
• Semi-barrier type kerbs
• Barrier type kerbs
• Submerged kerbs
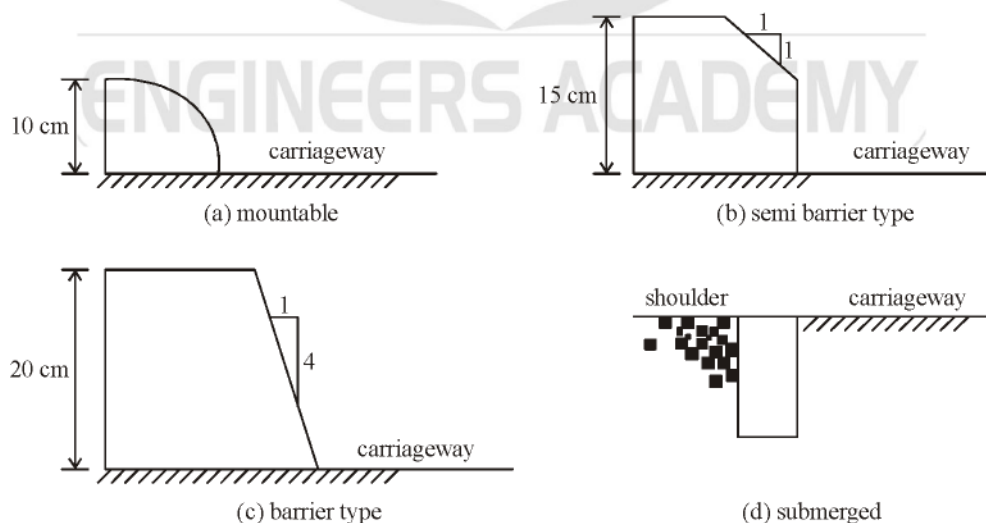
Fig. : Different types of kerbs

1.2.7 Shoulder

Shoulders are provided on both sides of the pavement all along the road in the case of undivided carriagewauy. Shoulders are provided along the outer edge of the carriageway in the case of divided carriageway. The earth shoulders should have sufficient stability to support even a loaded truck and therefore they are constructed using good quality material. In order to increase the capacity of the roadway, 'paved shoulders' are also laid on roads with high traffic flow. The minimum shoulder width recommeded by the IRC is 2.5 m.

1.2.8 Width of Formation

Width of formation or roadway width is the sum of the widths of pavements or carriage way including separators and shoulders. This does not include the extra land in formation/cutting. The values suggested by IRC are given in

Width of formation for various classed of roads

Road classification	Roadway width in m	
	Plain and rolling terrain	**Mountainous and steep terrain**
NH/SH	12	6.25-8.8
MDR	9	4.75
ODR	7.5-9.0	4.75
VR	7.5	4.0

1.2.9 Right of way

Right of way (ROW) or land width is the width of land acquired for the road, along its alignment. It should be adequate to accommodate all the cross-sectional elements of the highway and may reasonably provide for future development. To prevent ribbon development along highways, control lines and building lines may be provided. Control line is a line which represents the nearest limits of future uncontrolled building activity in relation to a road. Building line represents a line on either side of the road, between which and the road no building activity is permitted at all. The right of way width is governed by

- **Width of formation :** It depends on the category of the highway and width of roadway and road margins.
- **Height of embankment or depth of cutting :** It is governed by the topography and the vertical alignment.
- **Side slopes of embankment or cutting :** It depends on the height of the slope, soil type etc.
- **Drainage system :** Drainage system size which depends on rainfall, topography etc.
- **Sight distance considerations :** On curves etc. there is restriction to the visibility on the inner side of the curve due to the presence of some obstructions like building structures etc.
- **Reserve land for future widening :** Some land has to be acquired in advance anticipating future developments like widening of the road.
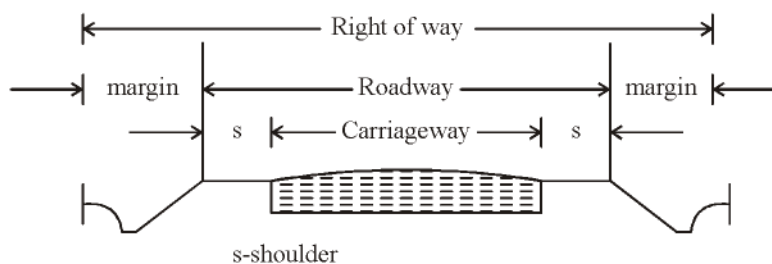

Fig. : Typical right of way (ROW)

1.2.10 Footpath

In order to provide safe facility to pedestrians to walk along the roadway, foot paths or side-walks are provided in urban areas where the pedestrain traffic is note worthy and the vehicular traffic is also heavy.

The absolute minimum width of foot path is 1.5 m and the desirable minimum width is 2.0 m; the width may be increased based on the pedestrian traffic volume.

1.2.11 Lay-byes

Lay-byes are provided near public conveniences with guide maps to enable drivers to stop clear off the carriageway. Lay-byes should normally be of 3.0 width and at least 30 m length with 15 m end tapers on both sides.

1.3 SIGHT DISTANCE

1.3.1 Types of Sight Distance

Sight distance available from a point is the actual distance along the road surface, over which a driver from a specified height above the carriage way has visibility of stationary or moving objects. Three sight distance situations are considered for design:

- Stopping sight distance (SSD) or the absolute minimum sight distance
- Intermediate sight distance (ISD) is the defined as twice SSD
- Overtaking sight distance (OSD) for safe overtaking operation
- Head light sight distance is the distance visible to a driver during night driving under the illumination of head light

1.3.2 Stopping Sight Distance

Safe sight distance to enter into an intersection
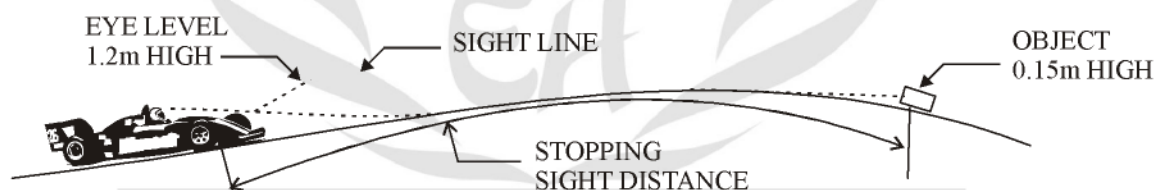
$$SSD = \text{lag distance} + \text{breaking distance}$$

$$SSD = vt + \frac{v^2}{2gf}$$

Where v = design speed in m/sec

t = reaction time in sec

g = acceleration due to gravity

and f = coefficient of friction.

The coefficient of friction f is given below for various design speed.

Coefficient of longitudinal friction

Speed, kmph	<30	40	50	60	>80
f	0.40	0.38	0.37	0.36	0.35

- Safe sight distance to enter into an intersection with gradient n%

$$SSD = vt + \frac{v^2}{2g(f \pm 0.01n)}$$

where,　n = gradient in %, take (+)ve sign for ascending gradient and take (–)ve sign for descending gradient.

1.3.2 Overtaking Sight Distance

The overtaking sight distance is the minimum distance open to the vision of the driver of a vehicle intending to overtake the slow vehicle ahead safely against the traffic in the opposite direction. The overtaking sight distance or passing sight distance is measured along the center line of the road over which a driver with his eye level 1.2 m above the road surface can see the top of an object 1.2 m above the road surface. The factors that affect the OSD are

- Velocities of the overtaking vehicle, overtaken vehicle and of the vehicle coming in the opposite direction.

- Spacing between vehicles, which in-turn depends on the speed

- Skill and reaction time of the driver

- Rate of acceleration of overtaking vehicle

- Gradient of the road
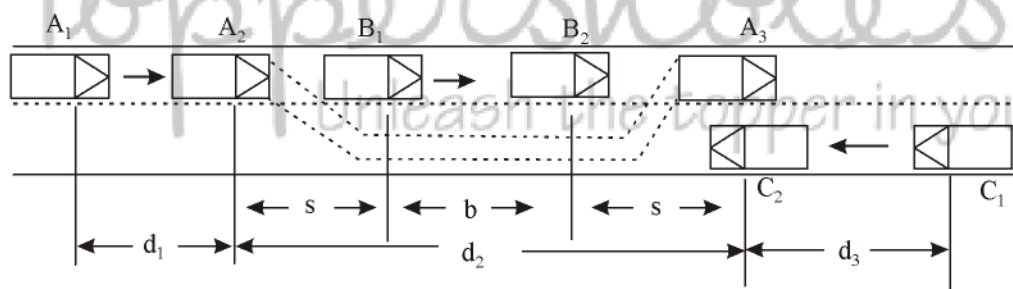
$$OSD = \underbrace{v_b t}_{(d_1)} + \underbrace{2s + v_b \sqrt{\frac{4s}{a}}}_{(d_2)} + \underbrace{vT}_{(d_3)}$$

Where　v_b = velocity of the slow moving vehicle in m/sec

　　　　t = the reaction time of the driver in (2 sec)

　　　　v = velocity of fast moving vehicle in m/sec

　　　　s = spacing between the two vehicle in meter = $(0.7v_b + l)$

and　　　a = overtaking vehicles acceleration in m/sec^2.

In case the speed of the overtaken vehicle v_b is not given, it can be assumed that it moves 16 kmph slower the design speed. The acceleration values of the faster vehicle depends on its speed.

The following table shows the relationship between them

**Maximum overtaking acceleration at different speeds**

Speed (kmph)	Maximum overtaking acceleration (m/sec^2)
25	1.41
30	1.30
40	1.24
50	1.11
65	0.92
80	0.72
100	0.53

Note :

- On divided highways, d_3 need not be considered.
- On divided highways with four or more lanes, IRC suggests that it is not necessary to provide the OSD, but only SSD is sufficient.

1.3.3 Overtaking Zones

Overtaking zones are provided when OSD cannot be provided throughout the length of the highway. These are zones dedicated for overtaking operation, marked with wide roads. The desirable length of overtaking zones is five time OSD and the minimum is three times OSD.

S1 : Overtaking zone begin
S2 : End of Overtaking zone
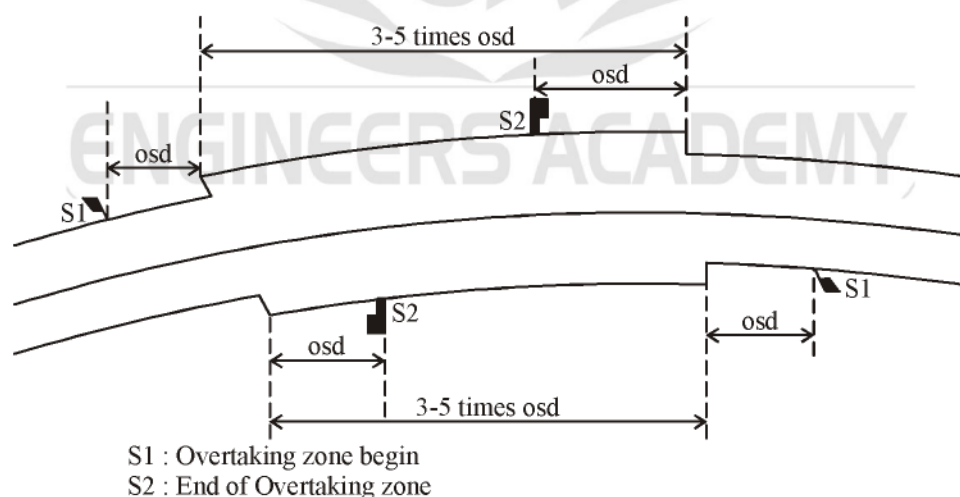
Fig. : Overtaking zones

1.3.4 Sight Distance at Intersections

At intersections where two or more roads meet, visibility should be provided for the drivers approaching the intersection from either sides. They should be able to perceive a hazard and stop the vehicle if required. Stopping sight distance for each road can be computed from the design speed. The sight distance should be provided such that the drivers on either side should be able to see each other :
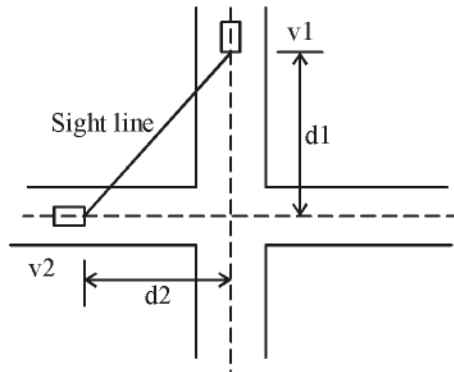

Fig. : Sight distance at intersections

1.4 HORIZONTAL ALIGNMENT

Horizontal alignment is one of the most important features influencing the efficiency and safety of a highway. A poor design will result in lower speeds and resultant reduction in highway performance in terms of safety and comfort.

1.4.1 Design Speed

Indian Road Congress (IRC) has classified the terrains into four categories, namely plain, rolling, mountainous, and steep based on the cross slope as given in table. Based on the type of road and type of terrain the design speed varies. The IRC has suggested desirable or ruling speed as well as minimum suggested design speed and is tabulated in table.

Terrain classification

Terrain classification	Cross slope (%)
Plain	0-10
Rolling	10-25
Mountainous	25-60
Steep	> 60

Design speed in km/hr as per IRC (ruling and minimum)

Type	Plain	Rolling	Hilly	Steep
NH & SH	100-80	80-65	50-40	40-30
MDR	80-65	65-50	40-30	30-20
ODR	65-50	50-40	30-25	25-20
VR	50-40	40-35	25-20	25-20

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Type	Design speed (kmph)
Arterial roads	80
Sub-arterial roads	60
Collector streets	50
Local streets	30

1.4.2 Horizontal Curve

The presence of horizontal curve imparts centrifugal force which is a reactive force acting outward on a vehicle negotiating it. Centrifugal force depends on speed and radius of the horizontal curve and is counteracted to a certain extent by transverse friction between the tyre and pavement surface. On a curved road, this force tends to cause the vehicle to overturn or to slide outward from the centre of road curvature. For proper design of the curve, an understanding of the forces acting on a vehicle taking a horizontal curve is necessary. Various forces acting on the vehicle is illustrated in the figure. They are the centrifugal force (P) acting outward, weight of the vehicle (W) acting downward, and the reaction of the ground on the wheels (R_A and R_B). The centrifugal force and the weight is assumed to be from the centre of gravity which is at h units above the ground. Let the wheel base be assumed as b units
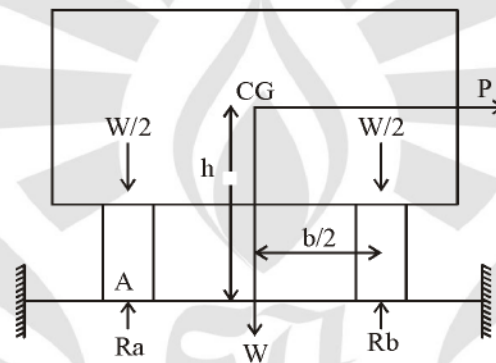
For safety against overturning the following condition must be satisfied

$$\frac{b}{2h} > \frac{v^2}{gR}$$

For safety against skidding the following condition must be satisfied

$$f > \frac{v^2}{gR}$$

1.4.3 Super-Elevation

Super-elevation or cant or banking is the transverse slope provided at horizontal curve to counteract the centrifugal force, by raising the outer edge of the pavement with respect to the inner edge, throughout the length of the horizontal curve. When the outer edge is raised, a component of the weight will be complimented in counteracting the effect of centrifugal force. In order to find out how much this raising should be, the following analysis may be done. The forces acting on a vehicle while taking a horizontal curve with superelevation is shown in figure
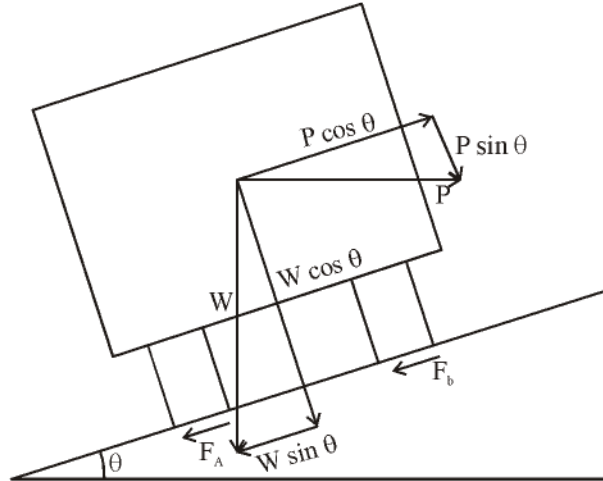
Fig. : Analysis of super-elevation

At equilibrium, by resolving the forces parallel to the surface of the pavement we get

$$P \cos \theta = W \sin \theta + f(R_A + R_B)$$

$$= W \sin \theta + f(W \cos \theta + P \sin \theta)$$

i.e., $P(\cos \theta + f \sin \theta) = W \sin \theta + f W \cos \theta$

Dividing by $W \cos \theta$,

$$\frac{P}{W}(1 - f \tan \theta) = \tan \theta + f$$

$$\frac{P}{W} = \frac{\tan \theta + f}{1 - f \tan \theta}$$

where, P = centrifugal force acting horizontally out-wards through the center of gravit

W = weight of the vehicle acting down-wards through the center of gravity

F = friction force between the wheels and the pavement, along the surface inward.

1.4.3.1 Design of super-elevation (e)

For fast moving vehicles, providing higher superelevation without considering coefficient of friction is safe, i.e. centrifugal force is fully counteracted by the weight of the vehicle or superelevation. For slow moving vehicles, providing lower superelevation considering coefficient of friction is safe, *i.e.* centrifugal force is counteracted by superelevation and coefficient of friction. IRC suggests following design procedure

Step 1 : Find e for 75 percent of design speed, neglecting f, *i.e.*, $e_1 = \dfrac{(0.75v)^2}{gR}$

Step 2 : If $e_1 \leq 0.07$, then $e = e_1 = \dfrac{(0.75v)^2}{gR}$, else if $e_1 > 0.07$ go to step 3.

Step 3 : Find f_1 for the design speed and max e, *i.e.*, $f_1 = \dfrac{v^2}{gR} - e = \dfrac{v^2}{gR} - 0.07$. If $f_1 < 0.15$, then the maximum e = 0.07 is safe for the design speed, else go to step 4.

Step 4 : Find the allowable speed v_a for the maximum e = 0.07 and f = 0.15, *i.e.* from equation $v_a = \sqrt{0.22gR}$ If $v_a \geq v$ then the design is adequate, otherwise use speed adopt control measures or look for speed control measures.

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1.4.3.2 Maximum and Minimum Superelevation

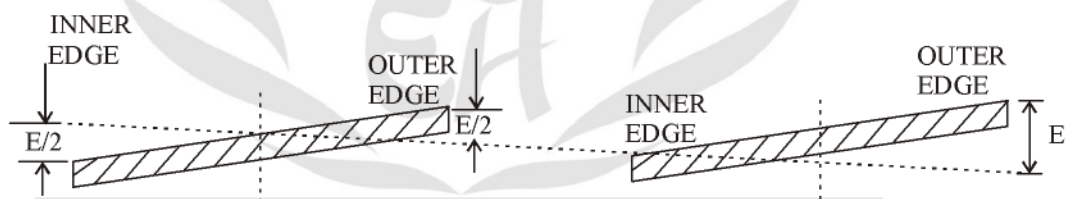**Depends on**

(a)   Slow moving vehicle

(b)   Heavy loaded trucks with high CG

IRC specifies a maximum super-elevation of 7 percent for plain and rolling terrain, while that of hilly terrain is 10 percent and urban road is 4 percent. The minimum super elevation is 2-4 percent for drainage purpose, especially for large radius of the horizontal curve

1.4.3.3 Attainment of Superelevation

1.   **Elimination of the crown of the cambered section by**

   (a)   **Rotating the outer edge about the crown :** The outer half of the cross slope is rotated about the crown at a desired rate such that this surface falls on the same plane as the inner half.

   (b)   **Shifting the position of the crown :** This method is also known as diagonal crown method. Here the position of the crown is progressively shifted outwards, thus increasing the width of the inner half of cross section progressively.

2.   **Rotation of the pavement cross section to attain full super elevation :** There are two methods of attaining superelevation by rotating the pavement

   (a)   **Rotation about the center line :** The pavement is rotated such that the inner edge is depressed and the outer edge is raised both by half the total amount of superelevation, i.e., by E/2 with respect to the centre.

   (b)   **Rotation about the inner edge :** Here the pavement is rotated raising the outer edge as well as the centre such that the outer edge is raised by the full amount of superelevation with respect to the inner edge.

1.4.4 Radius of Horizontal Curve

The radius of the horizontal curve is an important design aspect of the geometric design. The maximum comfortable speed on a horizontal curve depends on the radius of the curve. Although it is possible to design the curve with maximum superelevation and coefficient of friction but it is not desirable because re-alignment would be required if the design speed is increased in future

$$R_{ruling} = \frac{v_{ruling}^2}{g(e_{max} + f_{max})}$$

$$R_{minimum} = \frac{v_{minimum}^2}{g(e_{max} + f_{max})}$$

1.4.5 Extra Widening

Extra widening refers to the additional width of carriageway that is required on a curved section of a road over and above that required on a straight alignment. This widening is done due to two reasons: the first and most important is the additional width required for a vehicle taking a horizontal curve and the second is due to the tendency of the drivers to ply away from the edge of the carriageway as they drive on a curve. The first is referred as the mechanical widening and the second is called the psychological widening.

1.4.5.1 Mechanical Widening

When a vehicle negotiates a horizontal curve, the rear wheels follow a path of shorter radius than the front wheels as shown in given figure. This phenomenon is called off tracking, and has the effect of increasing the effective width of a road space required by the vehicle. Therefore, to provide the same clearance between vehicles travelling in opposite direction on curved roads as is provided on straight sections, there must be extra width of carriageway available.

$$W_m = \frac{nl^2}{2R_2}$$
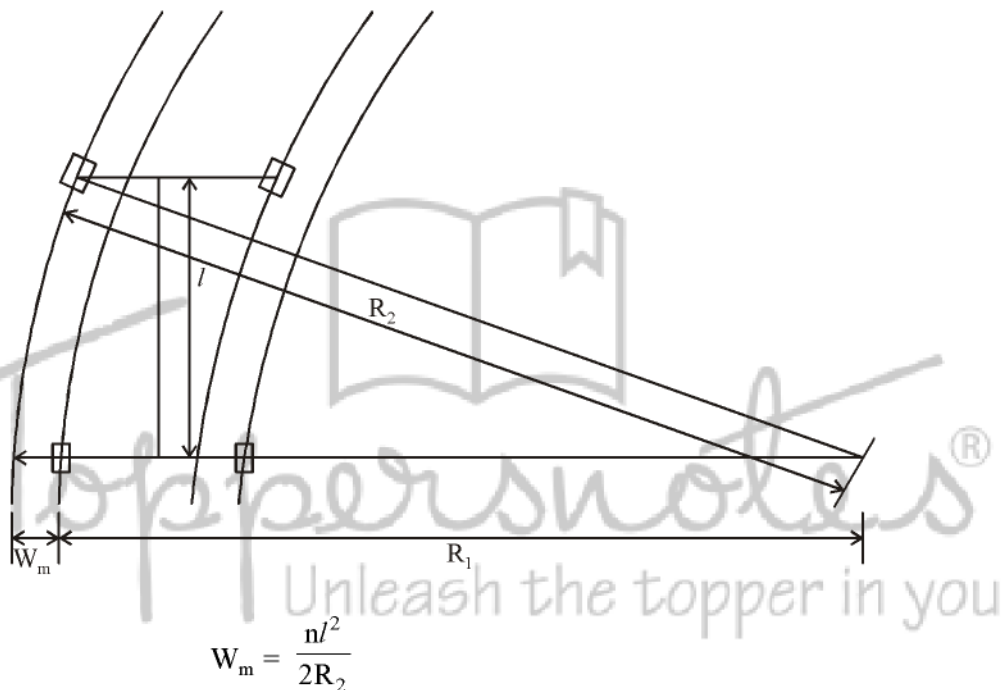
1.4.5.2 Psychological Widening

Widening of pavements has to be done for some psychological reasons also. There is a tendency for the drivers to drive close to the edges of the pavement on curves. Some extra space is to be provided for more clearance for the crossing and overtaking operations on curves. IRC proposed an empirical relation for the psychological widening at horizontal curves W_{ps}

$$W_{ps} = \frac{v}{2.64\sqrt{R}}$$

Therefore, the total widening needed at a horizontal curve W_e is

$$W_e = W_m + W_{ps}$$

$$= \frac{nl^2}{2R} + \frac{v}{2.64\sqrt{R}}$$

1.4.6 Transition Curves

Transition curve is provided to change the horizontal alignment from straight to circular curve gradually and has a radius which decreases from infinity at the straight end (tangent point) to the desired radius of the circular curve at the other end (curve point) There are five objectives for providing transition curve and are given below

- To introduce gradually the centrifugal force between the tangent point and the beginning of the circular curve, avoiding sudden jerk on the vehicle.This increases the comfort of passengers.

- To enable the driver turn the steering gradually for his own comfort and security,

- To provide gradual introduction of super-elevation and extra widening.

- To enhance the aesthetic appearance of the road.

1.4.6.1 Type of Transition Curve

Different types of transition curves are spiral or clothoid, cubic parabola, and Lemniscate. IRC recommends spiral as the transition curve because

(a)   Rate of change of centrifugal acceleration is consistent (smooth)

(b)   Radius of the transition curve is infinity at the straight edge and changes to R at the curve point

$\left(L_s \propto \dfrac{1}{R} \right)$ and calculation and field implementation is very easy.

1.4.6.2 Length of Transition Curve

The length of the transition curve should be determined as the maximum of the following three criteria:

(i)   rate of change of centrifugal acceleration

(ii) rate of change of superelevation

(iii) an empirical formula given by IRC.

(i)   Rate of Change of Centrifugal Acceleration :

At the tangent point, radius is infinity and hence centrifugal acceleration is zero. At the end of the transition, the radius R has minimum value R. The rate of change of centrifugal acceleration should be adopted such that the design should not cause discomfort to the drivers. If c is the rate of change of centrifugal acceleration, it can be written as :

$$c = \frac{\dfrac{v^2}{R} - 0}{t} = \frac{\dfrac{v^2}{R} - 0}{\dfrac{L_s}{v}} = \frac{v^3}{L_s R}$$

- Therefore, the length of the transition curve L_{s_1} in m is

$$L_{s_1} = \frac{v^3}{cR}$$

- Where c is the rate of change of centrifugal acceleration given by an empirical formula suggested by IRC as below

$$c = \frac{80}{75 + 3.6v}$$

$$0.5 \leq c \leq 0.8$$

where,
v = velocity in meter per second

R = radius of curve in meter

L_s = Length of transition curve in meter

(ii) Rate of Introduction of Super-Elevation : Raise (E) of the outer edge with respect to inner edge is given by $E = eB = e(W + W_e)$. the rate of change of this raise from 0 to E is achieved gradually with a gradient of 1 in N over the length of the transition curve (typical range of N is 60 to 150).

Therefore, the length of the transition curve L_{s_2} is

$$L_{s_2} = Ne(W + W_e) \qquad \text{(Rotated about inner edge)}$$

or

$$L_{s_2} = \frac{Ne(W + We)}{2} \qquad \text{(Rotated about center line)}$$

(iii) By Empirical Formula : IRC suggest the length of the transition curve is minimum for a plain and rolling terrain

$$L_{s_3} = \frac{35v^2}{R}$$

and for steep and hilly terrain is :

$$L_{s_3} = \frac{12.96v^2}{R}$$

and the shift s as :

$$s = \frac{L_s^2}{24R}$$

The length of the transition curve L_s is the maximum of equations i.e.,

$$L_s = \text{Max} : \left(L_{s_1}, L_{s_2}, L_{s_3}\right)$$

1.4.7 Setback Distance (m)

Setback distance m or the clearance distance is the distance required from the centerline of a horizontal curve to an obstruction on the inner side of the curve to provide adequate sight distance at a horizontal curve. The setback distance depends on

1. Sight distance (SSD, ISD and OSD)

2. Radius of the curve

3. Length of the curve

Case (a) $\qquad\qquad\qquad$ $L_s < L_c$

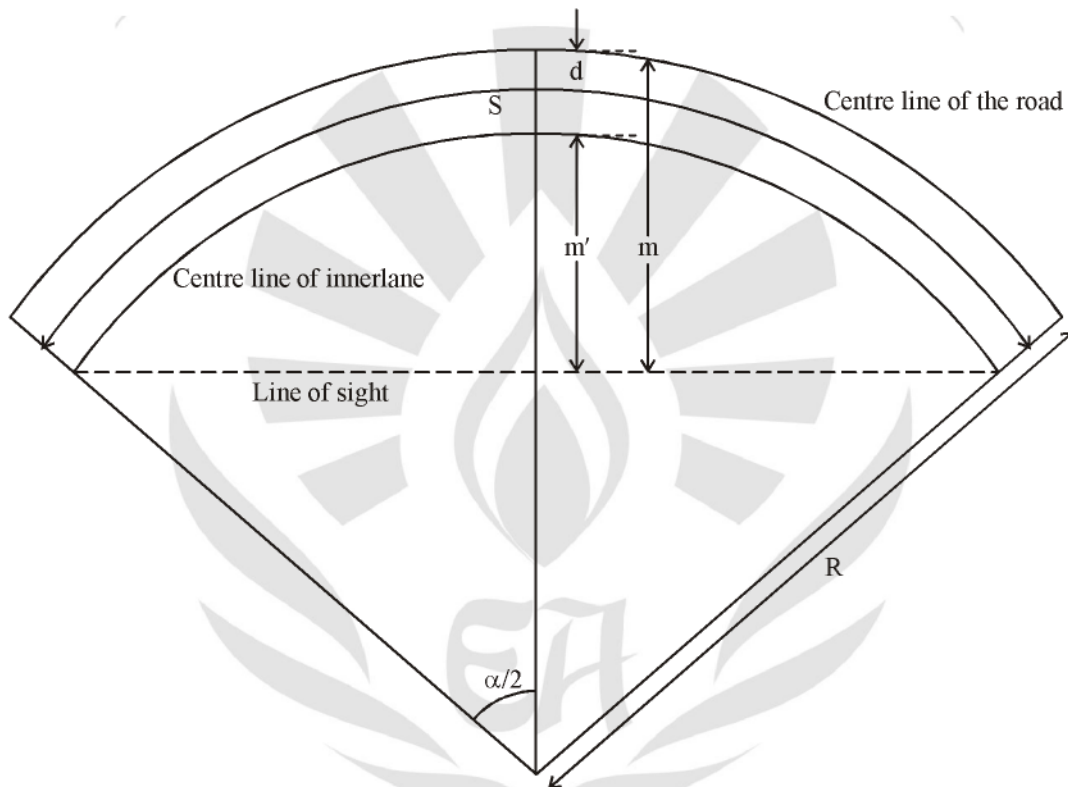For single lane roads $\qquad\qquad$ $\alpha = \dfrac{S}{R}$ Radians $= \dfrac{180S}{\pi R}$ Degrees

$$\dfrac{\alpha}{2} = \dfrac{180S}{2\pi R} \text{ Degrees}$$

Therefore, $\qquad\qquad\qquad$ $m = R - R\cos\left(\dfrac{\alpha}{2}\right)$

For multi lane roads, if d is the distance between centerline of the road and the centerline of the inner lane, then

Set-back for multi-lane roads $(L_s < L_c)$

$$m = R - (R-d)\cos\left(\dfrac{180S}{2\pi(R-d)}\right)$$

$$m = R - R\cos\left(\dfrac{\alpha}{2}\right)$$

Case (b) $\qquad\qquad\qquad$ $L_s > L_c$

For single lane $\qquad\qquad$ $m_1 = R - R\cos(\alpha/2)$

$$m_2 = \dfrac{(S - L_c)}{2}\sin\left(\dfrac{\alpha}{2}\right)$$

The set back is the sum of m_1 and m_2 given by

$$m = R - R\cos\left(\dfrac{\alpha}{2}\right) + \dfrac{(S - L_c)}{2}\sin\left(\dfrac{\alpha}{2}\right)$$