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SSC - JE

JUNIOR ENGINEER

Electrical Engineering

Staff Selection Commission

Volume - 4

Power Systems

SIERRA INNOVATIONS

CONTENTS

CHAPTER 2

MODELLING OF TRANSMISSION LINE

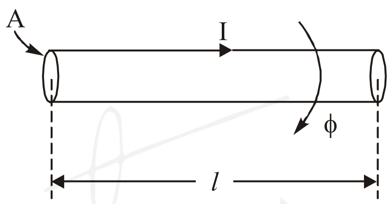
THEORY

2.1 PARAMETERS PERFORMANCE

A particular conductor of cross-sectional area 'A' and length 'l' having resistance R.

$$R = \rho \frac{l}{A}$$

$$\Rightarrow \quad R = \frac{l}{\sigma A}$$



$\rho \rightarrow$ Resistivity

$\sigma \rightarrow$ Conductivity

Whenever a current is passed through a conductor it produces a flux 'ϕ'.

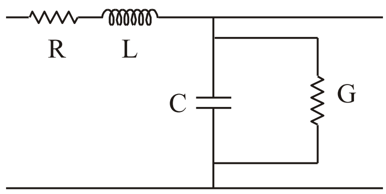
Where, $\lambda \propto I$

$\Rightarrow \quad \lambda = LI$

So there exists an inductance also

$$L = \frac{\lambda}{I}; \; L \rightarrow \text{Inductance}$$

There is some capacitance exists between two conductors where air behaves as insulator (dielectric). Practically ideal dielectric can't exist in nature, so there must be dielectric loss and losses are represented by shunt conductance



$G \rightarrow$ Shunt conductance

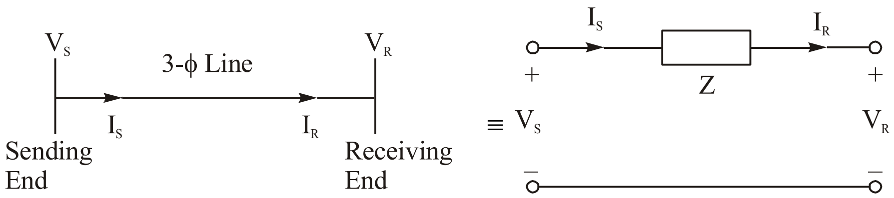
So there are four parameters R, L, C, G in power line. In power line transmitted power is represented in "MW" and dielectric loss will be in "Watt". So as compared to power transferred dielectric loss is negligible i.e. leakage is neglected.

All the transmission line contains resistance and inductance and in between the conductors there is capacitance and shunt conductance present through out the transmission line. So that R, L, C, G are called as "distributed parameters".

2.2 | Classification of Transmission Line

1. Short Line < 80 km
2. Medium Line $80 <$ km < 200
3. Long Line > 200 km

2.2.1 Short Line



Total series impedance

$$\bar{Z} = R + jX_L = R + j\omega L$$

$$V_S \rightarrow \text{Sending end voltage}$$

$$V_R \rightarrow \text{Receiving end voltage}$$

$$I_S \rightarrow \text{Sending end current}$$

$$I_R \rightarrow \text{Receiving end current}$$

According to the diagram

$$V_S = V_R + ZI_R \qquad \text{...(i)}$$

and

$$I_S = I_R \qquad \text{...(ii)}$$

Transmission Parameter

$$V_S = AV_R + BI_R \qquad \text{...(iii)}$$

$$I_S = CV_R + DI_R \qquad \text{...(iv)}$$

Equation (i) and (ii) also written as

$$V_S = 1 \cdot V_R + Z \cdot I_R$$

$$I_S = 0 \cdot V_R + 1 \cdot I_R$$

When compare with equation (iii) and (iv)

$$A = D = 1$$

$$B = Z$$

$$C = 0$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

In case of no load

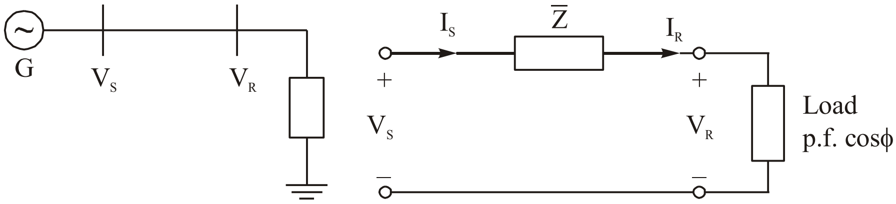
$$I_R = 0$$

$\therefore$ $$V_S = V_R$$

i.e. $$V_{R(\text{no load})} = V_S$$

Voltage regulation $$\text{V.R.} = \frac{V_{R(nl)} - V_{R(fl)}}{V_{R(fl)}} \times 100 = \frac{V_S - V_R}{V_R} \times 100$$

When load is connected



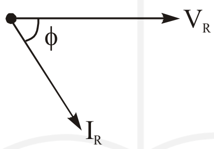
(i) (ii)

At lagging power factor $\cos\phi$, receiving end current I_R lags V_R by angle ϕ.

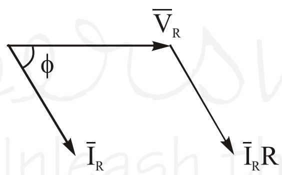
Phasor diagram $$\overline{V}_S = \overline{V}_R + \overline{I}_R R + j\,\overline{I}_R X$$

Step-(i) : Draw V_R along X-axis i.e. reference phase

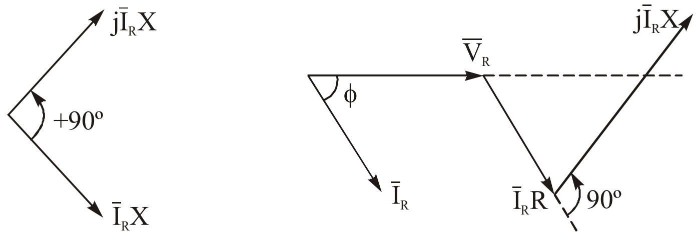
Step-(ii) : I_R lags V_R by angle.

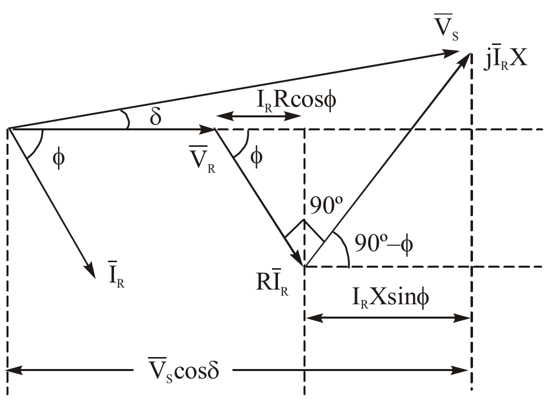


Step (iii) : Add $\overline{I}_R\ R$ with $\overline{V}_R$



Step (iv) : Add $j\,\overline{I}_R\ X_L$ with $\overline{I}_R R$





As per diagram $\quad |\bar{V}_S|\cos\delta = |\bar{V}_R| + |\bar{I}_R| R\cos\phi + |\bar{I}_R| X\sin\phi$

Due to transient stability criterion the value of 'δ' is small, so that $\cos\delta \approx 1$

$$\therefore \quad |\bar{V}_S| = |\bar{V}_R| + |\bar{I}_R|(R\cos\phi + X\sin\phi)$$

$$\Rightarrow \quad |V_S| - |\bar{V}_R| = |\bar{I}_R|(R\cos\phi + X\sin\phi)$$

$$\Rightarrow \quad \frac{|\bar{V}_S| - |\bar{V}_R|}{|\bar{V}_R|} = \frac{|\bar{I}_R|}{|\bar{V}_R|}(R\cos\phi \pm X\sin\phi)$$

$$\text{Voltage regulation} = \left(R_{pu}\cos\phi + X_{pu}\sin\phi\right)$$

i.e. + for lagging p.f.

– for leading p.f.

Case-1

For maximum voltage regulation

(It is worst regulation)

$$\frac{dV_R}{d\phi} = 0$$

At lagging power factor cosϕ

$$\Rightarrow \quad \frac{|\bar{I}_R|}{|\bar{V}_R|}\frac{d}{d\phi}[R\cos\phi + X\sin\phi] = 0$$

$$\Rightarrow \quad -R\sin\phi + X\cos\phi = 0$$

$$\Rightarrow \quad X\cos\phi = R\sin\phi$$

$$\Rightarrow \quad \tan\phi = \frac{X}{R}$$

$$\phi = \tan^{-1}\left(\frac{X}{R}\right)$$

Case-2

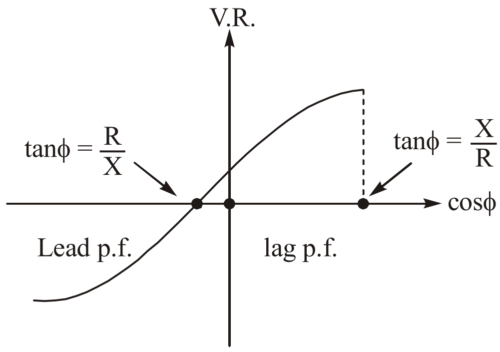
For zero voltage regulation

$$\frac{|\bar{I}_R|}{|\bar{V}_R|}[R\cos\phi - X\sin\phi] = 0$$

$$\Rightarrow \quad \tan\phi = \frac{R}{X}$$

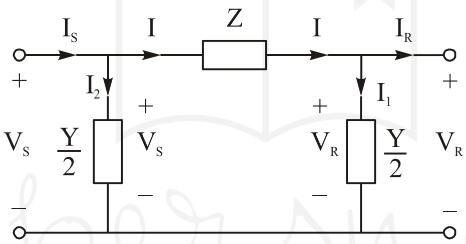
$$\phi = \tan^{-1}\left(\frac{R}{X}\right)$$

Voltage regulation and power factor curve



2.2.2 *Medium Line Model*

Nominal π-model:



According to the diagram

$$I_1 = \frac{Y}{2}V_R$$

$$I = I_R + I_1$$

$$V_S = V_R + IZ$$

$$\Rightarrow \quad V_S = V_R + Z\left[I_R + \frac{Y}{2}V_R\right]$$

$$\therefore \quad V_S = \left(1+\frac{YZ}{2}\right)V_R + ZI_R$$

$$I_2 = \frac{Y}{2}V_S; \quad I_S = I + I_2$$

$$I_S = I + \frac{Y}{2}V_S$$

$$\Rightarrow \quad I_S = Y\left[1+\frac{YZ}{4}\right]V_R + \left[1+\frac{YZ}{2}\right]I_R$$

$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y\left[1 + \frac{YZ}{4}\right]$$

Similarly in case of **T-model**

$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z\left[1 + \frac{YZ}{4}\right]$$

$$C = Y$$

2.3 | Characteristic Impedance or Surge Impedance

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

For lossless line $\quad R = 0, G = 0$

$$Z_0 = \sqrt{\frac{L}{C}}\ \Omega \ \text{(Real number)}$$

$\therefore \quad Z_0 = \text{Pure resistance}$

$3-\phi$ power $\quad P_R = 3|V_R|_{(ph)}|I_R|_{(ph)}\cos 0^o = 3\frac{|V_R|_{ph}^2}{Z_0}$

or, (Surge impedance loading) $\quad SIL = P_r(3\text{-}\phi) = 3\frac{|V_R|_{ph}^2}{Z_0} = 3\frac{\left(\frac{|V_R|_L}{\sqrt{3}}\right)^2}{Z_0}$

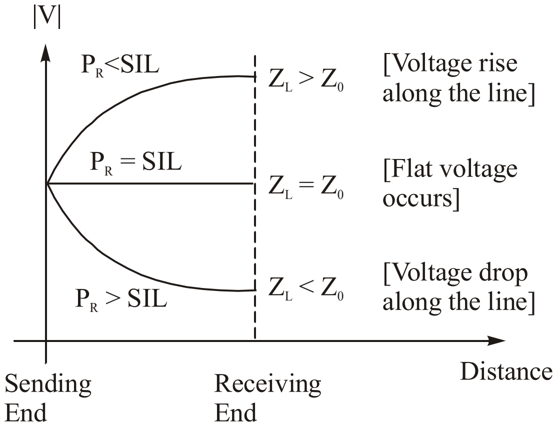
$$P_{R(3\text{-}\phi)} = \frac{|V_R|_L^2}{Z_0}$$

Graphical Analysis:

(i) If $\quad Z_L = Z_0$

$$|V_S| = |V_R|$$

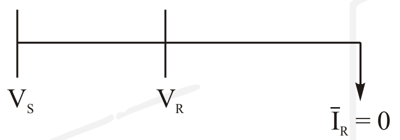
i.e. flat voltage profile occurs.

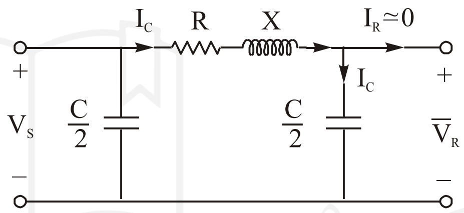


2.4 | FERRANTI EFFECT

(In case of medium line and long line)

At no load or light loads $I_R \approx 0$





Charging current

$$\bar{I}_C = j\omega\frac{C}{2}\bar{V}_R$$

$\bar{I}_C$ leads $\bar{V}_R$ by 90°.

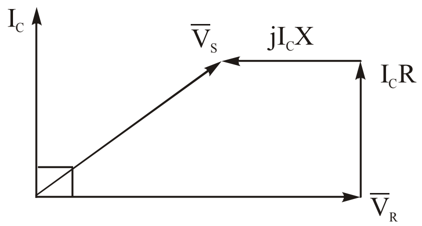
$$\bar{V}_S = I_C[R + jX] + \bar{V}_R$$

$$\bar{V}_S = \bar{V}_R + I_C R + jI_C X$$

At no load or light load $|\bar{V}_R| > |\bar{V}_S|$

$\Rightarrow$ $|\bar{V}_R|_{NL} > |\bar{V}_S|$

Phasor Diagram:



Note: At no load the receiving end voltage is greater than sending end voltage (due to charging current of the line) is called as Ferranti effect.

Example 1. Two loads of 10 MVA, 0.8 pf lag and 12 MW, 0.6 pf lead are connected to an 11 kV sub-station. The total load is

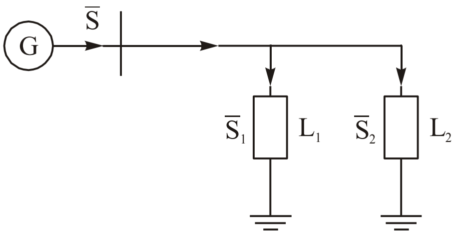
(a) 20 MW, 0.89 lead (b) 22.4 MW, 0.7 lag

(c) 22.4 MVAR, 0.9 lag (d) 20 MW, 0.8 lead

Ans. (*a*)

Solution : Given that

Load S_1 :



$$S_1 = 10 \text{ MVA}$$

$$\cos \phi_1 = 0.8 \text{ lag}$$

$\therefore$ $$\sin \phi_1 = 0.6$$

If pf is lagging i.e. reactive power $Q > 0$

$$\overline{S} = P + jQ$$

$\therefore$ $$P = S\cos \phi$$

$$Q = S\sin \phi$$

So that $$P_1 = S_1 \cos\phi_1 = 10 \times 0.8 = 8\text{MW}$$

$$Q_1 = S_1 \sin\phi_1 = 10 \times 0.6 = 6\text{MVAR}$$

$\therefore$ $$S_1 = 8 + j6$$

Load L_2 : $P_2 = 12$ MW

[Load is in MW i.e. P is given]

$$\cos \phi_2 = 0.6 \text{ lead}$$

$\therefore$ $$\sin \phi_2 = 0.8$$

Note :

(i) If given loads is in "MVA" i.e. complex power is given.

(ii) If load is in "MW" i.e. active power "P" is given.

(iii) It load is in "MVAR" i.e. reactive power "Q" is given.

If power factor is leading i.e. reactive power Q is negative i.e. $Q < 0$

$$P_2 = S_2 \cos \phi_2$$

$\Rightarrow$ $$S_2 = \frac{P_2}{\cos\phi_2} = \frac{12}{0.6} = 20\text{MVA}$$

$\therefore$ $$S_2 = 20 \text{ MVA}$$

$$Q_2 = -S_2 \sin \phi_2$$

$\Rightarrow$ $$Q_2 = -20 \times 0.8$$

$\therefore$ $$Q_2 = -16 \text{ MVAR}$$

Here Q_2 is negative as leading pf

$$\therefore \quad S_2 = 12 - j16$$

$$\text{Total load} = \bar{S}_1 + \bar{S}_2 = 8 + j6 + 12 - j16 = 20 - j10$$

$$= 22.36\angle -26.56^o$$

i.e.

$$P = 20 \text{ MW}$$

$$Q = -10 \text{ MVAR i.e. } 10 \text{ MVAR \& lead}$$

$$\cos\phi = 0.895 \text{ leading}$$

or

$$P = S\cos\phi;\ Q = \pm S\sin\phi$$

$$|Q| = S\sin\phi$$

$$\therefore \quad \tan\phi = \frac{|Q|}{P} = \frac{10}{20} = 0.5$$

$$\phi = 26.56^o$$

$$\cos\phi = \cos 26.56^o = 0.895$$

∴ Total load is 20 MW, 0.895 lead.

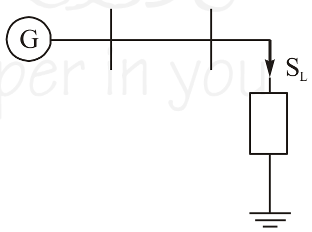
or 22.36 MVA, 0.895 lead.

Example 2. A 400 V, 50 Hz, 3–ϕ balanced source supplies power to a star connected load whose rating is $12\sqrt{3}$ kVA, 0.8 pf (lag). The rating (in VAR) of the delta connected (capacitance) reactive power bank necessary to bring the power factor to unity is

(a) 28.78 (b) 21.60 (c) 15.60 (d) 12.47

Ans. (***d***)

Solution : Rated kVA = $12\sqrt{3}$ KVA



$$\cos\phi = 0.8 \text{ (lagging)}$$

$$\therefore \quad \sin\phi = 0.6$$

And

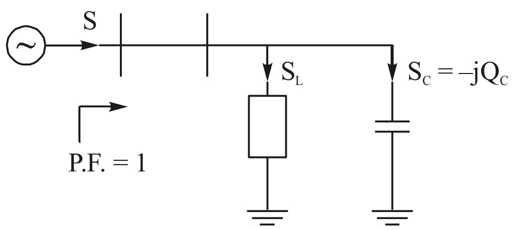
$$\phi = 36.9^o$$

$$\therefore \quad S_L = 12\sqrt{3} \text{ kVA}$$

$$P_L = S_L\cos\phi = 12\sqrt{3}\times 0.8 = 16.62\text{kW}$$

$$Q_L = S_L\sin\phi = 12\sqrt{3}\times 0.6 = 12.47\text{kVAR}$$

$$S_L = P_L + jQ_L = 16.62 + j12.47$$



∵ Capacitance power bank is attached with load so that

$$S_C = -jQ_C$$

$$\therefore \quad \bar{S} = P + j(Q_L - Q_C) = 16.62 + j(12.47 - Q_C)$$

After attachment of reactive power bank power factor becomes unity.

$$\therefore \quad \cos\phi = 1 \text{ and } \sin\phi = 0$$

$$Q = S \sin\phi = 0$$

$$\therefore \quad 12.47 - Q_C = 0$$

$$Q_C = 12.47 \text{ MVAR}$$

$$(Q_C)_{3-\phi} = 12.47 \text{ MVAR}$$

2.5 | Line Parameters

An Electric Transmission line has four parameters namely:

(I) Resistance (R) per unit length

(II) Inductance (L) per unit length

(III) Capacitance (C) per unit length

(IV) Shunt conductance (G) per unit length

Electrical design and performance of a transmission line depends on these parameters. These four parameters are uniformly distributed along the whole line. So it is called distributed parameters.

2.6 | Resistance (R)

Effective AC resistance is given by

$$R = \frac{\text{Average power loss in conductor}}{I^2} \text{ ohms}$$

Where, I = r.m.s current

Ohmic or DC resistance is given by

$$R_0 = \frac{\rho l}{A} \text{ ohms}$$

Where, ρ = Resistivity of the conductor (ohm-m)

l = Length (m)

A = Area of cross-section (m^2)

If current distribution is uniform throughout the conductor (i.e. skin effect is neglected).

$R = R_0$ i.e. effective AC resistance is equal to DC resistance.

i.e. $R_{AC} = R_{DC}$

For small change in temperature R increasing with temperature.

$$R_t = R(1+\alpha_0 t)$$

Where α_0 = temperature coefficient at 0ºC

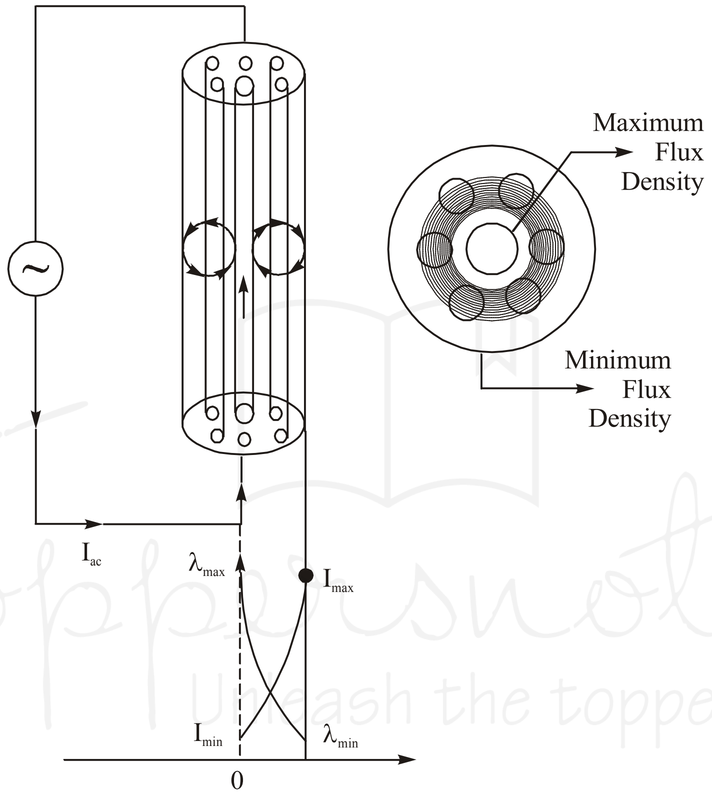
If current distribution is not uniform

$$R_{AC} > R_{DC}$$

$$R_{AC} = 1.6\ R_{DC}$$

it is due to skin effect and proximity effect.

Skin Effect : If we consider solid conductor i.e. stranded conductor



$$\text{Flux linkage } (\lambda) = LI$$

$$L = \text{Inductance (H)}$$

$$I = \text{Current (A)}$$

$$L = \frac{\lambda}{I} = \frac{\text{Flux linkage}}{\text{Current}}$$

Outside $\lambda_{small} \Rightarrow L_{min} \Rightarrow X_{small}$ i.e. I maximum

At centre $\lambda_{Large} \Rightarrow L_{max} \Rightarrow X_{max} \Rightarrow I_{low}\ (X = 2\pi fL)$

We can say flux linkage λ is maximum at the centre compare to outside so that current density will be minimum at the centre and maximum at the outside.

If DC pass through the cross section of a conductor current density will be uniform.

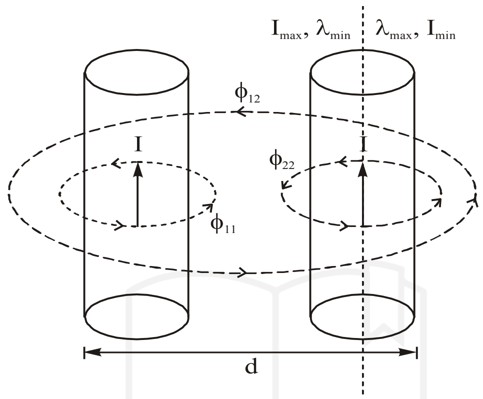
When AC is flowing current density is not uniform it is higher at surface of the conductor & minimum at centre. This phenomenon is called skin effect.

In case of HVDC skin effect is negligible.

Skin effect is depends upon

- Frequency of supply
- Diameter of conductor
- Distance between the conductors
- Permeability of conductor material
- Conductivity of the conductor.

Proximity Effect : Let conductor A & B



The current flowing in one conductor influence by flux linkage it surrounding conductor so that effective areas in the neighbour conductor is changed which causes change in the resistance & hence $R_{AC} > R_{DC}$.

Proximity effect depends upon

- Supply frequency
- Diameter of conductor
- Distance between conductors
- Permeability μ_r
- Conductivity

Proximity effect is more in cable due to less gap between conductor.

2.7 | INDUCTANCE (L)

Inductance $$L = \frac{\text{Flux linkage}}{\text{Current}} = \frac{\lambda}{I} = \frac{N\phi}{I}$$

Where λ = Flux linkage per unit length.

Inductance of a conductor

Assumption :

1. Skin effect is neglected
2. Stranded conductor is assumed as a solid conductor.

   For standard / solid conductor

Inductance per meter $L = L_{int} + L_{ext}$

L_{int} : Inductance obtaining consisting the total internal flux linkage due to all flux inside the conductor.

L_{ext} : The inductance due to all external flux linkage.

Internal inductance (L_{int})

$$L_{int} = \frac{1}{2}\times 10^{-7}\ \text{H/m for } (\mu_r = 1)$$

For solid conductor internal inductance does not depend on size of conductor

$$L_{int} \propto \mu_r$$

For hollow conductor $L_{int} = 0$

External inductance (L_{ext}) :

$$L_{ext} = 2\times 10^{-7}\, l\text{n}\frac{d}{r}\ \text{H/m}$$

Total inductance :

$$L = L_{ext} + L_{int}$$

Total inductance $L = 2\times 10^{-7}\ln\frac{d}{r'}\text{H/m}$

where $r' = 0.7788\,r$

r' = Effective radius

r = Physical radius

d = Distance between conductors

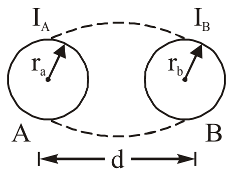
For hollow conductor $L = 2\times 10^{-7}\ln\frac{d}{r}\ \text{H/m}$

For hollow	For solid conductor
$L_{int} = 0$	$L_{int} = \frac{1}{2}\times 10^{-7}$
$L = 2\times 10^{-7}\ln\frac{d}{r}$	$L = 2\times 10^{-7}\ln\frac{d}{r'}$
$L \propto \frac{1}{r}$	$L \propto \frac{1}{r'}$
X_L low	X_L high $(r' > r)$
Power capacity is high.	Power capacity is low.

Note : Max. power limit through conductor

$$P_{em} = \frac{EV}{X_L}$$

Inductance of Single Phase 2-Wire Line



Where, d = distance between two conductor A and B.

r_a, r_b = Radius of A and B respectively.

I_A, I_B = Current in conductor A and B respectively.

Loop inductance $L = L_A + L_B$

$$L = 4 \times 10^{-7} ln\left(\frac{d}{r'}\right) H/m$$

PRACTICE SHEET

OBJECTIVE QUESTIONS

1. The effect of capacitance can be neglected when the length of overhead transmission line does not exceed

(a) 20 km (b) 60 km

(c) 120 km (d) 300 km

2. Which of the following is neglected while analyzing a short transmission line?

(a) Shunt admittances

(b) Power losses

(c) Series impedance

(d) None of these

3. For a 500 Hz frequency excitation, a 50km long power line will be modelled as

(a) Short line.

(b) Medium line

(c) Long line

(d) Data insufficient for decision

4. Percentage regulation of a short transmission line is given by the expression

(a) $\frac{V_R - V_S}{V_R} \times 100$

(b) $\frac{V_R - V_S}{V_S} \times 100$

(c) $\frac{V_S - V_R}{V_R} \times 100$

(d) $\frac{V_S - V_R}{V_S} \times 100$

5. As compared to sending-end voltage, the receiving-end voltage of a short line under no-load condition is

(a) higher

(b) Lower

(c) Remains the same

(d) None of these

6. If a short transmission line is delivering to lagging pf load, the sending-end pf would be (notations having their usual meaning)

(a) $\frac{V_R \cos\phi + IR\sin\phi}{V_S}$

(b) $\frac{V_R \cos\phi + IR}{V_S}$

(c) $\frac{V_R \sin\phi + IR}{V_S}$

(d) $\frac{V_R \sin\phi + IR\cos\phi}{V_S}$

7. Which of the following voltage regulation is considered to be the best

(a) 2% (b) 30%

(c) 70% (d) 98%

8. For a short line if the receiving-end voltage is = sending end voltage under loaded conditions

(a) The sending-end power factor is unity

(b) The receiving-end power factor is unity

(c) The sending-end power factor is leading

(d) The receiving-end power factor is leading

9. The regulation of a line at full load 0.8 pf lagging is 12%. The regulation at full-load 0.8 pf leading can be

(a) 24% (b) 18%

(c) 12% (d) 4%

10. If in a short transmission line, resistance and inductive reactance are found to be equal and regulation appears to be zero, then the load will

(a) Have unity power factor

(b) Have zero power factor

(c) be 0.707 leading

(d) None of these

11. A single phase transmission line of impedance j 0.8 ohm supplies a resistive load of 500 A at 300 V. The sending-end power factor is

(a) Unity (b) 0.8 lagging

(c) 0.8 leading (d) 0.6 lagging

12. For an ac transmission line of length not exceeding 80 km, it is usual to lump the line capacitance at

(a) The sending end (b) The receiving end

(c) The mid point (d) Any convenient point

13. Transmission efficiency of a transmission line increases with the

(a) Decrease in power factor and voltage

(b) Increase in power factor and voltage

(c) Increase in power factor but decrease in voltage

(d) Increase in voltage but decrease in power factor

14. Under no load conditions, the current in a transmission line is because of

(a) Capacitance effect

(b) Corona effect

(c) Proximity effect

(d) Back flow from earth

15. Which of the following statements are correct?

(a) Flow of unduly heavy current is Ferranti effect

(b) Ferranti effect occurs under unloaded condition of line.

(c) The rise in receiving-end voltage then sending end voltage is Ferranti effect

(d) Both (b) and (c) combined is Ferranti effect

16. The A B C D constants of a 3 phase transposed transmission line with linear and passive elements

(a) Are always equal

(b) Never equal

(c) A and D are equal

(d) B and C are equal

17. The values of A, B, C and D constants for a short transmission line are respectively

(a) Z, 0, 1 and 1 (b) 0, 1, 1 and Z

(c) 1, Z, 0 and 1 (d) 1, 1, Z and 0

18. For a transmission line with resistance R reactance X and negligible capacitance, the generalised constant A is

(a) 0 (b) 1

(c) R + j X (d) R + X

19. The square root of the ratio of line impedance and shunt admittance is known as the line

(a) Surge impedance (b) Conductance

(c) Susceptance (d) Admittance

20. Which of the following statements is correct?

(a) Surge impedance is the impedance at the time of breakdown of voltage

(b) Surge impedance and characteristic impedance for a transmission line are the same

(c) Surge impedance is the impedance o transmission line when corona takes place

(d) None of the above

21. The characteristic impedance of a transmission line depends upon

(a) Shape of the conductor

(b) Conductivity of the conductor material

(c) Geometrical configuration of the conductors

(d) None of the above

22. In a transmission line of negligible resistance, the surge impedance will be

(a) $\sqrt{L/C}$ (b) $\sqrt{C/L}$

(c) $\frac{1}{\sqrt{LC}}$ (d) $\sqrt{LC}$

23. Characteristic impedance of an overhead transmission line is usually in the range of

(a) 100 to 200 Ω (b) 200 to 300 Ω

(c) 0 to 100 Ω (d) 400 to 500 Ω

24. Surge impedance of a transformer is in the range of

(a) 80 - 100 Ω

(b) 400-500 Ω

(c) 1,000-2,000 Ω

(d) None of these