



SSC - JE

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Electrical Measurement



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CHAPTER

2

Analog Instruments

THEORY

ELECTRICAL INSTRUMENTS

2.1 THE EFFECTS UTILIZED FOR THE OPERATION OF INSTRUMENTS

- (i) Magnetic effect (PMMC, MI, W)
- (ii) Induction effect (Energy meter)
- (iii) Hall effect (Transducer)
- (iv) Thermal effect (For thermocouple instrument)
- (v) Chemical effect (for D.C. Ampere hour meter)
- (vi) Electrostatic effect (for electrostatic voltmeters only)

2.2 CLASSIFICATION OF INSTRUMENTS

2.2.1 Absolute Instruments :

Absolute instruments are those instruments which give the value of the quantity to be measured in terms of constant, of the instruments & their deflection only. No previous calibration is required, example: tangent galvanometer, **Rayleigh's** current balance method for current & Lorentz method for resistance.

2.2.2 Secondary Instruments :

One those in which the value of electrical quantity to be measured can be determined from the deflection of the instruments only where they have been pre-calibrated by absolute instruments. Ordinary ammeter, voltmeter & watt meters belong to this category.

(i) *Indicating Instruments* :

Indicating Instruments are those which indicate the instantaneous value of the electrical quantity being measured at the time at which it is being measured. Example: Ammeters, voltmeter etc.

(ii) *Recording Instruments* :

Recording Instruments are those which instead of indicating by means of pointer and scale. The instantaneous value of an electrical quantity give a continuous record of the variations of such a quantity over a selected period of time.

(iii) *Integrating Instruments* :

Integrating Instruments are those which measures & registers by a set of dials & pointers either the total quantity of electricity (ampere-hour) or the total amount of electrical energy (kWh) supplied to a circuit in a given time.

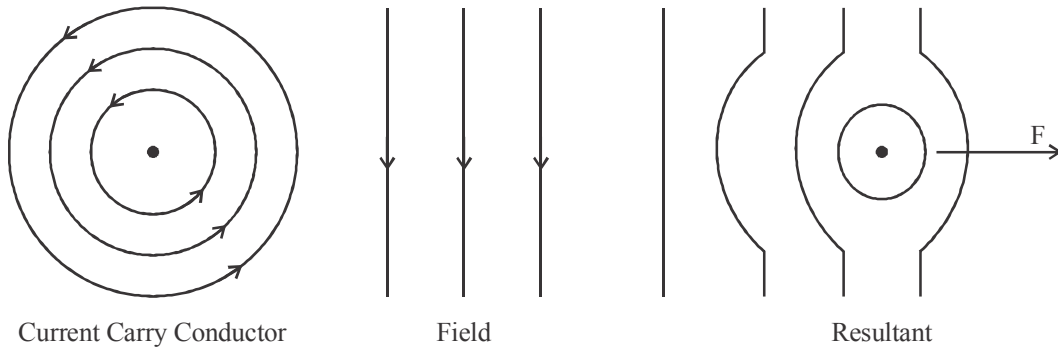
Example: Ampere hour & watt hour meter.

2.3 ANALOG INSTRUMENTS

- (i) **Analog** : Continuous function of time.
- (ii) **Digital** : Discrete or in steps.

2.3.1 Principles of Operations :

(a) *Magnetic Effect* :



Current carrying conductor produced field. Dot shows upward current field is anti clockwise. If it is placed in magnetic field as shown then a force exerts as per left hand law or motor law.

$F = BIL$ Newton Used in Ammeters and voltmeters.

If magnet is permanent then it is used for D.C. (PMMC polarized). If magnet is electromagnet then it can be used for A.C./D.C. (Moving Iron) unpolarized.

(b) *Electrodynamic effect* :

It has two coils (fixed and moving), magnitude field produced due to the interaction of them (A.C./D.C.) unpolarized.

Used in Ammeters, Voltmeters and wattmeter.

(c) *Electromagnetic effect* :

Ammeter, Voltmeter, wattmeter, kWh meter.

(d) *Thermal effect* :

Due to heating effect, it measures current or voltage, Power AC (high frequency)/D.C.

(e) *Chemical effect* :

D.C. Amp hour meter.

(f) *Induction effect* :

AC (Voltage, Current, Power or Energy).

(g) *Electrostatic effect* :

High voltage D.C./A.C.

(h) *Hall effect* :

If a current carrying conductor is placed in a transverse magnetic field then an emf is produced which depends on current, flux density and material constant of conductor, called Hall effect.

2.3.2 Analysis of Motion :

$$F = \frac{Md^2x}{dt^2} + \frac{fdx}{dt} + kx$$

$$F(s) = Ms^2x + fsx + kx$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + fs + k} = \frac{\frac{1}{M}}{s^2 + \frac{f}{M}s + \frac{k}{M}}$$

$$\omega_n = \sqrt{k/M}$$

$$\xi = \frac{f}{2\sqrt{kM}}$$

Where,
For torque,

ω_n = Natural frequency

ξ = Damping ratio

$$\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Ds + k}$$

Where,

J = Moment of Inertia

$$\omega_n = \sqrt{\frac{k}{J}}$$

D = Viscous friction

$$\varepsilon = \frac{D}{2\sqrt{KJ}}$$

K = Spring constant

2.3.3 Construction

(i) Moving System :

Should be light and friction should be less, weight can be reduced by using aluminum and friction can be reduced by using spindle mounted between jeweled bearings.

(ii) Supports :

Used to support the moving system.

TYPES OF SUPPORTS :

- (a) **Suspension:** has fine, ribbon shaped metal filament for upper suspension and coil of fine wire for lower part. This is used for high sensitivity work like in labs.
- (b) **Jewel bearings:** Moving system shaft mounted on bearings, friction is very less, arrangement is delicate, not very reliable.
- (c) **Taut suspension:** Taut suspension has a flat ribbon suspension above and below moving element. Not only supports but also provides controlling torque, advantages low friction, can be used either in vertical or horizontal position, good sensitivity

(iii) Torque/Weight ratio :

J, F, K should be very small, so that weight is reduced hence inertia is less and torque is more Torque/weight ratio reflects the sensitivity, Performance and reliability of instrument.

2.3.4 Types of Torque

(i) Deflecting Torque :

The deflection of the instrument (Deflecting Torque, T_d) is produced by utilizing the various effects like magnetic, electrodynamic, thermal or inductive effect The deflecting torque causes the moving system to move from its zero position

(ii) Controlling Torque :

The deflection of the moving system would be indefinite if there were no controlling or restoring torque. This torque opposes the deflecting torque & increases with the deflection of the moving system.

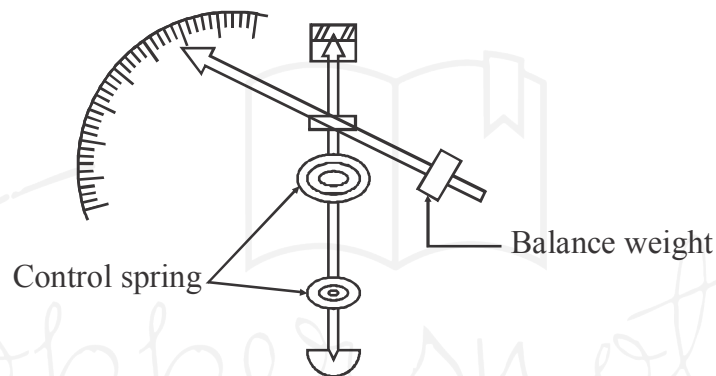
The pointer is brought to rest at a position where the two opposing torque's are equal. In the absence of controlling torque pointer would swing over to the maximum deflected position irrespective of whatever the magnitude of measured quantities.

There are two types of mechanism by which controlling torque are obtained.

(a) Spring Control :

A hair spring usually of phosphor bronze is attached to the moving system of the instrument. With the deflection of the pointer, the spring is twisted in the opposite direction.

This twist in spring produces controlling torque which is directly proportional to the angle of deflection of the moving system. The pointer comes to position of rest when deflecting torque & controlling torque are equal.



$$T_C = \frac{Ebt^3\theta}{12l} \text{ N-m}$$

Here,

E = Young's modulus of spring material

b = Width of Spring (m)

t = Thickness of Spring (m)

l = Length of Spring (m)

θ = Angular deflection (in radian)

$$T_C = K\theta$$

$$K = \frac{Ebt^3}{12l} \text{ N-m/rad}$$

Where,

K = Spring constant or torsion constant

Note : Spring material should be annealed to avoid fatigue and two springs in opposite direction are used to nullify temperature effect

Example: Spring has following dimensions, length = 370 mm, thickness = 0.073 mm, width = 0.51 mm, $E = 112.8 \text{ GN/m}^2$, $\theta = 90^\circ$. Find torque exerted.

Solution : Given data

$$T_C = \frac{Ebt^3}{12l} \theta$$

$$= \frac{112.8 \times 10^9 (.51 \times 10^{-3}) \times (.073 \times 10^{-3})^3}{12 \times 370 \times 10^{-3}} \times \frac{\pi}{2}$$

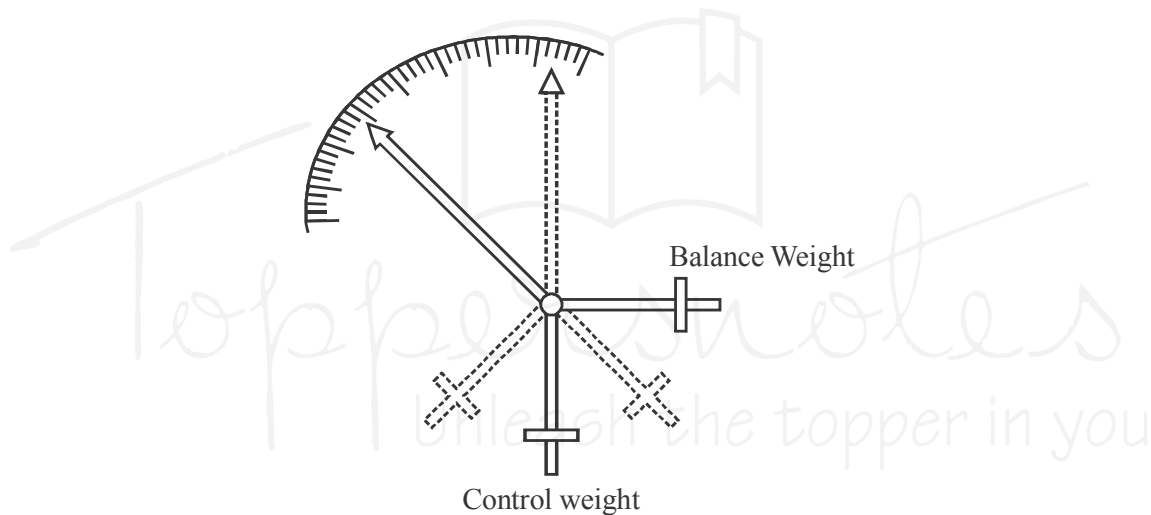
$$= 7.9 \times 10^{-6} \text{ N-m}$$

Requirement of Spring :

- (1) It should be nonmagnetic
- (2) Should not be subjected to much fatigue
- (3) Should have low temperature resistance coefficient
- (4) It should have large number of turns

(b) Gravity Control :

Gravity control is obtained by attaching a small adjustable weight, to some part of the moving system such that it exerts torque in the opposite direction.



Controlling torque (T_C) is proportional to $\sin\theta$.

$$T_C \propto \sin\theta$$

If

$$T_d \propto i$$

and

$$T_d = T_C \text{ (At equilibrium condition)}$$

then

$$i \propto \sin\theta$$

Advantage of Gravity Control :

- (1) It is cheap.
- (2) It is unaffected by temperature
- (3) It is not subject to fatigue.

Disadvantages

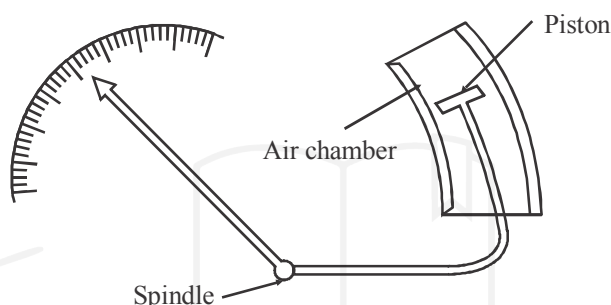
- (1) It gives cramped scale.
- (2) The instrument has to be kept in vertical position.

(iii) Damping Torque :

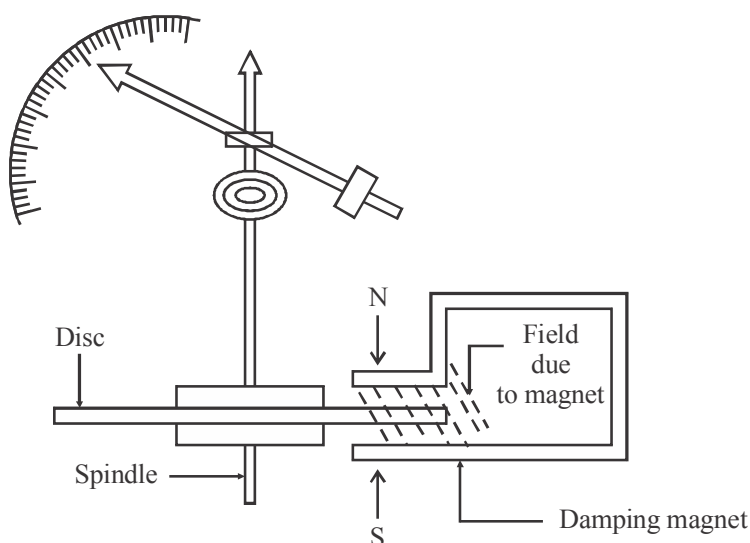
A damping force is one which acts on the moving system of the instrument when it is moving & always opposes its motion. Such damping force is necessary to bring the pointer to rest quickly otherwise due to the inertia of the moving system. The pointer will oscillate about its final position for some time before coming to rest in the steady position. (Damping is less than 1, it should not be critically damped).

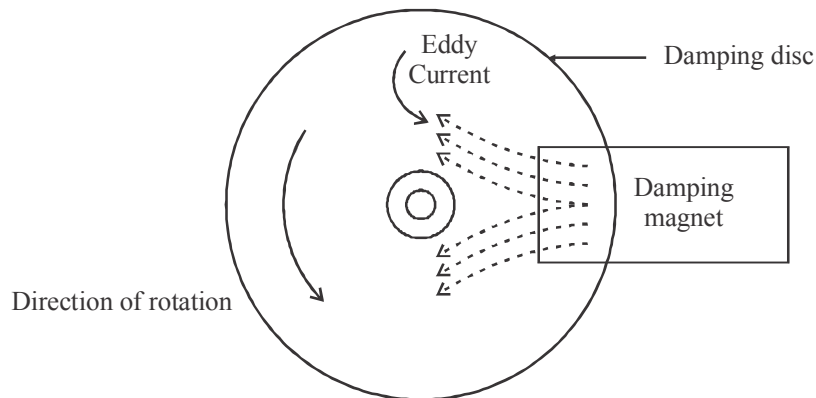
TYPES OF DAMPING SYSTEMS**(a) Air Friction :**

As shown in figure a light aluminum piston is attached to the moving system of the instrument is arranged to travel in a fixed air chamber whose cross-section may be rectangular or circular, oscillations are damped out by compression & suction action of the piston on the air enclosed in the chamber.

**(b) Eddy Current Damping :**

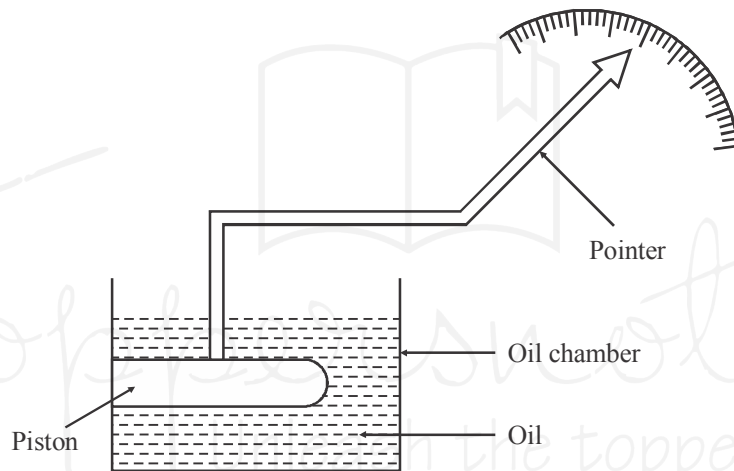
As shown in figure a thin disc of conducting but non magnetic material like copper/aluminum mounted on the spindle which carries the moving system. The disc is so positioned that its edges when in rotation at the magnetic flux between the poles of a permanent magnet hence eddy current are produced in the disc which produces a damping force in such a direction as to oppose the very cause which is producing this (Lenz's Law). The cause is the rotation of the disc hence the force retards the motion of the disc and the moving system on the whole.





(c) **Fluid Friction :**

This damping consists of the force which damped out the vibration by viscosity of the oil. It is not preferable because of the creeping (disintegration) of the oil. Necessity of the instrument to be kept in vertical position.



Example: The torque of an ammeter varies as the square of the current through it. If current of 10 Amp produces a deflection of 90° . What deflection will occur for a current of 6 amperes.

When the instrument is

- (i) Spring controlled
- (ii) Gravity controlled.

Solution : (i) Spring controlled:

for

$$i = 10 \text{ Amp}$$

$$\theta = 90^\circ$$

$$i^2 \propto \theta$$

$$\theta \propto i^2$$

$$90^\circ \propto 10^2$$

So,

$$\frac{\theta_2}{\theta_1} \propto \frac{i_2^2}{i_1^2}$$

$$\theta_2 = \theta_1 \cdot \frac{i_2^2}{i_1^2} = 90^\circ \cdot \frac{6^2}{10^2} = 32.4^\circ$$

(ii) Gravity Controlled

$$I^2 \propto \sin\theta$$

$$\sin\theta \propto I^2$$

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{I_2^2}{I_1^2}$$

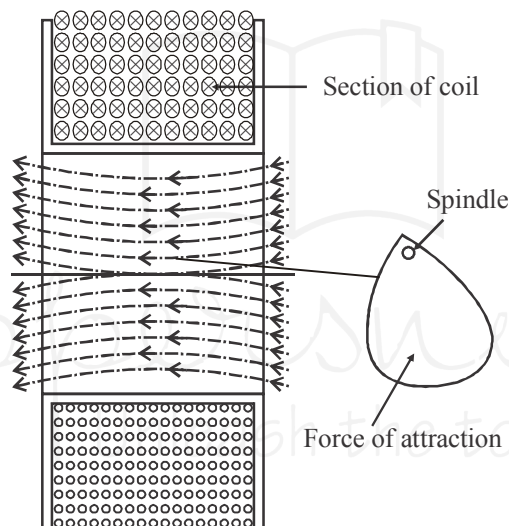
$$\sin\theta_2 = \sin\theta_1 \cdot \frac{I_2^2}{I_1^2}$$

$$\sin\theta_2 = \sin 90^\circ \frac{36}{100} = 1 \cdot \frac{9}{25} = 0.36$$

$$\theta_2 = \sin^{-1} 0.36 = 21.1^\circ$$

2.4 MOVING IRON INSTRUMENTS (AMMETER & VOLTMETER)

Moving iron type instrument may be attraction type or repulsion type. These are based on the principles of **minimum reluctance**.



2.4.1 Working Principle of an Attraction Type Moving Iron (M.I.) Instrument

As shown in figure. It consists of two electromagnets which produce a magnetic field according to the current given and it consists of an oval-shaped disc of soft iron on a spindle between bearings near coils. The iron disc will swing into the coil when the electric current passing through it. The pointer is fixed to a spindle. When the passage of current through the coil will cause the pointer to deflect.

The deflection is proportional to the current flow in the coil.

These instruments can be used on A.C. as well as D.C. [Power frequency (0-125) Hz, Cheapest. Square response]

Deflecting Torque

$$T_d \propto I^2$$

If spring control is used, then

Controlling torque

$$T_C = K \cdot \theta$$

$$T_d = T_C \text{ (At steady state)}$$

Hence

$$\theta \propto I^2$$

If gravity control is used, then $I^2 \propto \sin\theta$

- In both the cases the scales would be uneven.
- Air friction damping is provided.
- This instrument is of unpolarized type.

2.4.2 Sources of Error :

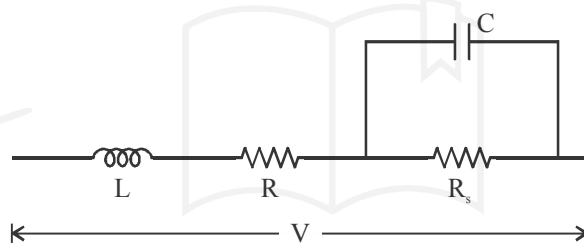
(A) Error with both AC & DC

Temperature error : Swamping resistance (manganin) is connected in series with meter.

(B) Error with AC only:

- Change in frequency produced
- Change in impedance of the coil
- Change in the magnitude of eddy currents.
- Deflecting torque is not exactly proportional to the square of current due to nonlinear characteristics of iron metal

Note : Impedance of the circuit becomes independent of the frequency,



$$C = 0.41 \frac{L}{R_s^2}$$

Where,

C = Capacitance

L = Inductance of circuit

R_s = Swamping resistance

Advantages

- Cheap
- Robust
- Reliable
- Can be used on A.C. & D.C. (Universal use)
- Low friction error (High τ/w Ratio)
- Scale range is 0 - 240°

Disadvantages

- Nonlinear scale
- Can not be calibrated with high degree of precision with D.C.

2.4.3 Deflecting Torque in Terms of change in Self-induction

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \text{ N-m}$$

2.4.4 Extension of Range

(a) **Ammeter** : The range of ammeter can be extended by a parallel low resistance (shunt).

$$R_s = \frac{R_m}{N-1}$$

Here,

R_s = Shunt resistance

R_m = Meter resistance

N = Multiplication factor

$$N = \frac{I}{I_m}$$

Here,

I = Current to be measured

I_m = Meter current

(b) **Voltmeter** : The range of voltmeter can be extended by a series high resistance (multiplier)

Required : Temperature coefficient of resistance should not change with time, low thermoelectric emf Manganin (D.C.) and constantan (D.C.) are used in voltmeter.

$$R_{sc} = (N - 1) \times R_m$$

\therefore

$$N = \frac{V}{V_m}$$

Where,

V = Voltage to be measured

V_m = Voltage of the meter

R_m = Meter Resistance

Example: The inductance of a moving iron ammeter is $\left(8 + 4\theta - \frac{1}{2}\theta^2\right)$ micro henry where θ is deflection in radians. The controlling torque is (12×10^{-6}) N-meter/radian (spring controlled is used). Calculate the deflection for current of 4 Amp.

Solution :

$$T = \frac{1}{2} I^2 \frac{dL}{d\theta}, \quad \left[\because \frac{dL}{d\theta} = \frac{d}{d\theta} \left(8 + 4\theta - \frac{1}{2}\theta^2 \right) \right]$$

$$12 \times 10^{-6} = \frac{1}{2} \times I^2 \times (4 - \theta) \times 10^{-6}$$

$$12 \times 10^{-6} = \frac{1}{2} \times 4 \times 4 \times (4 - \theta) \times 10^{-6}$$

$$\frac{12}{8} = 4 - \theta$$

$$1.5 = 4 - \theta$$

$$\theta = 4 - 1.5$$

$$\theta = 2.5 \text{ radian}$$

$$\left(\text{radian} = \frac{180}{\pi} \text{ deg} \right)$$

PRACTICE SHEET

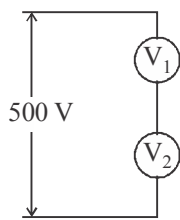
OBJECTIVE QUESTIONS

- A true rms reading voltmeter uses two thermocouples in order:
 - to increase sensitivity
 - that the second thermocouple cancels out the non-linear effects of the first thermocouple
 - to prevent drift in the d.c. amplifier
 - none of the above
- A thin aluminium film is usually deposited on the non viewing side of the CRO phosphor screen because:
 - it acts as a heat sink and prevents phosphor burn.
 - the light scatter from the phosphor is reduced.
 - it does not allow the screen to be negatively charged.
 - all of the above
- Two meters X and Y required 40 mA and 50 mA respectively for full scale deflection. Then,
 - X is more sensitive
 - Y is more sensitive
 - Both are equally sensitive
 - can be inclined to each other at any angle
- Which of the following instrument is free from hysteresis and eddy current errors?
 - Moving iron instrument
 - Electrostatic instrument
 - Moving coil-permanent magnet type instrument
 - Moving coil dynamometer type instrument
- The term 'artificial ageing' in instrument is associated with
 - springs
 - permanent magnets
 - controlling torques
 - damping
- Two voltmeters of 0-300 V range are connected in parallel to a ac circuit. One voltmeter is moving iron type and reads 200V. If the other instrument is moving coil type, its reading will be
 - $200\sqrt{3}$ Volts
 - 200×1.41 Volts
 - slightly less than 200 V
 - zero
- 50 Hz ac voltage is measured with a moving iron voltmeter and a rectifier type ac voltmeter connected in parallel. If the meter readings are V_1 and V_2 respectively and the meters are free from calibration errors, then the form factor of the ac voltage may be estimated as
 - $\frac{V_1}{V_2}$
 - $1.11 \frac{V_1}{V_2}$
 - $2 \frac{V_1}{V_2}$
 - $\pi \frac{V_1}{2V_2}$
- A 35 V dc supply is connected across a combined resistance of 600 ohms and an unknown resistance of R ohms in series. A voltmeter having a resistance of 1200 Ω is connected across 600 ohms resistor and reads 5 V. The resistance R will be


```

graph TD
    S((35 V)) --- R1[600Ω]
    R1 --- R2[R]
    R2 --- GND
    V((1200Ω  
5V)) --- R1
    V --- R2
      
```

 - 120 ohms
 - 500 ohms
 - 1.7 ohms
 - 2.4 k ohms
- If two 300V full-scale voltmeters V_1 and V_2 having sensitivities of 100k Ω /V and 150 k Ω /V are connected in series to measure 500 V, then



- (a) V_1 and V_2 will read 250 V each
 (b) V_1 and V_2 will read 250 V each
 (c) V_1 will read 200 V and V_2 will read 300 V
 (d) V_1 and V_2 will read 0 V each
10. A meter has full-scale deflection of 90° at a current of 1 A. The response of the meter is square law. Assuming spring control, the current for a deflection of 45° will be
 (a) 0.25 A (b) 0.50 A
 (c) 0.67 A (d) 0.707 A
11. A permanent magnet moving coil type ammeter and a moving iron type ammeter are connected in series in a resistive circuit fed from output of a half wave rectifier voltage source.
 If the moving iron type instrument reads 5 A, the permanent magnet moving coil type instrument is likely to read.
 (a) zero (b) 2.5 A
 (c) 3.18 A (d) 5 A
12. Match List-I (expression for torque) with List-II (Type of instrument) and select the correct answer using the codes given below the lists:

List-IA. $I_1 I_2 \cos \phi$ B. $\phi_1 \phi_2 \cos \phi_1$ C. Ki **List-II**

1. Ferrodynamic instruments

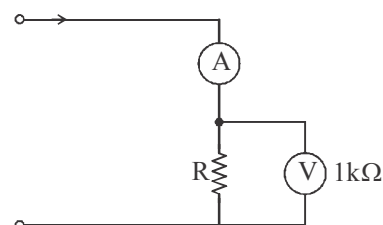
2. D' Arsonval instruments

3. Electrodynamic instruments

Codes:

	A	B	C
(a)	2	1	3
(b)	3	2	1
(c)	1	3	2
(d)	3	1	2

13. A PMMC zero to 10 A ammeter is not provided with any controlling mechanism and the moving part are free to rotate. If a current of 1 A dc is passed through the moving coil, then the instrument
 (a) will read 1 A
 (b) will read 10 A
 (c) pointer will continuously rotate
 (d) pointer will remain stationary.
14. A PMMC instrument uses a coil of resistance 0.2 ohm and produces full-scale deflection with a steady current of 10 A. If this instrument is used as an ammeter without the addition of any external resistance, then the range of the instrument will be zero to
 (a) 0.2 A (b) 1 A
 (c) 2 A (d) 10 A
15. Swamping resistance is a resistance which is added to the moving coil of a meter to
 (a) reduce the full-scale current
 (b) reduce the temperature error
 (c) increase the sensitivity
 (d) increase the field strength
16. A high frequency ac signal is applied to a PMMC instrument. If the rms value of the ac signal is 2 V, then the reading of the instrument will be
 (a) Zero (b) 2 V
 (c) 1.41 V (d) 5.656 V
17. In the circuit shown in the figure, if the ammeter indicates 1 A, and the voltmeter having an internal resistance of 1 k Ω indicated 100 V, then the value of R would be



- (a) 111.11 Ω (b) 105.2 Ω
 (c) 100 Ω (d) 90.9 Ω