



SSC - JE

JUNIOR ENGINEER

Civil Engineering

Staff Selection Commission

Volume - 11

Strength of Materials (SOM)



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CHAPTER

1

INTRODUCTION

THEORY

1.1 MATERIAL CLASSIFICATION

According to behaviour on loading, material can be classified as:

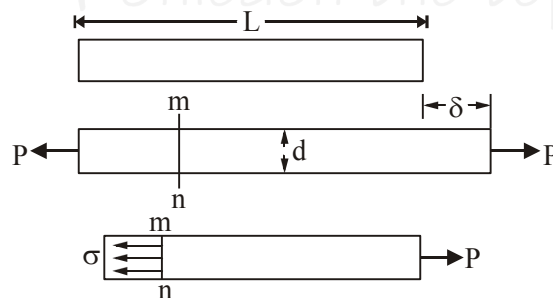
- (1) **Elastic** : Undergoes deformation when subjected to the external loading and comes back to its original state after removal of load.
- (2) **Plastic** : Material do not regain its original dimensions and the deformation is permanent.
- (3) **Rigid** : Does not undergo any deformation when loaded externally.

In statics and dynamics, we dealt with forces and motions associated with particles and rigid bodies. In strength of materials, we examine the stresses and strains that occur inside real bodies those deform under loads. here, you must appreciate the difference between rigid body and real body.

1.2 STRESS AND STRAIN

1.2.1 Normal Stress :

Consider a prismatic bar loaded by axial forces P at the ends. A prismatic bar is straight structural member having constant cross section throughout its length. The axial force produce a uniform stretching of bar. Here, bar is said to be in tension.



A section taken perpendicular to longitudinal axis of bar is cross-section. Considering free-body diagram, the tensile force P acts on right hand of free body at the other end is force representing the action of removed part of bar upon the part that remains. These forces are continuously distributed over the cross-section. The intensity of force (i.e. force per unit area) is called the stress and is denoted by, s

Hence, under equilibrium, $F = \sigma A$

$$\Rightarrow \sigma = \frac{F}{A}$$

The stress is the force of resistance per unit area offered by a body against the deformation.

When the bar is stretched by force P, as shown in figure, the resulting stresses are tensile stresses and if forces are reversed in direction, causing the bar to be compressed, the stresses are compressive stresses.

As stress acts in direction perpendicular to cut surface, it is referred as normal stress. The normal stresses may be tensile or compressive. The shear stress act parallel to the surface. Conventionally the tensile stresses are taken as positive and compressive stresses are negative.

The unit of stress is N/m^2 also referred as pascal.

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

It can also be expressed as MPa. i.e. N/mm^2 .

1.2.2 Normal Strain :

An axially loaded bar undergoes a change in length, becoming longer in tension and shorter when in compression, strain is defined as change in length per unit length.

$$(\text{strain}) \epsilon = \frac{\delta}{L}$$

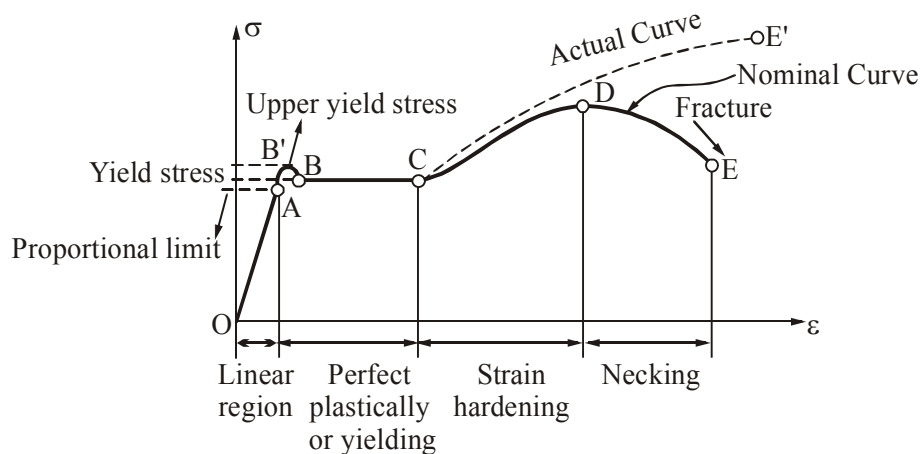
If bar is in tension, the strain is called a tensile strain, representing stretching of material. If the bar is in compression, the strain is compressive strain. The tensile strain is taken as positive and compressive strain is negative. The strain ϵ is called normal strain because it is associated with normal stresses. As normal strain ϵ is ratio of two lengths it is a dimensionless quantity i.e. it has no units.

Note: The definition of normal stress and normal strain are based purely on statical and geometrical consideration. It can be used for load of any magnitude and for any material.

1.3 STRESS STRAIN DIAGRAM OF MILD STEEL IN TENSION

The mechanical properties of material are determined by test performed on small specimen of the material. The most common material tension is tension test, in which tensile loads are applied on cylindrical specimen. The ASTM (American society for testing and materials) standard tension specimen has diameter of 0.5 in and a gauge length of 2.0 in. The machine used in test is UTM (universal testing machine).

In a static test, the load is applied very slowly and in dynamic test, the rate of loading may be very high. Here, we are analyzing properties based on static test.



The typical stress strain diagram of mild steel is shown in figure. Here, the stress is nominal stress or engineering stress and strain is nominal strain or engineering strain.

$$\text{Nominal (engineering stress)} = \frac{\text{Load}}{\text{Initial cross section area}} = \frac{P}{A_0}$$

$$\text{True stress} = \frac{\text{Load}}{\text{Actual Area}} = \frac{P}{A_a}$$

$$\text{Nominal strain} = \frac{\Delta L}{L_0}$$

$$\text{True strain} = \frac{\Delta L}{L_a}$$

The nominal stress is obtained by dividing the load P by initial cross sectional area A. the true stress is calculated by using the actual area of the bar.

Similarly, for calculation of strain, if initial gauge length is used nominal strain is obtained. If the actual length is used, true strain is obtained.

- (1) The diagram begin in with straight line from O to A. In this region the stress and strain are directly propotional and behaviour of material is linearally elastic.
- (2) Beyond point A, linear relationship between stress and strain no longer exists. A is called proportional limit.
- (3) When load is increased beyond A, the slope of curve become smaller and smaller, unit at point B, the curve becomes horizontal.
- (4) From B to C, considerable elongation occurs with no increase in tensile force. The phenomenon is known as yielding, reigon BC is called as yield platue.
- (5) In region CD, the material begin to strain hardening, the material undergoes change in its atomic and crystalline structure, resulting in increased resistance of material to further deformation.
- (6) The load reach its maximum value and corresponding stress is called ultimate stress.
- (7) The fracture finally occur at point E as shown in figure. Various properties of material can be deduced from stress-strain diagram, stress crresponding to E is called fracture / rupture stress.

1.4 SOME IMPORTANT PROPERTIES OF MATERIAL

1.4.1 Ductility :

The ductility of material by which it can be drawn as wire of small cross-section upon tensioning forces.

The percent elongation is defined as

$$\text{Percent elongation} = \frac{L_f - L_o}{L_o} \times 100$$

$$\text{Percent change in Area} = \frac{A_f - A_o}{A_o} \times 100$$

Where,

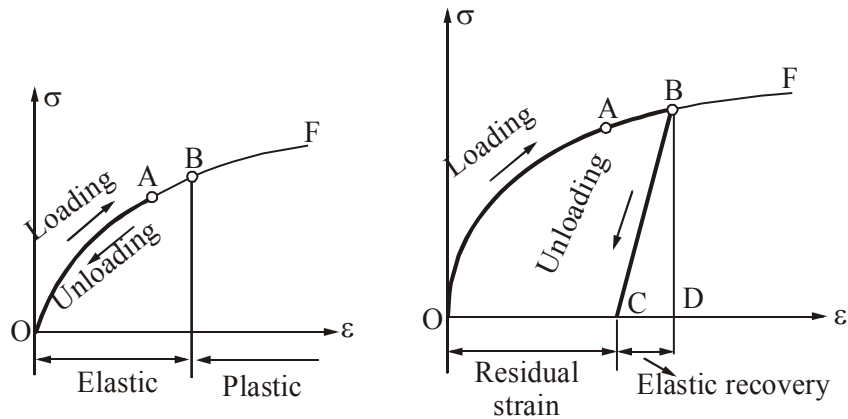
A_0 = Original cross-section area

A_f = Final area at fracture

The material which fails in tension at lower values of strain are classified as brittle materials.

1.4.2 Elasticity and Plasticity :

When load is slowly removed, two different situation may occur as shown in figure.



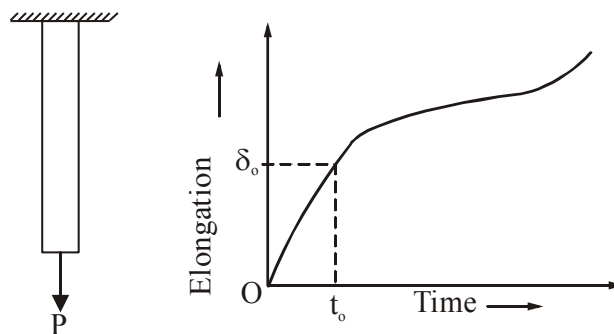
Suppose material is loaded and strain go from O to A. When load is removed, the material follow exactly the same curve back to origin O. This Property of material by which it returns to its original dimension during unloading is called elasticity and material is said to be elastic.

When material is loaded to higher level and when unloading occurs, the material follows line BC (which is parallel to initial slope of OA) on the diagram. When point C is reached, the load has entirely been removed but a residual strain or permanent strain, OC remain in the material. The corresponding residual elongation of bar is called permanent set. Thus, the bar returns partially to original shape, hence the material is said to be partially elastic. Then stress level above which all the strain is not recovered is called elastic limit of material.

The characieristic of material by which it undergoes inelastic strain beyond those at the elastic limit is known as plasticity.

1.4.3 Creep :

When a constant load is applied on a material, over a time, a permanent deformation occures. Which is called creep.



Consider a vertical bar loaded by constant force P. When load P is applied initially, the bar elongates by amount δ_0 during time t_0 . Subsequent to time t_0 , the load remains constant. However, due to creep, the bar elongatics even though load does not change. Creep is more at higher temperature than at ordinary temperature, i.e., creep is propotional to temperature.

1.5 HOOKE'S LAW

According to Hooke's law within the proportional limit, the stress is proportional directly to the strain.

i.e. $\sigma \propto \epsilon$

or
$$\frac{\sigma}{\epsilon} = E$$

E = Young's modulus or modulus of elasticity

When a material behaves elastically and also exhibits a linear relationship between stress and strain it is said to be linearly elastic. The linear relationship between stress and strain for a bar in simple tension and compression is expressed as:

$$\sigma = E \epsilon$$

Where E is a constant of proportionality known as the modulus of elasticity for the material. The modulus of elasticity is the slope of the stress-strain diagram in the linearly elastic region. The unit of E is the same as the unit of stress.

This equation is known as Hooke's law. The modulus of elasticity E has relatively large values for materials that are stiff. The elastic modulus of common materials are:

- (A) $E_{\text{Steel}} = 200 \text{ GPa}$
- (B) $E_{\text{Aluminium}} = 80 \text{ GPa}$
- (C) $E_{\text{Wood}} = 11 \text{ GPa}$

The modulus of elasticity is often called Young's modulus.

1.5.1 Poisson's Ratio :

When a prismatic bar is loaded in tension, the axial elongation is accompanied by lateral contraction (direction normal to applied load). The lateral strain is proportional to axial strain in the linear elastic range when the material is both homogeneous and isotropic.

The ratio of strain in lateral direction to strain in axial direction is known as Poisson's ratio.

$$\nu = \frac{\text{lateral strain}}{\text{axial strain}}$$

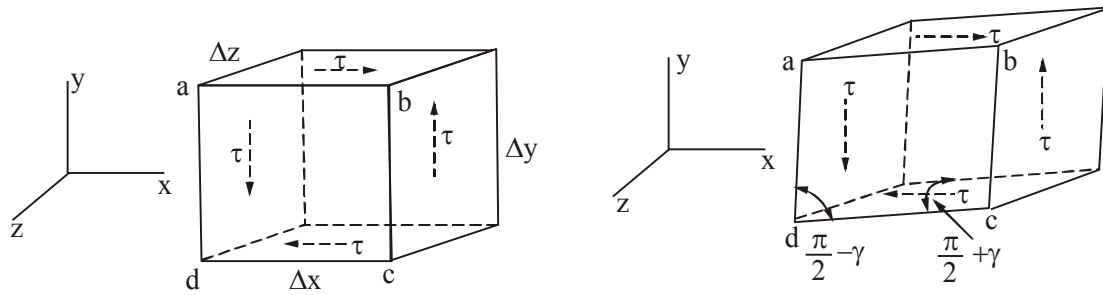
A material is homogeneous if it has the same composition throughout the body and isotropic material has the same properties in all directions.

1.5.2 Shear Stress and Strain :

Shear stresses act parallel or tangential to the surface. The shear stress is given as

$$\tau = \frac{F}{A}$$

To understand the nature of shear stress, let us consider a stress element in the form of a rectangular parallel block having sides of Δx , Δy and Δz .



Considering force equilibrium total shear force on the top face is $\tau\Delta x\Delta z$ and this force is balanced by equal and opposite force on bottom face. These two force form a couple having moment about z-axis of magnitude $\tau\Delta x\Delta y$. Equilibrium of element requires this moment to be balanced by equal and opposite moment resulting from shear stress acting on side face of element.

This requires the magnitude of shear stress on opposite face of an element to be equal in magnitude and opposite in direction.

Under the action of these stresses, the material is deformed, The angle γ is measure of distortion or change in shape of element and is called shear strain the unit of shear strains are radians.

A shear stress acting on positive face of element is positive if it acts in positive direction of one of the coordinate axis and negative if it acts in the negative direction of the axis. A shear stress acting on a negative face of an element is positive if it acts in the negative direction of an axis and negative if it acts in the positive direction.

Shear strain in an element is positive when the angle between two positive (or negative) faces is reduced. The strain is negative when angle between two positive faces (or two negative) faces is increased.

The shear stress-strain diagram can be plotted in same way as in tension-test diagram. Hooke's law in shear is

$$\tau = G\gamma$$

Where,

G = Shear modulus of elasticity or modulus of rigidity.

1.5.3 Bulk Modulus :

If material is subjected to similar and equal triaxial stresses, then ratio of stress to volumetric strain is called bulk modulus:

$$\begin{aligned} K &= \frac{\sigma}{\epsilon_v} \\ &= \frac{\sigma E}{3\sigma(1-2\nu)} \\ E &= 3K(1-2\nu) \end{aligned}$$

For material to be incompressible

Either $\sigma_x + \sigma_y + \sigma_z = 0$

or $\nu = 0.5 \Rightarrow$ Not possible

1.5.4 Relation Between Elastic Constants :

$$E = 2G(1 + \nu)$$

$$E = 3K(1 - 2\nu)$$

$$E = \frac{9KG}{3K + G}$$

$$\nu = \frac{3K - 2G}{6K + 2G}$$

Where,

E = Modulus of elasticity

K = Bulk modulus

G = Modulus of rigidity

ν = Poisson's ratio

Example: A long wire of specific mass m hang freely under its own weight. Derive formula for tensile stress in the wire.

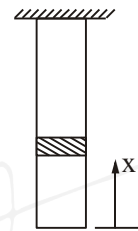
Solution: Let the weight per unit volume of the rope be w .

Force on the cross section of the elemental part

$$w \times \text{volume below} = wAx$$

$\therefore$

$$\text{stress} = \frac{wAx}{A} = wx$$



Note: $w = \frac{mg}{v}$ where m = total mass of the whole wire & v = total volume of the whole = Al

PRACTICE SHEET

OBJECTIVE QUESTIONS

- If Poisson's ratio of a material is 0.5, then the elastic modulus for the material is
 - three times its shear modulus
 - four times its shear modulus
 - equal to its shear modulus
 - indeterminate
- Match List-I (Elastic properties of an isotropic elastic material) with List-II (Natural of strain produced) and select the correct answer using the codes given below the lists :

List-I

- Young's modulus
- Modulus of rigidity
- Bulk modulus
- Poisson's ratio

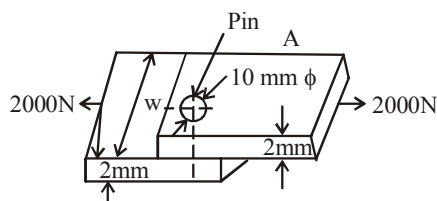
List-II

- Shear strain
- Normal strain
- Transverse strain
- Volumetric strain

Codes:

- | | A | B | C | D |
|-----|---|---|---|---|
| (a) | 1 | 2 | 3 | 4 |
| (b) | 2 | 1 | 3 | 4 |
| (c) | 2 | 1 | 4 | 3 |
| (d) | 1 | 2 | 4 | 3 |

- If permissible stress in plates of joint through a pin as shown in the figure is 200 MPa, then the width w will be



- 15 mm
- 20 mm
- 18 mm
- 25 mm

- The number of independent elastic constants required to express the stress-strain relationship for a linearly elastic isotropic material is
 - one
 - two
 - three
 - four
- The number of elastic constants for a completely anisotropic elastic material is
 - 3
 - 4
 - 21
 - 25
- The maximum value of Poisson's ratio for an elastic material is
 - 0.25
 - 0.5
 - 0.75
 - 1.0
- The unit of elastic modulus is the same as those of
 - Stress, shear modulus and pressure
 - Strain, shear modulus and force
 - Shear modulus, stress and force
 - Stress, strain and pressure.
- Young's modulus of elasticity and Poisson's ratio of a material are 1.25×10^5 MPa and 0.34 respectively. The modulus of rigidity of the material is:
 - 0.4025×10^5 MPa
 - 0.4664×10^5 MPa
 - 0.8375×10^5 MPa
 - 0.9469×10^5 MPa
- The independent elastic constants for a homogenous and isotropic material are
 - E, G, K, ν
 - E, G, K
 - E, G, ν
 - E, G
- In a homogenous, isotropic elastic material, the modulus of elasticity E in terms of G and K is equal to

- $\frac{G + 3K}{9KG}$
- $\frac{3G + K}{9KG}$
- $\frac{9KG}{G + 3K}$
- $\frac{9KG}{K + 3G}$

11. Match List I with List II and select the correct answer using the codes given below the lists

List I (Property)

- A. Tensile strength
- B. Impact strength
- C. Bending strength
- D. Fatigue strength

List II (Testing Machine)

- 1. Rotating Bending machine
- 2. Three-Point Loading Machine
- 3. Universal Testing Machine
- 4. Izod Testing Machine

Codes: A B C D

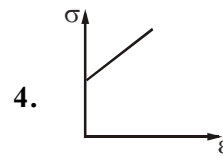
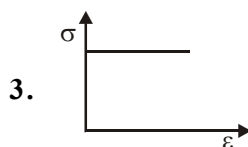
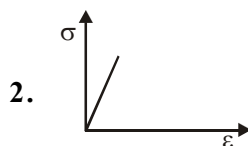
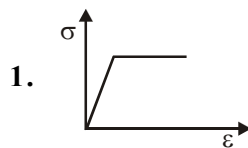
- (a) 4 3 2 1
- (b) 3 2 1 4
- (c) 2 1 4 3
- (d) 3 4 2 1

12. Match List-I with List-II and select the correct answer using the codes given the lists:

List-I

- A. Rigid-Perfectly plastic
- B. Elastic-Perfectly plastic
- C. Rigid-Strain hardening
- D. Linearly elastic

List-II



Codes: A B C D

- (a) 3 1 4 2
- (b) 1 3 2 4
- (c) 3 1 2 4
- (d) 1 3 4 2

13. Which one of the following materials is highly elastic?

- (a) Rubber
- (b) Brass
- (c) Steel
- (d) Glass

14. The elastic Constants E and K are related as (μ is the Poisson's ratio)

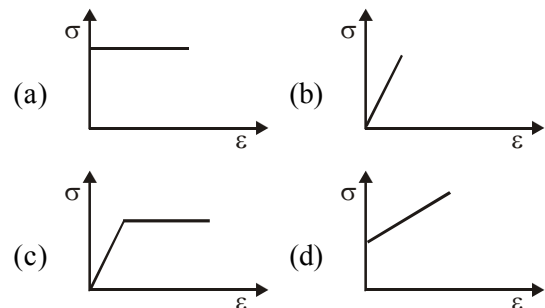
- (a) $E = 2K (1 - 2\mu)$
- (b) $E = 3K (1 - 2\mu)$
- (c) $E = 3K (1 + \mu)$
- (d) $E = 2K (1 + 2\mu)$

15. **Assertion (A)** : Stress-strain curves for brittle material do not exhibit yield point.

Reason (R): Brittle materials fail without yielding.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

16. An idealised stress-strain curve for a perfectly plastic material is given by



17. **Assertion (A):** Poisson's ratio is a measure of the lateral strain in all direction perpendicular to and in terms of the linear strain.

Reason (R): The nature of lateral strain in a uni-axially loaded bar is opposite to that of the linear strain.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true
18. A weight falls on a plunger fitted in a container filled with oil thereby producing a pressure of 1.5 N/mm^2 in the oil. The Bulk Modulus of oil is 2800 N/mm^2 . Given this situation, the volumetric compressive strain produced in the oil will be:
(a) 400×10^{-6} (b) 800×10^{-6}
(c) 268×10^{-6} (d) 535×10^{-6}
19. The Young's modulus of elasticity of a material is 2.5 times its modulus of rigidity. The Poisson's ratio for the material will be:
(a) 0.25 (b) 0.33
(c) 0.50 (d) 0.75
20. Lueder's lines on steel specimen under simple tension test is a direct indication of yielding of material due to slip along the plane
(a) Of maximum principal stress
(b) Of maximum shear
(c) Of loading
(d) Perpendicular to the direction of loading
21. A tensile bar is stressed to 250 N/mm^2 which is beyond its elastic limit. At this stage the strain produced in the bar is observed to be 0.0014. If the modulus of elasticity of the material of the bar is 205000 N/mm^2 then the elastic component of the strain is very close to
(a) 0.0004 (b) 0.0002
(c) 0.0001 (d) 0.00005

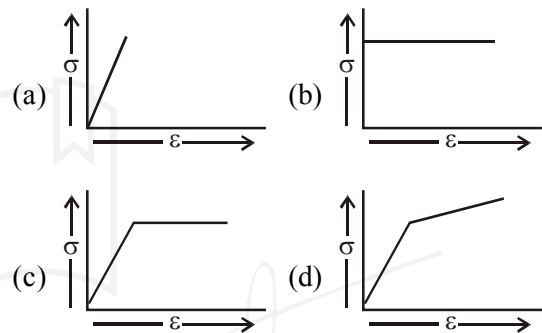
22. In a simple tension test, Hooke's law is valid upto the.

(a) Elastic limit (b) Limit of proportionality
(c) Ultimate stress (d) Breaking point

23. The percentage elongation of a material as obtained from static tension test depends upon the.

(a) Diameter of the test specimen
(b) Gauge length of the specimen
(c) Nature of end-grips of the testing machine
(d) Geometry of the test specimen

24. The stress-strain curve of an ideal elastic strain hardening material will be as



25. **Assertion (A):** Poisson's ratio of a material is a measure of its ductility:

Reason (R): For every linear strain in the direction of force, Poisson's ratio of the material gives the lateral strain in directions perpendicular to the direction of force.

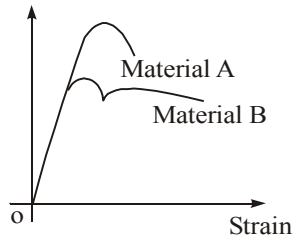
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

26. Which one of the following pairs is NOT correctly matched?

(a) Uniformly distributed stress Force passed through the centroid of the cross-section
(b) Elastic deformation Work done by external forces during deformation is dissipated fully as heat

- (c) Potential energy of strain Body is in a state of elastic deformation
- (d) Hooke's law Relation between stress and strain

27. The stress-strain diagram for two materials A and B is shown below:



The following statements are made based on this diagram

- (I) Material A is more brittle than material B
- (II) The ultimate strength of material B is more than that of A

With reference to the above statements, which of the following applies?

- (a) Both the statements are false
- (b) Both the statements are true
- (c) I is true but II is false
- (d) I is false but II is true

28. Match list I (Materials) with List II (Stress-strain curves) and select the correct answer using the codes given below the Lists:

List - I

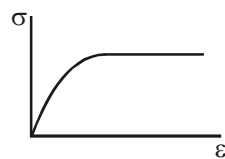
A. Mild Steel

B. Pure copper

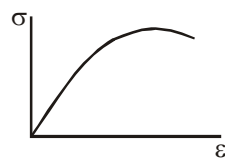
C. Cast iron

List - II

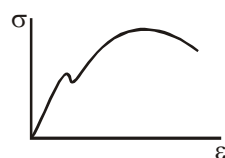
1.



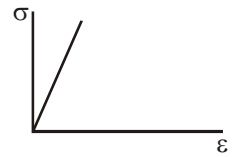
2.



3.



D. Pure aluminium



Codes: A B C D

- (a) 3 1 4 1
- (b) 3 2 4 2
- (c) 2 4 3 1
- (d) 4 1 3 2

29. Match List I with List II and select the correct answer using the codes given below the Lists:

List - I

- A. Ultimate strength
- B. Natural strain
- C. Conventional strain
- D. Stress

List - II

1. Internal structure
2. Change of length per unit instantaneous length
3. Change of length per unit gauge length
4. Load per unit area

Codes: A B C D

- (a) 1 2 3 4
- (b) 4 3 2 1
- (c) 1 3 2 4
- (d) 4 2 3 1

30. **Assertion (A):** Hooke's law is the constitutive law for a linear elastic material.

Reason (R): Formulation of the theory of elasticity requires the hypothesis that there exists a unique unstressed state of the body, to which the body returns whenever all the forces are removed.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

31. Consider the following statements:

1. There are only two independent elastic constants.
2. Elastic constants are different in orthogonal directions.
3. Material properties are same everywhere.
4. Elastic constants are same in all loading directions.
5. The material has ability to withstand shock loading.

Which of the above statements are true for a linearly elastic, homogeneous and isotropic material?

- (a) 1, 3, 4 and 5 (b) 2, 3, and 4
(c) 1, 3, and 4 (d) 2 and 5

32. **Assertion (A):** For a ductile material stress-strain curve is a straight line up to the yield point.

Reason (R): The material follows Hooke's law up to the point of proportionality.

- (a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

33. Match List-I (Types of Tests and Materials) with List-II (Types of Fractures) and select the correct answer using the codes given below the lists:

List - I (Types of Tests and Materials)

- A. Tensile test on CI
B. Torsion test on MS
C. Tensile test on MS
D. Torsion test on CI

List-II (Types of Fractures)

1. Plain fracture on a transverse plane
2. Granular helecoidal fracture
3. Plain granular at 45° to the axis
4. Cup and Cone
5. Granular fracture on a transverse plane

Codes: A B C D

- (a) 4 2 3 1
(b) 4 1 3 2
(c) 5 1 4 2
(d) 5 2 4 1

34. The moduli of elasticity and rigidity of a material are 200 GPa and 80 GPa, respectively. What is the value of the Poisson's ratio of the material?

- (a) 0.30 (b) 0.26
(c) 0.25 (d) 0.24

35. A mild steel specimen is tested in tension up to fracture in a Universal Testing Machine. Which of the following machanical properties of the material can be evaluated from such a test?

1. Modulus of elasticity
2. Yield stress
3. Ductility
4. Tensile strength
5. Modulus of rigidity

Select the correct answer using the code given below:

- (a) 1, 3 and 5 (b) 2, 3 and 4
(c) 1, 2 and 5 (d) 1, 2, 3 and 4

36. What is the cause of failure of a short MS strut under an axial load?

- (a) Fracture stress (b) Shear stress
(c) Buckling (d) Yielding

37. A steel cube of volume 8000 cc is subjected to all round stress of 1330 kg/cm^2 . The bulk modulus of the material is $1.33 \times 10^6 \text{ kg/cm}^2$. The volumetric change is

- (a) 8 cc (b) 6 cc
(c) 0.8 cc (d) 10^{-3} cc

38. In terms of bulk modulus (K) and modulus of rigidity (G), Poisson's ratio acn be expressed as

- (a) $\frac{3K-4G}{6K+4G}$ (b) $\frac{3K+4G}{6K-4G}$
(c) $\frac{3K-2G}{6K+2G}$ (d) $\frac{3K+2G}{6K-2G}$