Toppersnotes
Unleash the topper in you

BPSC - AE

ASSISTANT ENGINEER

Mechanical Engineering

Bihar Public Service Commission

Volume - 8

Industrial Engineering

SIERRA INNOVATIONS

Table of Content

S No.	Chapter Title	Page No.
1	Forecasting	1
2	Routing, Scheduling, etc.	12
3	Line Balancing	35
4	Break Even Analysis	56
5	PERT and PCM	68
6	Inventory Control	91
7	Materials Requirement Planning	122
8	Work Study	127
9	Plant Layout	149
10	Quality Analysis and Control	158
11	Process Capability	169
12	Graphical Method	183
13	Simplex Method	191
14	Transportation Model	201
15	Assignment Model	215
16	Queuing Model	222
17	Value Analysis for CostValue	241
18	Miscellaneous	247
19	Practice Sheet	269

1 CHAPTER Forecasting

THEORY

1.1 | Forecasting

Forecasting means estimation of type, quantity and quality of future works e.g. sales etc. It is a calculated economic analysis.

1.1.1 Basic Elements of Forecasting

- Trends
- Cycles
- Seasonal Variations
- Irregular Variations

1.2 | Sales Forecasting Techniques

- Historic Estimation
- Sales Force Estimation
- Trend Line (or Time-series Analysis) Technique
- Market Survey
- Delphi Method
- Judge Mental Techniques
- Prior Knowledge
- Forecasting by Past Average
- Forecasting from Last Period's Sales
- Forecasting by Moving Average
- Forecasting by Weighted Moving Average
- Forecasting by Exponential Smoothing
- Correlation Analysis
- Linear Regression Analysis.

1.2.1 Average Method

Forecast sales for next period = Average sales for previous period

$$F_{t+1} = \frac{S_t + S_{t-1} + ... + S_1}{t}$$

Where, F_{t+1} = Forecast sales for next period

$S_t, S_{t-1}, ..., S_1$ are sales of t number of periods.

Example:

Period No.	1	2	3	4	5	6
Sales	7	5	9	8	5	8

Forecast sales for Period No

$$F_6 + 1 = \frac{7+5+9+8+5+8}{6} = 7$$

1.2.2 Forecast by Moving Average

In this method the forecast is neither influenced by very old data nor does it solely reflect the figures of the previous period.

Example:

Year	Period	Sales	Four-period average forecasting
1987	1	50	
	2	60	
	3	50	
	4	40	
1988	1	50	Forecast for 1988 period $1 = \frac{50+60+50+40}{4} = 50$
	2	55	Forecast for 1988 period $2 = \frac{60+50+40+50}{40} = 50$

1.2.3 Weighted Moving Average

A weighted moving Average allows any weights to be placed on each element, providing of course, that the sum of all weights equals one.

Example:

Period	Sales
Month-1	100
Month-2	90
Month-3	105
Month-4	95
Month-5	110

Forecast (weights 40%, 30%, 20%, 10% of most recent month)

Forecast for month-5 would be:

$$F_5 = 0.4 \times 95 + 0.3 \times 105 + 0.2 \times 90 + 0.1 \times 100 = 97.5$$

Forecast for month-6 would be:

$$F_6 = 0.4 \times 110 + 0.3 \times 95 + 0.2 \times 105 + 0.1 \times 90 = 102.5$$

1.2.4 *Exponential Smoothing*

New forecast = α (latest sales figure) + (1 – α) (old forecast) [VIMP]

$$F_{t+1} = \alpha d_t + (1 - \alpha) F_t$$

Where, α is known as the smoothing constant.

The size of α should be chosen in the light of the stability or variability of actual sales, and is normally from 0.1 to 0.3.

The smoothing constant, α , that gives the equivalent of an N-period moving average can be calculated as follows, $\alpha = \dfrac{2}{N+1}$

For e.g. if we wish to adopt an exponential smoothing technique equivalent to a nine period moving average then, $\alpha = \dfrac{2}{9+1} = 0.2$

Basically, exponential smoothing is an average method and is useful for forecasting one period ahead. In this approach, the most recent past period demand is weighted most heavily. In a continuing manner the weights assigned to successively past period demands decrease according to exponential law.

1.2.5 *Generalized Equation*

$$F_t = \alpha.(1 - \alpha)^0 d_{t-1} + \alpha.(1 - \alpha)^1 d_{t-2} + \alpha.(1 - \alpha)^2 d_{t-3} + ... + \alpha(1 - \alpha)^{k-1} d_{t-k} + (1 - \alpha)^k F_{t-k}$$

[Where k is the number of past periods]

It can be seen from above equation that the weights associated with each demand of equation are not equal but rather the successively older demand weights decrease by factor (1 – α). In other words, the successive terms $\alpha(1-\alpha)^0$, $\alpha(1-\alpha)^1$, $\alpha(1-\alpha)^2$, $\alpha(1-\alpha)^3$ decreases exponentially.

This means that the more recent demands are more heavily weighted than the remote demands.

Exponential smoothing method of Demand Forecasting:

- Demand for the most recent data is given more weightage.
- This method requires only the current demand and forecast demand.
- This method assigns weight to all the previous data.

Regression Analysis:

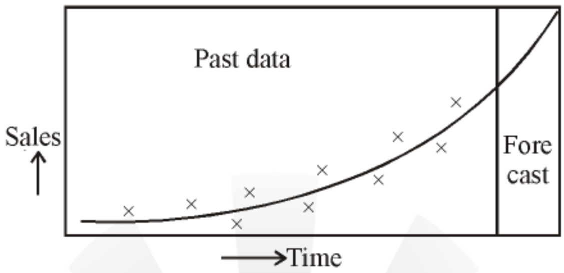
Regression analysis is also known as method of curve fitting. On this method the data on the past sales is plotted against time, and the best curve called the 'Trend line' or 'Regression line' or 'Trend curve'. The forecast is obtained by extrapolating this trend line or curve.

For linear regression

$$y = a + bx$$

$$a = \frac{\sum y - b\sum x}{n}$$

$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$



$$\text{Standard error} = \sqrt{\frac{\sum(x - y_1)^2}{(n-2)}}$$

***Example*:** What are moving average and exponential smoothing models for forecasting? A dealership for Honda city cars sells a particular model of the car in various months of the year. Using the moving average method, find the exponential smoothing forecast for the month of October 2010. Take exponential smoothing constant as 0.2:

Jan.	2010	80 cars
Feb.	2010	65 cars
March	2010	90 cars
April	2010	70 cars
May	2010	80 cars
June	2010	100 cars
July	2010	85 cars
Aug.	2010	65 cars
Sept.	2010	75 cars

Solution: (i) ***Moving average model for forecasting:*** Refer theory part of this book.

(ii) ***Exponential smoothing model for forecasting:*** Refer theory part of this book

Months	***Sells cars***	***Forecast demand (n = 3)***
Jan.	80	
Feb.	65	
March	90	
April	70	(80 + 65 + 90)/3 =78.33
May	80	(65 + 90 + 70)/3 = 75

June	100	(90 + 70 + 80)/3 = 80
July	85	(70 + 80 100)/3 = 83.33
Aug.	60	(80 + 100 + 85)/3 = 88.33
Sep.	75	(100 + 85 + 60)/3 = 81.67

Forecast of oct. by exponential smoothing method

$$F_{oct} = F_{sep} + \alpha\ (D_{sep} - F_{sep.})$$

$$\alpha = 0.2\ F_{sep} = 81.67\ D_{spt.} = 75$$

$$F_{oct} = 81.67 + 0.2\ (75 - 81.67)$$

$$F_{Oct} = 80.33 \simeq 81$$

Forecast for the month of October using moving average

$$F_{oct} = \frac{D_{July} + D_{Aug} + D_{Sep}}{3}$$

$$= \frac{75 + 60 + 80}{3} = 71.67$$

Example: Explain the need for sales forecasting. How are forecasting methods classified?

The past data about the load on a machine centre is as given below:

Month	*Load, Machine-Hours*
1	585
2	611
3	656
4	748
5	863
6	914
7	964

(i) If a five month moving average is used to forecast the next month's demand, compute the forecast of the load on the centre in the 8^{th} month.

(ii) Compute a weighted three month moving average for the 8^{th} month, where the weights are 0.5 for the latest month, 0.3 and 0.2 for the other months, respectively.

Solution: Most organizations are not in a position to wait unit orders are received before they begin to determine what production facilities, process, equipment, manpower, or materials are required and in what quantities. Most successful organization anticipate the future and for their products and translate that information into factor inputs required to satisfy expected demand. Forecasting provides a blue print for managerial planning. Forecasting is the estimation of the future on the basis of the past.

In many organizations, sales forecasts are used to establish production levels, facilitate scheduling, set inventory levels, determine man power loading, make purchasing decisions, establish sales conditions (pricing and advertising) and aid financial planning (cash budgeting and capital budgeting).

A good forecast should have the following attributes. It should be accurate, simple, easy, economical, quick and upto date. Following are the basic steps involved in a systematic demand forecast.

(i) State objectives

(ii) Select method

(iii) Identify variables

(iv) Arrange data

(v) Develop relationship

(vi) Prepare forecast and interpret

(vii) Forecast in specific units.

(i) Forecast for 8th month on the basis of five month moving average

$$= (964 + 914 + 863 + 748 + 656)/5 = 829$$

(ii) Forecast for 8th month on the basis of weighted average

$$= 0.5 \times 964 + 0.3 \times 914 + 0.2 \times 863 = 928.8$$

Example: **(i)** List common time-series forecasting models. Explain simple exponential smoothing method of forecasting demand. What are its limitations?

(ii) The monthly forecast and demand values of a firm are given below:

Month	*Forecast units*	*Demand units*
Jan	100	97
Feb	100	93
Mar	100	110
Apr	100	98
May	102	130
Jun	104	133
Jul	106	129
Aug	108	138
Sep	110	136
Oct	112	124
Nov	114	139
Dec	116	125

Calculate Tracking Signal for each month. Comment on the forecast model.

Solution: **(i)** Component of time series models

(1) Trend (T)

(2) Cyclic variation (C)

(3) Seasonal variation (S)

(4) Random variation (R)

Exponential Smoothing

This is similar to the weighted average method. The recent data is given more weightage and the weightages for the earlier periods are successfully being reduced. Let x_t is the actual (historical) data of demand during the period t. Let ± is the weightage given for the period t and F_t is the forecast for the time t then forecast for the time (t + 1) will be given as

(ii) Tracking signal = $\dfrac{Cumulative\ deviation}{MAD}$

$$= \frac{\Sigma(x_t - F_t)}{MAD}$$

Where, MAD = Mean Absolute deviation

$$= \frac{Sum\ of\ absolute\ deviations}{Total\ number\ of\ datas} = \frac{\Sigma(x_t - F_t)}{n}$$

Month	Forecast	Demand	$(x_t - F_t)$	MAD		T.S. $= \frac{\sum(x_t - f_t)}{MAD}$
January	100	97	-3	3	-3	-1
February	100	93	-7	5	-10	-2
March	100	110	10	6.67	0	0
April	100	98	-2	5.5	-2	-0.3636
May	102	130	28	10	26	2.6
June	104	133	29	13.167	55	4.177
July	106	129	23	14.571	78	5.353
August	108	138	30	16.5	108	6.545
September	110	136	26	17.55	134	7.635
October	112	124	12	17	146	8.588
November	114	139	25	17.727	171	9.646
December	116	125	9	17	180	10.588

Mean square error (MSE) = $\dfrac{\Sigma|x_t - F_t|^2}{n} = \dfrac{4742}{12}$

$= 395.167$

Upper limit = $3 \times \sqrt{MSE} = 3 \times \sqrt{395.167}$

$= 59.636$

Since upper limit of T.S < 59.636 hence model should not be revised.

Example: Demand for a certain item has been as shown below:

The forecast for April was 100 units with a smoothing constant of 0.20 and using first order exponential smoothing what is the July forecast? What do you think about a 0.20 smoothing constant?

Time	*Actual Demand*
April	200
May	50
June	150

Solution: Using exponential smoothing average:

$$F_{may} = \alpha \times D_{April} + (1 - \alpha)\ F_{April}$$
$$= 0.2 \times 200 + (1 - 0.2) \times 100 = 120$$
$$F_{June} = \alpha \times D_{May} + (1 - \alpha)F_{may}$$
$$= 0.2 \times 50 + (1 - .2) \times 120 = 106$$
$$F_{july} = \alpha \times D_{june} + (1 - \alpha) \times F_{june}$$
$$= 0.2 \times 150 + 0.8 \times 106 = 114.8 = 115$$

Example: In a time series forecasting model, the demand for five time periods was 10, 13, 15 18 and 22. A linear regression fit results in as equation F = 6.9 + 2.9 t where F is the forecast for period t. The sum of absolute deviation for the five data is?

Solution: Sum of absolute deviation

$$= |D_1 - F_1| + |D_2 - F_2| + |D_3 - F_3| + |D_4 - F_4| + |D_5 - F_5|$$
$$= |10 - 6.9 - 2.9 \times 1| + |13 - 6.9 - 2.9 \times 2| + |15 - 6.9 - 2.9 \times 3|$$
$$+ |18 - 6.9 - 2.9 - 2.9 \times 4| + |22 - 6.9 - 2.9 \times 5|$$
$$= 0.2 + 0.3 + 0.6 + 0.5 + 0.6 = 2.2$$

❑❑❑

PRACTICE SHEET

1. Which one of the following forecasting techniques is not suited for making forecasts for planning production schedules in the short range?

(a) Moving average

(b) Exponential moving average

(c) Regression analysis

(d) Delphi

2. When using a simple moving average to forecast demand, one would.

(a) Give equal weight to all demand data

(b) Assign more weight to the recent demand data

(c) Include new demand data in the average without discarding the earlier data

(d) Include new demand data in the average after discarding some of the earlier demand data

3. In a forecasting model, at the end of period 13, the forecasted value for period 14 is 75. Actual value in the periods 14 to 16 are constant at 100. If the assumed simple exponential smoothing parameter is 0.5, then the MSE at the end of period 16 is:

(a) 820.31 (b) 273.44

(c) 43.75 (d) 14.58

4. The most commonly used criteria for measuring forecast error is:

(a) Mean absolute deviation

(b) Mean absolute percentage error

(c) Mean standard error

(d) Mean square error

5. In a time series forecasting model, the demand for five time periods was 10, 13, 15, 18 and 22. A linear regression fit resulted in an equation F = 6.9 + 2.9 t where F is the forecast for period t. The sum of absolute deviations for the five data is:

(a) 2.2 (b) 0.2

(c) –1.2 (d) 24.3

6. ***Assertion (A):*** Time series analysis technique of sales-forecasting can be applied to only medium and short-range forecasting.

Reason (R): Qualitative information about the market is necessary for long-range forecasting.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is **not** the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

7. Given T = Underlying trend, C = Cyclic variations within the trend, S = Seasonal variation within the trend and R = Residual, remaining or random variation, as per the time series analysis of sales forecasting, the demand will be a function of:

(a) T and C (b) R and S

(c) T, C and S (d) T, C, S and R

8. Which one of the following methods can be used for forecasting the sales potential of a new product?

(a) Time series analysis

(b) Jury of executive opinion method

(c) Sales force composite method

(d) Direct survey method

9. Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

A. Decision making under complete certainty

B. Decision making under risk

C. Decision making under complete uncertainly

D. Decision making based on expert opinion

List-II

1. Delphi approach

2. Maximax criterion

3. Transportation mode

4. Decision tree

Codes:	**A**	**B**	**C**	**D**
(a)	3	4	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	4	3	1	2

10. Match List-I (Methods) with List-II (Problems) and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	Moving average	**1.**	Assembly
B.	Line balancing	**2.**	Purchase
C.	Economic batch size	**3.**	Forecasting
D.	Johnson algorithm	**4.**	Sequencing

Codes:	**A**	**B**	**C**	**D**
(a)	1	3	2	4
(b)	1	3	4	2
(c)	3	1	4	2
(d)	3	1	2	4

11. A company intends to use exponential smoothing technique for making a forecast for one of its products. The previous year's forecast has been 78 units and the actual demand for the corresponding period turned out to be 73 units. If the value of the smoothening constant ± is 0.2, the forecast for the next period will be:

(a) 73 units (b) 75 units

(c) 77 units (d) 78 units

12. For sales forecasting, pooling of expert opinions is made use of in.

(a) Statistical correlation

(b) Delphi technique

(c) Moving average method

(d) Exponential smoothing

13. To meet short range changes in demand of a product, which of the following strategies can be considered?

1. Overtime

2. Subcontracting

3. Building up inventory

4. New investments

Select the correct answer from the codes given below:

(a) 1, 2 and 3 (b) 1, 3 and 4

(c) 2 and 3 (d) 1 and 2

❑❑❑

ANSWER SHEET

1. ***Ans. (d)***

   Moving, average, Exponential moving average is used for short range. Regression is used for short and medium range. Delphi is used for long range forecasting.

2. ***Ans. (d)***

3. ***Ans. (b)***

Period	14.0	15.00	16.000
X_t	100.0	100.00	100.000
F_t	75.0	87.50	93.750
$(X_t - F_t)$	25.0	12.50	6.250
$\alpha(X_t - F_t)$	12.5	6.25	3.125
F_{t+1}	87.5	93.75	96.875
$(X_t - F_t)^2$	625	156.25	39.0625

$\Sigma(X_t - F_t)^2$ 820.31

Mean squared error, MSE = $\frac{820.31}{3}$ = 273.44

4. ***Ans. (d)***

5. ***Ans. (a)***

   Sum of absolute deviation

   = $(D_1 - F_1) + (D_2 - F_2) + (D_3 - F_3) + (D_4 - F_4)$

   $+ (D_5 - F_5)$

   = (10 – 6.9 – 2.9x1) + (13 – 6.9 – 2.9x2)

   + (15 – 6.9 – 2.9x3) + …………….

6. ***Ans. (b)***

7. ***Ans. (c)***

   Sales forecasting should not be influenced by the random variations in demand.

8. ***Ans. (d)***

9. ***Ans. (c)***

10. ***Ans. (d)***

11. ***Ans. (c)***

    New forecast

    = Old forecast + α(actual demand – old forecast)

    = 78 + 0.2 (73 – 78) = 77

12. ***Ans. (b)***

13. ***Ans. (b)***

❑❑❑

2 CHAPTER Routing, Scheduling, Etc.

THEORY

2.1 | Routing

Includes the planning of what work shall be done on the material to produce the product or part, where and by whom the work shall be done. It also includes the determination of path that the work shall follow and the necessary sequence of operations which must be done on the material to make the product.

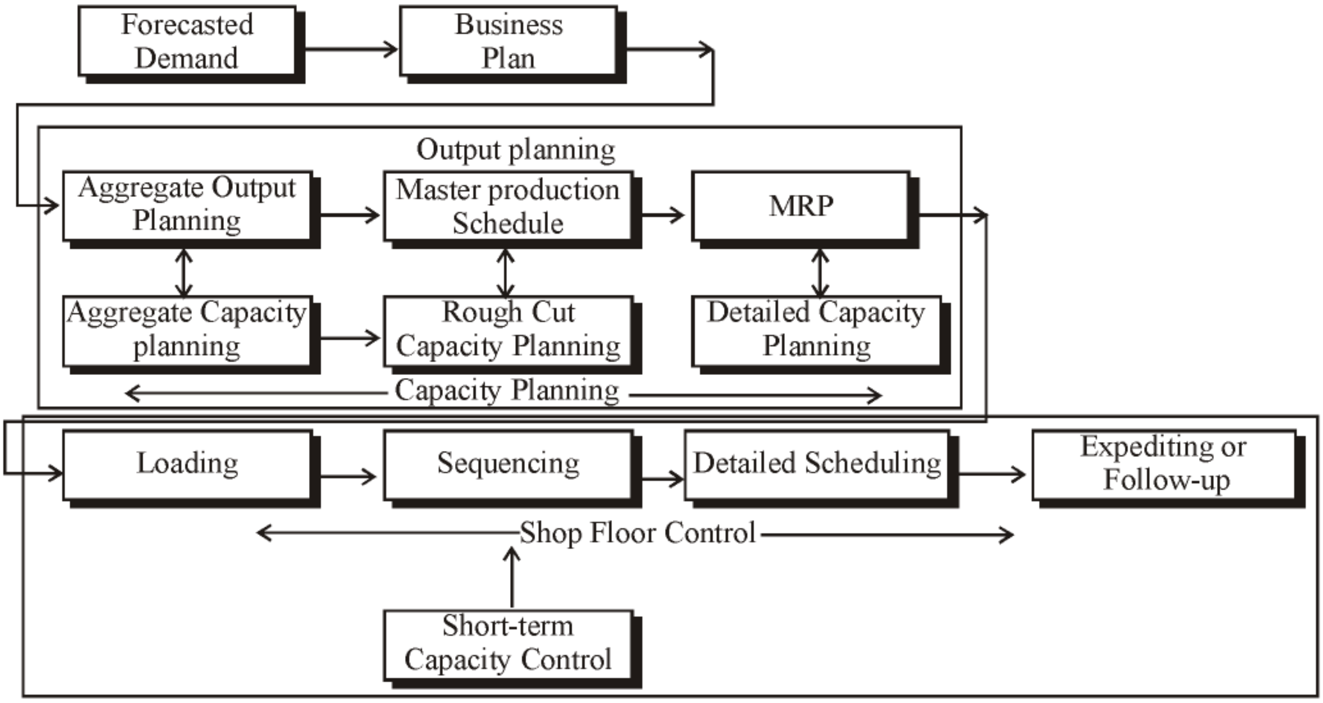
2.1.1 Routing Procedure Consist of the Following Steps

The finished product is analysed thoroughly from the manufacturing stand point, including the determination of components if it is an assembly product. Such an analysis must include:

- The following activities are to be performed in a particular sequence for routing a product.
- Analysis of the product and breaking it down into components.
- Taking makes or buys decisions.
- Determination of operations and processing time requirement.
- Determination of the lot size.
- Material or parts needed.
- Whether the parts are to be manufactured, are to be found in stores (either as raw materials or worked materials), or whether they are-to be purchased.
- Quantity of materials needed for each part and for the entire order.

2.2 | Scheduling

Scheduling is used to allocate resources over time to accomplish specific tasks. It should take account of technical requirement of task, available capacity and forecasted demand. Forecasted demand determines plan for the output, which tells us when products are needed. The output-plan should be translated into operations, timing and schedule on the shop-floor. This involves loading, sequencing, detailed scheduling, expediting and input/output control.



The Planning and Scheduling Function

2.3 | Sequencing

When number of jobs are waiting in queue before an operational facility (such as, a milling machine), there is a need to decide the sequence of processing all the waiting jobs.Sequencing is basically an order in which the jobs, waiting before an operational facility, are processed. For this, priority rule, processing time, etc. are needed.

The decision regarding order in which jobs-in-waiting are processed on an operational facility or work-centre is called as sequencing.

2.4 | Detailed Scheduling

Once the priority rule of job sequencing is known, we can sequence the jobs in a particular order. This order would determine which job is done first, then which the next one is and so on. However, sequencing does not tell us the day and time at which a particular job is to be done. This aspect is covered in detailed scheduling. In this, estimates are prepared regarding setup and processing time at which a job is due to start and finish. Detailed

2.5 | Loading

The customer order for each job has certain job contents, which need to be performed on various work centers of facilities. During each planning period, jobs orders are assigned on facilities. This ultimately determines the work-load or jobs to be performed in a planned period.

The assignment of specific jobs to each operational facility during a planning period is known as loading.

2.6 | EXPEDITING

Once the detailed schedule is operationalized, we need to keep a watch over the progress in the shop-floor. This is necessary to avoid a deviation from the schedule. In case of deviation from the schedule, the causes of deviation are immediately attended to. For example, machine breakdown, non-availability of a tool, etc., cause disruption in schedule. Therefore, continuous follow up or expediting is needed to overcome the deviations from schedule.

Expediting or follow-up *involves continuous tracking of the job's progress and taking specific action if there is a deviation from the detailed schedule. The objective of expediting is to complete the jobs as per the detailed schedule and overcome any special case causing delay, failure, break-down, non-availability of material and disruption of detailed schedule.*

2.7 | SHORT-TERM CAPACITY (INPUT-OUTPUT) CONTROL

Schedules are made so that jobs are completed at a specific time on every facility. For this, each facility has certain capacity to perform. In real situation, the utilization of the capacity of each facility may be different from the planned one. This difference should be monitored carefully because under-utilization of capacity means waste resource and overutilization may cause disruption, failure, delays, or even breakdown. Therefore, in case of discrepancy in input and output of the capacities, some adjustments in schedule are needed.

Short-term capacity control involves monitoring of deviation between actual and planned utilization of the capacity of an operational facility.

There are two types of schedules used: Master Schedules and Shop or Production Schedule.

2.7.1 Master Schedule

The first step in scheduling is to prepare the Master Schedule. ***A master schedule specifies the product to be manufactured, the quality to be produced and the delivery date to the customer.*** It also indicates the relative importance or manufacturing orders. The scheduling periods used in the master schedule are usually months.

Whenever a new order is received, it is scheduled on the master schedule taking into account the production capacity of the plant. Based on the master schedule, individual components and sub-assemblies that make up each product are planned:

- Orders are placed for purchasing raw materials to manufacture the various components.
- Orders are placed for purchasing components from outside vendors.
- Shop or production schedules are prepared for parts to be manufactured within the plant.

The Objectives of Master Schedule are:

- It helps in keeping a running total of the production requirements.
- With its help, the production manager can plan in advance for any necessity of shifting from one product to another or for a possible overall increase or decrease in production requirements.
- It provides the necessary data for calculating the back log of work or load ahead of each major machine.
- After an order is placed in the master schedule, the customer can be supplied with probable or definite date of delivery.

2.7.2 Shop or Production Schedule

After preparing the master schedule, the next step is to prepare shop or production schedule. This includes the department machine and labourload schedules, and the start dates and finish dates for the various components to be manufactured within the plant. A scheduling clerk does this job so that all processing and shipping requirements are relatively met. For this, the following are the major considerations to be taken case of:

- Due date of the order.
- Whether and where the machine and labour capacity are available.
- Relative urgency of the order with respect to the other orders.

Objectives of Production Schedule:

- It meets the output goals of the master schedule and fulfils delivery promises.
- It keeps a constant supply of work ahead of each machine.
- It puts manufacturing orders in the shortest possible time consistent with economy.

2.8 | The Scheduling Problem

2.8.1 List Scheduling Algorithms

This class of algorithms arranges jobs on a list according to some rule. The next job on the list is then assigned to the first available machine.

2.8.2 Random List

This list is made according to a random permutation.

2.8.3 Longest Processing Time (LPT)

The longest processing time rule orders the jobs in the order of decreasing processing times. Whenever a machine is free, the largest job ready at the time will begin processing.

This algorithm is a heuristic used for finding the minimum make span of a schedule. It schedules the longest jobs first so that no one large job will "stick out" at the end of the schedule and dramatically lengthen the completion time of the last job.

2.8.4 Shortest Processing Time (SPT)

The shortest processing time rule orders the jobs in the order of increasing processing times. Whenever a machine is free, the shortest job ready at the time will begin processing. This algorithm is optimal for finding the minimum total completion time and weighted completion time.

In the single machine environment with ready time at 0 for all jobs, this algorithm is optimal in minimizing the mean flow time, minimizing the mean number of jobs in the system, minimizing the mean waiting time of the jobs from the time of arrival to the start of processing, minimizing the maximum waiting time and the mean lateness.

2.8.5 Weighted Shortest Processing Time (WSPT)

The weighted shortest processing time rule is a variation of the SPT rule. Let t[i] and w[i] denote the processing time and the weight associated with the job to be done in the sequence ordered by the WSPT rule. WSPT sequences jobs such that the following inequality holds.

$$t[1]/w[1] \Leftarrow t[2]/w[2] \Leftarrow \dots \Leftarrow t[n]/w[n]$$

In the single machine environment with ready time set at 0 for all jobs, the WSPT minimizes the weighted mean flow time.

2.8.6 Earliest Due Date (EDD)

In the single machine environment with ready time set at 0 for all jobs, the earliest due date rule orders the sequence of jobs to be done from the job with the earliest due date to the job with the latest due date. Let d[i] denote the due date of the i^{th} job in the ordered sequence . EDD sequences jobs such that the following inequality holds,

$$d[1] \Leftarrow d[2] \Leftarrow \ldots d[n]$$

EDD, in the above setting, finds the optimal schedule when one wants to minimize the maximum lateness, or to minimize the maximum tardiness.

2.8.7 Minimum Slack Time (MST)

The minimum slack time rule measures the "urgency" of a job by its slack time. Let $d[i]$ and $t[i]$ denote the due date and the processing time associated with the *ith* job to be done in the ordered sequence. MST sequences jobs such that the following inequality holds,

$$d[1] - t[1] \Leftarrow d[2] - t[2] \Leftarrow \ldots \Leftarrow d[n] - t[n]$$

In the single machine environment with ready time set at 0, MST maximizes the minimum lateness.

2.8.8 Other Algorithms

HODGSON'S ALGORITHM

Hodgson's Algorithm minimizes the number of tardy jobs in the single machine environment with ready time equal to zero.

Let E denote the set of early jobs and L denote the set of late jobs. Initially, all jobs are in set E and set L is empty.

Step 1: Order all jobs in the set E using EDD rule.

Step 2 : If no jobs in E are late, stop; E must be optimal. Otherwise, find the first late job in E. Let this first late job be the k^{th} job in set E, job $[k]$.

Step 3: Out of the first k jobs, find the longest job. Remove this job from E and put it in L. Return to step 2.

Example: There are five jobs in waiting for getting processed on a machine. Their sequence of arrival, processing time and due-date are given in the table below. Schedule the jobs using FCFS, SPT, D Date, LCFS, Random, and STR rules. Compare the results.

Job (In Sequence of Arrival)	**Processing Time (Days)**	**Due Date (i.e. Days From Now)**
J1	4	6
J2	5	7
J3	3	8
J4	7	10
J5	2	3