Toppersnotes
Unleash the topper in you

JKPSC

Prelims

Jammu and Kashmir Public Service Commission

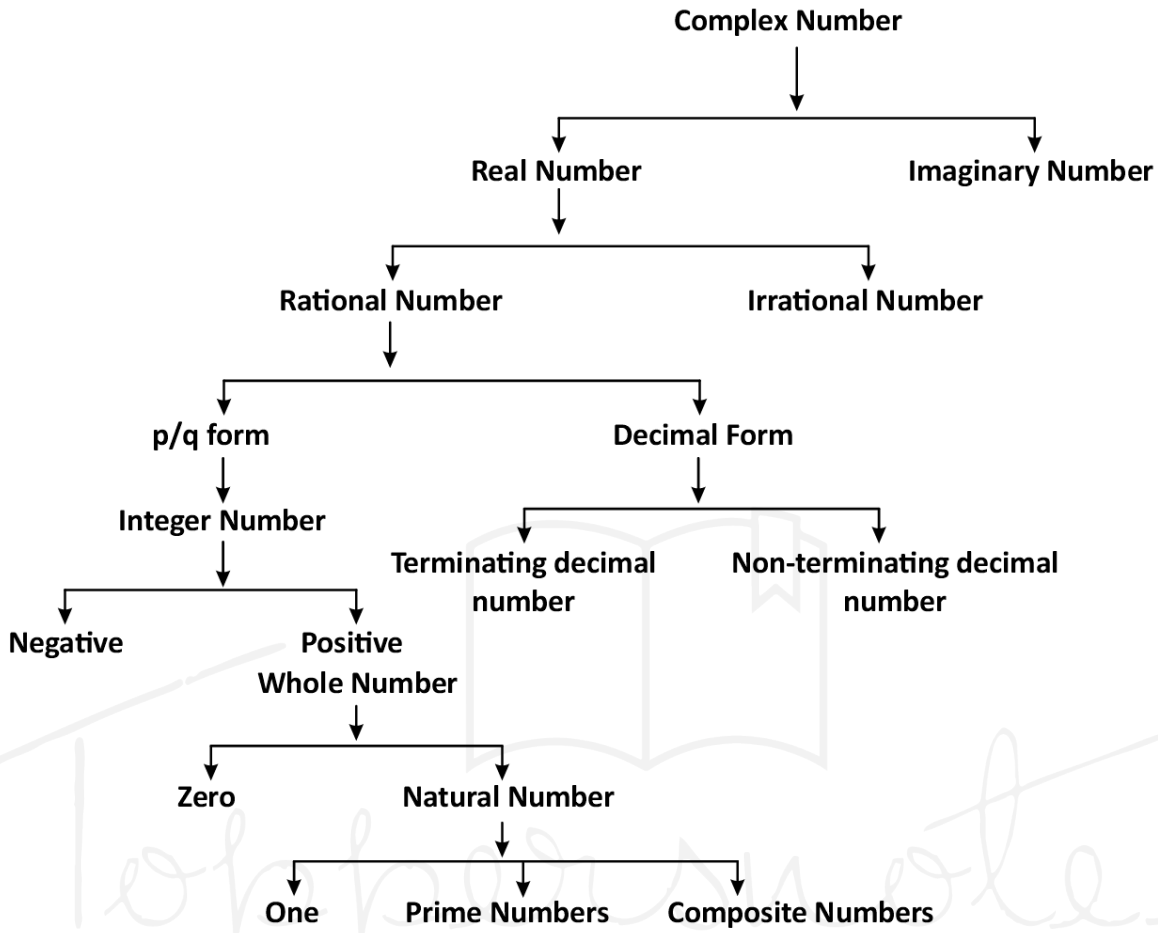
GS Paper 2 || Volume - 8

CSAT

SIERRA INNOVATIONS

विषयसूची

1 CHAPTER Number System



Complex Number (Z)

Z = Real numbers + Imaginary numbers

$$Z = a + ib$$

Where, a = Real numbers.
b = Imaginary numbers.

Real Numbers

Rational and irrational numbers together are called real numbers. These can be represented on the number line.

Imaginary Numbers

Numbers that can not be represented on the number line.

Integer Numbers

A set of numbers which includes whole numbers as well as negative numbers, is called integer numbers, it is denoted by I.

I = {–4, –3, –2, –1, 0, 1, 2, 3, 4,.....}

Natural Numbers

The numbers which are used to count things are called natural numbers.

N = {1, 2, 3, 4, 5, ……}

Whole Numbers

When 0 is also included in the family of natural numbers, then they are called whole numbers.

W = {0, 1, 2, 3, 4, 5, …..}

The product of four consecutive natural numbers is always exactly divisible by 24.

Even Numbers

Numbers which are completely divisible by 2 are called even numbers.

n^{th} term = 2n

Sum of first n even natural numbers = n(n+1)

Sum of square of first n even natural numbers = $\frac{2n(n+1)(2n+1)}{3}$

$$\left\{n = \frac{\text{Last term}}{2}\right\}$$

Odd Numbers

The numbers which are not divisible by 2 are odd numbers.

Sum of first n odd numbers = n^2

$$\left\{n = \frac{\text{Last term} + 1}{2}\right\}$$

Natural Numbers

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

Sum of square of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

Sum of cube of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

The difference of the squares of two consecutive natural numbers is equal to their sum.

Example - $11^2 = 121$

$12^2 = 144$

$11 + 12 \rightarrow 23$

Difference 144 – 121 = 23

Prime Numbers – Which have only two forms - 1 × numbers

E.g. - {2, 3, 5, 7, 11, 13, 17, 19……..}

Where, 1 isn't a Prime Number.

- The digit 2 is only even prime number.
- 3, 5, 7 is the only pair of consecutive odd prime numbers.
- Total prime numbers between 1 to 25 = 9
- Total prime numbers between 25 to 50 = 6
- There are total of 15 prime numbers between 1-50.
- There are total of 10 prime numbers between 51 – 100.
  So there are total 25 prime numbers from 1-100.
- Total prime numbers from 1 to 200 = 46
- Total prime numbers from 1 to 300 = 62
- Total prime numbers from 1 to 400 = 78
- Total prime numbers from 1 to 500 = 95

Co-prime Numbers

Numbers whose HCF is only 1.

E.g. - (4,9), (15, 22), (39, 40)

HCF = 1

Perfect Number

A number whose sum of its factors is equal to that number (except the number itself in the factors)

E.g. - 6 → 1, 2, 3 → Here 1 + 2 + 3 → 6

28 → 1, 2, 4, 7, 14 → 1 + 2 + 4 + 7 + 14 → 28

Rational Numbers

Numbers that can be written in the form of P/Q, but where Q must not be zero and P and Q must be integers.

E.g. - 2/3, 4/5, $\frac{10}{-11}, \frac{7}{8}$

Irrational Numbers

These cannot be displayed in P/Q form.

E.g. - $\sqrt{2}, \sqrt{3}, \sqrt{11}, \sqrt{19}, \sqrt{26}$...

Perfect square numbers

↓

Unit Digit which can be of square	Which can't be square
0	2 ——
1	3 ——
4	7 ——
5 or 25	8 ——
6	
9	

- The last two digits of the square of any number will be the same as the last two digits of the square of numbers 1-24.

Note: Therefore, everyone must remember the squares of 1-25.

Convert to Binary and Decimal –

1. **Convert Decimal Number to Binary Number**

   To find the binary number equivalent to a decimal number, we continuously divide the given decimal number by 2 until we get 1 as the final quotient.

   E.g.

2	89	2 × 44 = 88 ; 89 – 88 = 1
2	44	2 × 22 = 44; 44 – 44 = 0
2	22	2 × 11 = 22 ; 22 – 22 = 0
2	11	2 × 5 = 10 ; 11 – 10 = 1
2	5	2 × 2 = 4 ; 5 – 4 = 1
2	2	2 × 1 = 2 ; 2 – 2 = 0
	1	Final quotient

   Hence, binary number equivalent to 89 = $(1011001)_2$

2. **Convert Binary to Decimal Nubmer**

   In binary system the value of 1 when it moves one place to its left every time it doubles itself and wherever 0 comes its value is 0.

   E.g.

1	0	1	1	0	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

   Now

   $(1011001)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 \times 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

   $= 64 + 0 + 16 + 8 + 8 + 0 + 1 \quad \{2^0 = 1\}$

   $= 89$

Finding the Number of Divisors or Number of Factors

First we will do the prime factorization of the number and write it as Power and multiply by adding

One to each power, then the number of divisors will be obtained.

Ex: By how many total numbers can 2280 be completely divided?

Sol. $2280 = 2^3 \times 3^1 \times 5^1 \times 19^1$

Number of divisors = (3 + 1) (1 + 1) (1 + 1) (1 + 1)

$= 4 \times 2 \times 2 \times 2 = 32$

Find the unit's digit

1. **When the number is in the form of power –**

   When the unit digit of Base is 0, 1, 5 or 6, the unit digit of the result remains the same for any natural power. When the unit digit of base is 2, 3, 4, 7, 8, or 9, divide the power by 4 and put the same power on the unit digit of the base as the remainder. When the power is rounded off to 4, then the 4^{th} power will be placed on the unit digit of the base.

2. **In the form of simplification –**

   Write the unit digit of each number and simplify it according to the symbol, the result that will come will be its unit digit answer.

Divide by Power of Numbers (Finding the Divisor)

1. If $a^n + b^n$ is given –
   If n is odd, then (a+b) will be its divisor.
2. If $a^n - b^n$ is given –
   Divisor (when n is odd) $\rightarrow$ (a–b)
   Divisor (when n is even) $\rightarrow$ (a – b) or (a + b) or both.

1. If $a^n \div (a-1)$ then the remainder always be 1.
2. $a^n \div (a+1)$ { If n is an even then the remainder always be 1. / If n is an odd then the remainder always be a.
3. If $(a^n + a) \div (a-1)$ then the remainder always be 2 .
4. $(a^n + a) \div (a+1)$ { If n is an even then the remainder always be zero (0). / If n is an odd then the remainder always be (a – 1)

Terminating Decimal

Those numbers which end after a few digits after the decimal like - 0.25, 0.15, 0.375 can be written in a fraction number.

Non-Terminating Decimal

Those numbers which continue after the decimal and can be of two types.

0.3333, 0.7777, 0.183183183…….

Repeating	Numbers that never end after the decimal, but repeat, till infinity. It can be written in fractions.
Non Repeating Decimal	Numbers that never end after the decimal point, but they do not repeat their numbers.

Recurring Decimal Fraction

That decimal fraction is the repetition of one or more digits after the decimal point, then one or more digits are repeated after the dot.

Eg. $\frac{1}{3} = 0.333..., \frac{22}{7} = 3.14285714.....$ To represent such fractions, a line is drawn over the repeating digit.

$$0.3\overline{524} = \frac{3524-35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$$

$$\frac{22}{7} = 3.14285714.... = 3.14\overline{2857}$$

It is called bar.

- **Convert pure recurring decimal fraction to simple fraction as follows –**

$$0.\overline{P} = \frac{P}{9} \qquad 0.\overline{pq} = \frac{pq}{99} \qquad 0.\overline{pqr} = \frac{pqr}{999}$$

- **Convert a mixed recurring decimal fraction to an ordinary fraction as follows –**

$$0.p\overline{q} = \frac{pq-p}{90} \qquad 0.pq\overline{r} = \frac{pqr-pq}{900}$$

$$0.p\overline{qr} = \frac{pqr-p}{990} \qquad 0.pq\overline{rs} = \frac{pqrs-pq}{9900}$$

Example -

(i) $0.\overline{39} = \frac{39}{99} = \frac{13}{33}$

(ii) $0.6\overline{25} = \frac{625-6}{990} = \frac{619}{990}$

(iii) $0.3\overline{524} = \frac{3524-35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$

Symbol of the Roman Method

1	→	I
2	→	II
3	→	III
4	→	IV
5	→	V
6	→	VI
7	→	VII
8	→	VIII
9	→	IX
10	→	X
20	→	XX
30	→	XXX
40	→	XL
50	→	L
100	→	C
500	→	D
1000	→	M

Rule of Divisibility

Rule of 2	The last digit is an even number or zero (0) as - 236, 150, 1000004
Rule of 3	If the sum of the digits of a number is divisible by 3, then the whole number will be divisible by 3. E.g. 729, 12342, 5631
Rule of 4	Last two digits are zero or divisible by 4. E.g. 1024, 58764, 567800
Rule of 5	The last digit is zero or 5. E.g. 3125, 625, 1250
Rule of 6	If a number is divisible by both 2 and 3 then it is also divisible by 6. E.g. 3060, 42462, 10242
Rule of 7	After multiplying the last digit of a number by 2 and subtracting it from the remaining number, if the number is a multiple of 0 or 7 or if any digit is repeated in a multiple of 6**,** then the number will be divisible by 7**.** E.g. 222222, 44444444444, 7854
Rule of 8	If the last three digits of a number are divisible by 8 or the last three digits are '000' (zero). E.g. 9872, 347000
Rule of 9	If the sum of the digits of a number is divisible by 9, then the whole number will be divisible by 9.
Rule of 10	The last digit should be zero (0).
Rule of 11	If the difference between the sum of digits at odd places and sum of digits at even places is zero (0) or 11 or a multiple of 11. E.g. 1331, 5643, 8172659
Rule of 12	Composite form of divisible by 3 and 4.
Rule of 13	Repeating the digit 6 times, or multiplying the last digit by 4 and adding it to the remaining number, if the number is divisible by 13, then the whole number will be divisible by 13**.** E.g. 222222, 17784

Practice Questions

Q.1 If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of that number, then what will be $\frac{5}{3}$ of that number?

(a) 12 (b) 18

(c) 15 (d) 20

Q.2 If the sum of two numbers is a and their product is a then their reciprocals will be –

(a) $\frac{1}{a}+\frac{1}{b}$ (b) $\frac{b}{a}$

(c) $\frac{a}{b}$ (d) $\frac{a}{ab}$

Q.3 The sum of two numbers is 75 and their difference is 25, then what will be the product of those two numbers?

(a) 1350 (b) 1250

(c) 1000 (d) 125

Q.4 Divide 150 into two parts such that the sum of their reciprocal is $\frac{3}{112}$. Calculate both parts.

(a) 50, 90 (b) 70, 80

(c) 60, 90 (d) 50, 100

Q.5 If the sum of any three consecutive odd natural numbers is 147, then the middle number will be –

(a) 47 (b) 48

(c) 49 (d) 51

Q.6 If the product of first three and last three of 4 consecutive prime numbers is 385 and 1001, then find the greatest prime number.

Q.7 What will be the sum of the even numbers between 50 and 100?

Q.8 What will be the sum of odd numbers between 50 and 100?

Q.9 In a division method, the divisor is 12 times the quotient and 5 times the remainder. Accordingly, if the remainder is 36, then what will be the dividend?

(a) 2706 (b) 2796

(c) 2736 (d) 2826

Q.10 What is the unit digits of $(3694)^{1739} \times (615)^{317} \times (841)^{491}$

(a) 0 (b) 2

(c) 3 (d) 5

Q.11 What will be written in the form of $\frac{p}{q}$ of 18.484848....?

(a) $\frac{462}{25}$ (b) $\frac{610}{33}$

(c) $\frac{200}{11}$ (d) $\frac{609}{33}$

Q.12 Put $\frac{0.\overline{936}-0.\overline{568}}{0.\overline{45}+2.\overline{67}}$ in the form of rational number.

Q.13 What will be the common factor of $\left\{(127)^{127}+(97)^{127}\right\}$ and $\left\{(127)^{97}+(97)^{97}\right\}$?

(a) 127 (b) 97

(c) 30 (d) 224

Answer Key

Q.1 (d) **Q.2 (c)** **Q.3 (b)** **Q.4 (b)**

Q.5 (c) **Q.6 13** **Q.7 1800** **Q.8 1875**

Q.9 (c) **Q.10 (a)** **Q.11 (b)** **Q.12** $\frac{2024}{17205}$

Q.13 (d)

2 CHAPTER Mathematical Operations

- In this chapter, by mathematical operations, mean simple mathematical operations. Under this, the expression is solved by changing the mathematical symbols of the given expression according to the instructions given in the question.
- In this chapter, we study the problems related to average, age related, mathematical symbols, mathematical puzzles, logical mathematical operations, permutation-combination etc.

For example, '$' means '+', '✕' means '-', '@' means 'x', '©' means '÷' then the following equation means -

[5 $ 20 © 4 ✕ 2] @ 5

Solution [5 + 20 ÷ 4 -2] x 5

= (5 + 5 -2) x 5 = 40 Ans.

Q.1 By changing which two signs the given equation will be correct.

24 ÷ 8 - 5 x 5 + 3 = 13

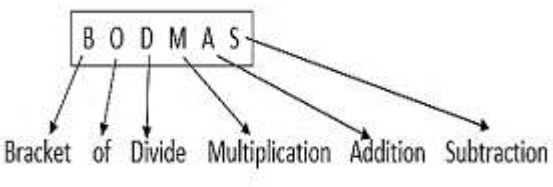
(A) '×' and '-' (B) '+' and '×'

(C) '÷' and '×' (D) '÷' and '+'

Solution - Option B will be correct.

$=\frac{24}{8}-5+5x3$

= 3 - 5+15 = 13



Blending Problems

Q.1 The weight of four boxes is 90, 30, 20 and 50 kg, a box made of two or more boxes, if one box is used only once, which of the following is not possible for the weight of the boxes?

(A) 190

(B) 170

(C) 100

(D) 150

Age Related Question

Q.1 Ram's age is twice the age of Shyam but 10 years ago Ram's age was 5 years more than three times Shyam's age, then find Shyam's age.

Q.2 Vikram's present age is 4 times Mohan's present age, 10 years from now Vikram's age will be twice Mohan's age, then what will be Vikram's present age?

(A) 30 (B) 20

(C) 10 (D) 5

Sign Language Equations

Q.1 If 9 # 3 = 6, 15 # 3 = 9, 60 # 4 = 32
Then what will be the value of 27 # 3.

Q.2 10 $ 25 = 8, 12 $ 25 = 10, then find 14 $ 53 = ?

(A) 11 (B) 12

(C) 13 (D) 14

Q.3 If two symbols are interchanged, the given equation will be correct.

40 + 10 ÷ 2 x 8 -17 = 17

(A) ÷ and - (B) x and +

(C) + and - (D) ÷ and x

Average Related Questions

Q.1 The total age of 12 students is 180, then find the sum of their ages 3 years ago.

(A) 140 (B) 144

(C) 142 (D) 148

Q.2 There are cows and some persons in a group, the number of legs in this group is 14 more than twice the number of heads, then find the number of cows.

(A) 5 (B) 7

(C) 10 (D) 12

Permutation and Simple Equation Based Problems

Q.1 If we write the count from 1 to 100 then how many times will 3 be written?

(A) 10 (B) 20

(C) 15 (D) 19

Q.2 Harish wants to go from Jaipur to Delhi. He gets 3 stoppages on going from Jaipur to Delhi, the first stoppage is found in Alwar and he can go to Alwar through four different routes. The second stoppage is Rewari and he can go from Alwar to Rewari by 3 routes and the third stoppage is Gurgaon and he can go from Rewari to Gurgaon by 3 routes and the last stoppage is from Gurgaon to Delhi and he can go by 2 routes then tell how many different ways he can go One can go from Jaipur to Delhi by different routes.

(A) 70 (B) 72

(C) 75 (D) 82

Q.3 If there are 250 rooms in a hotel then find the total number of digits in the numerical values used in those rooms.

(A) 642 (B) 585

(C) 662 (D) 872

Q.4 In a conference 15 people shake hands with each other, each person shakes hands only once, then tell how many times hands were shook in that conference?

(A) 110 (B) 105

(C) 120 (D) 90

Q.5 The present age of a father is three times that of his daughter, 5 years ago his age was 4 times that of his daughter, then find the age of his daughter.

(A) 64 (B) 24

(C) 75 (D) 15

Q.6 There are 1200 employees in a company including their leaders if 1 leader works for every 15 employees then find the total number of leaders.

(A) 80 (B) 90

(C) 75 (D) 85

Q.7 There are 15 boys in a class which are all 170 cm tall if it is 3/4 of the total number of boys and the total number of boys is 2/3 of the total number of students then find the number of girls in the class.

(A) 12 (B) 15

(C) 10 (D) 14

Q.8 Some friends plan to go for lunch, their total budget is Rs 192 but at the last moment 4 of them cancel the plan so that per person Rs. 8 increases, so tell how many friends made plans to go to lunch.

(A) 14 (B) 12
(C) 16 (D) 18

Question related to LCM

Q.1 5 bells start ringing together and they ring at intervals of 5, 6, 7, 10, 12 seconds respectively, then how many times will they ring together in 1 hour if the starting bell is not included.

(A) 7 (B) 8
(C) 11 (D) 12

Q.2 The ratio of the present ages of P and R is 9 : 4, the difference of their ages is 20 years. The sum of their ages after 10 years will be.

(A) 62 (B) 66
(C) 72 (D) 76

Signs and Symbols

Q.1 If x means '-', '-' means x, '+' means '÷' and '÷' means '+', then (15-10) ÷ (130 + 10) x 50 = ?

(A) 1800 (B) 113
(C) 2000 (D) 123

Q.2 If '÷' means '-', '-' means x, 'x' means '+' and '+' means '÷', then20 x 60 ÷ 40 - 20 + 10 = ?

(A) 80 (B) 60
(C) 40 (D) 0

Q.3 What will come in place of question mark (?) in the given equation?

39 ? 11 ? 33 ? 117

(A) x ,÷ and = (B) ÷, x and =
(C) +, = and x (D)x, ÷ and =

Q.4 If '+' denotes 'x', '-' denotes +, 'X' denotes '÷' and '÷' denotes '-', then 12 x 2 + 6 - 7 ÷ 5 = ?

(A) 38 (B) 39
(C) 40 (D) 37

Q.5 If '+' means 'x', '-' means '÷', 'x' means '-' and '÷' means '+', then what will be the value of the following equation

9 + 8 ÷ 8 - 4 x 6 = ?

(A) 68 (B) 36
(C) 65 (D) 11

Q.6 Question mark in the given equation? What will come in place of -

10 ? 0 ? 9 ? 5 = 10

(A) -, + and ÷ (B) +, - and x
(C) +, x and ÷ (D) x, + and ÷

Q.7 If ‘x’ means ‘÷’, ‘-‘ means ‘x’, ‘+’ means ‘-‘, ÷ means ‘+’, then 16 ÷ 4 + 9 - 3 = ?

(A) -17 (B) -7
(C) 17 (D) -9

Q.8 Question mark in the given equation? What will come in place of

19 ? 2 ? 4 ? 2 ? 1 = 39

(A) ÷ , -, + and x
(B) -, +, x and x
(C) x, +, ÷ and -
(D) +, x, ÷ and -

Q.9 If ÷ means +, x means -, - means ÷ and + means x, then
49 + 2 – 7 ÷ 11 x 4 = ?
(A) 20 (B) 21
(C) 19 (D) 16

Q.10 11 - If 6 x 2 ÷ 6 = 2 and 7 x1 ÷ 3 =5, then
11 x1 ÷ 13=?
(A) 9 (B) 0
(C) 1 (D) -1

Answer Key

Directions - Find out the related words from the alternatives given below.

Blending Problems

Q.1 (D)

Age Related Questions

Q.1 Shyam = 15
Q.2 Vikram = 20

Sign Language Equations

Q.1 15 **Q.2 13** **Q.3 17**

Average Related Questions

Q.1 (B) **Q.2 (B)**

Permutation and Simple Equation Based Problems

Q.1 (B) **Q.2 (B)** **Q.3 (A)** **Q.4 (B)**
Q.5 (D) **Q.6 (C)** **Q.7 (C)** **Q.8 (B)**

Question related to LCM

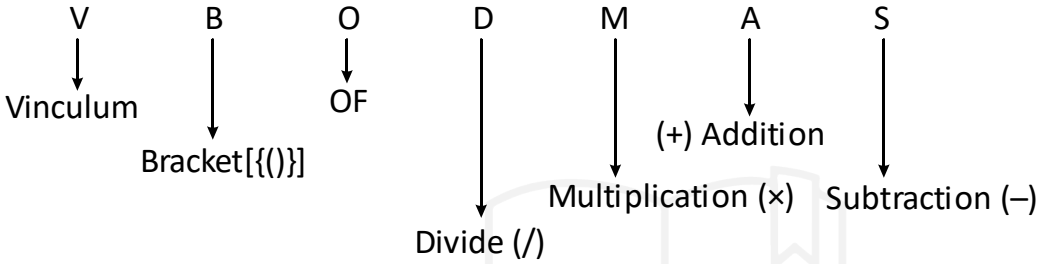
Q.1 (A) **Q.2 (C)**

Signs and Symbols

Q.1 (B) **Q.2 (D)** **Q.3 (B)** **Q.4 (A)**
Q.5 (A) **Q.6 (C)** **Q.7 (B)** **Q.8 (C)**
Q.9 (B) **Q.10 (D)**

3 CHAPTER Simplification

- In simplification, we represent the given data in a simple form, such as the data is done in fraction, in decimal, in division, in power and by solving or changing the mathematical operation.
- If different types of operations are given on some number, then how can we solve it so that the answer to the question is correct, for that there is a rule which we call the rule of VBODMAS.
- Which operation we should do first, it decides the rule of VBODMAS.



- The first of all these mathematical operations is V which means Vinculum (line bracket). If there is a line bracket in the question, then first we will solve it and then (BODMAS) Rule will work in it.
- B (Bracket) in the second place means brackets which can be –
  1. Small bracket ( )
  2. Middle/curly bracket { }
  3. Big bracket/ [ ]
- First the small brackets, then the curly bracket, and then the big brackets are solved.
- In the third place is "O" which is formed from "of" or "order", which means "multiply" or "of".
- In the fourth place is "D" which means "Division", in the given expression do the first division in different actions if given.
- There is "M" in the fifth place which means "Multiplication", in the given expression after "Division" we will do "Multiplication".
- Sixth position is held by "A" which is related to "Addition". Addition action takes place after division and multiplication.
- There is "S" in the seventh place which is made of "Subtraction".

Q. Simplify –

$$\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\,4\frac{1}{3}\right)$$

Sol: Step 1 – Convert the mixed fraction into simple fraction

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\,\frac{13}{3}\right)$$

Now, according to VBODMAS –

Step 2 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{3-2}{12}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\,\frac{13}{3}\right)$$

Step 3 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{1}{12}\right)\right\}\right]\div\frac{13}{6}$$

Step 4 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\left(\frac{30-1}{12}\right)\right\}\right]\div\frac{13}{6}$$

Step 5 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\frac{29}{12}\right\}\right]\div\frac{13}{6}$$

Step 6 –

$$\left[\frac{13}{4}\div\left\{\frac{30-29}{24}\right\}\right]\div\frac{13}{6}$$

Step 7 –

$$\left[\frac{13}{4}\div\frac{1}{24}\right]\div\frac{13}{6}$$

Step 8 –

$$\left[\frac{13}{4}\times 24\right]\div\frac{13}{6}$$

Step 9 –

$$13\times 6\times\frac{6}{13}$$

= 36 Ans.

Algebraic Formulas –

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a – b)^2 = a^2 – 2ab + b^2$
3. $(a + b)^2 + (a – b)^2 = 2(a^2 + b^2)$
4. $(a^2 – b^2) = (a + b)(a – b)$
5. $a^2 + b^2 + c^2 = (a + b + c)^2 – 2(ab + bc + ca)$
6. $a^2+\frac{1}{a^2}=\left(a+\frac{1}{a}\right)^2-2$
7. $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}\left[(a-b)^2+(b+c)^2+(c-a)^2\right]$
8. $a^3 + b^3 = (a + b)^3 – 3ab(a + b) = (a + b)(a^2 – ab + b^2)$
9. $a^3 – b^3 = (a – b)^3 + 3ab(a – b) = (a – b)(a^2 + ab + b^2)$
10. $a^3 + b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ca)$

$$=\frac{1}{2}(a+b+c)\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}$$

If $a + b + c = 0$, then

$a^3 + b^3 + c^3 = 3abc$

11. $a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$
12. $a^3-\frac{1}{a^3}=\left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)$

Square and Square Root Table

Square	Square Root	Square	Square Root
$1^2 = 1$	$\sqrt{1}=1$	$16^2 = 256$	$\sqrt{256}=16$
$2^2 = 4$	$\sqrt{4}=2$	$17^2 = 289$	$\sqrt{289}=17$
$3^2 = 9$	$\sqrt{9}=3$	$18^2 = 324$	$\sqrt{324}=18$

$4^2 = 16$	$\sqrt{16} = 4$	$19^2 = 361$	$\sqrt{361} = 19$
$5^2 = 25$	$\sqrt{25} = 5$	$20^2 = 400$	$\sqrt{400} = 20$
$6^2 = 36$	$\sqrt{36} = 6$	$21^2 = 441$	$\sqrt{441} = 21$
$7^2 = 49$	$\sqrt{49} = 7$	$22^2 = 484$	$\sqrt{484} = 22$
$8^2 = 64$	$\sqrt{64} = 8$	$23^2 = 529$	$\sqrt{529} = 23$
$9^2 = 81$	$\sqrt{81} = 9$	$24^2 = 576$	$\sqrt{576} = 24$
$10^2 = 100$	$\sqrt{100} = 10$	$25^2 = 625$	$\sqrt{625} = 25$
$11^2 = 121$	$\sqrt{121} = 11$	$26^2 = 676$	$\sqrt{676} = 26$
$12^2 = 144$	$\sqrt{144} = 12$	$27^2 = 729$	$\sqrt{729} = 27$
$13^2 = 169$	$\sqrt{169} = 13$	$28^2 = 784$	$\sqrt{784} = 28$
$14^2 = 196$	$\sqrt{196} = 14$	$29^2 = 841$	$\sqrt{841} = 29$
$15^2 = 225$	$\sqrt{225} = 15$	$30^2 = 900$	$\sqrt{900} = 30$

Cube and Cube Root Table

Cube	Cube Root	Cube	Cube Root
$1^3 = 1$	$\sqrt[3]{1} = 1$	$16^3 = 4096$	$\sqrt[3]{4096} = 16$
$2^3 = 8$	$\sqrt[3]{8} = 2$	$17^3 = 4913$	$\sqrt[3]{4913} = 17$
$3^3 = 27$	$\sqrt[3]{27} = 3$	$18^3 = 5832$	$\sqrt[3]{5832} = 18$
$4^3 = 64$	$\sqrt[3]{64} = 4$	$19^3 = 6859$	$\sqrt[3]{6859} = 19$
$5^3 = 125$	$\sqrt[3]{125} = 5$	$20^3 = 8000$	$\sqrt[3]{8000} = 20$
$6^3 = 216$	$\sqrt[3]{216} = 6$	$21^3 = 9261$	$\sqrt[3]{9261} = 21$
$7^3 = 343$	$\sqrt[3]{343} = 7$	$22^3 = 10648$	$\sqrt[3]{10648} = 22$
$8^3 = 512$	$\sqrt[3]{512} = 8$	$23^3 = 12167$	$\sqrt[3]{12167} = 23$
$9^3 = 729$	$\sqrt[3]{729} = 9$	$24^3 = 13824$	$\sqrt[3]{13824} = 24$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$	$25^3 = 15625$	$\sqrt[3]{15625} = 25$
$11^3 = 1331$	$\sqrt[3]{1331} = 11$	$26^3 = 17576$	$\sqrt[3]{17576} = 26$
$12^3 = 1728$	$\sqrt[3]{1728} = 12$	$27^3 = 19683$	$\sqrt[3]{19683} = 27$
$13^3 = 2197$	$\sqrt[3]{2197} = 13$	$28^3 = 21952$	$\sqrt[3]{21952} = 28$
$14^3 = 2744$	$\sqrt[3]{2744} = 14$	$29^3 = 24389$	$\sqrt[3]{24389} = 29$
$15^3 = 3375$	$\sqrt[3]{3375} = 15$	$30^3 = 27000$	$\sqrt[3]{27000} = 30$

Arithmetic Progression

The series in which each term can be found by adding or subtracting with its preceding term is
called the arithmetic progression.
E.g. 2, 5, 8, 11, …….

n^{th} term of an Arithmetic Progression

$T_n = a + (n - 1)\, d$

Where, a = First term

d = Common difference (2^{nd} term – 1^{st} term)

n = Number of all terms.

Addition of n^{th} terms of an Arithmetic Progression –

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

If the first and last term is known –

$$S_n = \frac{n}{2}[a+\ell]$$

Where, $\ell =$ Last term

Arithmetic progression between the two variables

$A = \frac{a+b}{2}$ [The arithmetic progression of a & b is A]

Geometric Progression

If the ratio of each term of the series to its preceding term is a certain variable, then it is called a geometric series. This fixed variable is called the common ratio.

n^{th} term of Geometric Series –

$T_n = a.r^{n-1}$

Where, a = First term
r = Common ratio
n = Number of terms

Addition of n^{th} terms of Geometric Series –

$$S_n = a\left(\frac{1-r^n}{1-r}\right); \text{ When } r<1$$

$$S_n = a\left(\frac{r^n-1}{r-1}\right); \text{ when } r>1$$

1. Geometric series between two variables $G = \sqrt{ab}$
2. If the arithmetic mean and geometric mean between two positive quantities a and b are A and G, then A > G, $\frac{a+b}{2} > \sqrt{ab}$

Harmonic Progression

If the reciprocals of the terms of a series are written in the same order and it is in arithmetic progression, then this is known as harmonic series.

n^{th} term of a Harmonic Progression –

$$T_n = \frac{1}{a+(n-1)d}$$

Harmonic series (H) = $\frac{2ab}{a+b}$

Relation between Arithmetic Mean, Geometric Mean and Harmonic Mean

Let A, G and H be the arithmetic mean, geometric mean and harmonic mean between two
quantities a and b respectively, then

$\boxed{G^2 = AH}$ and $\boxed{A > G > H}$

Practice Question

Q.1 The value of $24 \times 2 \div 12 + 12 \div 6$ of 2 $\div (15 \div 8 \times 4)$ of $(28 \div 7$ of 5) is –

(a) $4\frac{32}{75}$ (b) $4\frac{8}{75}$
(c) $4\frac{2}{3}$ (d) $4\frac{1}{6}$

Q.2 Simplify –

$$\left[3\frac{1}{4} \div \left\{1\frac{1}{4} - \frac{1}{2}\left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6}\right)\right\}\right] \div \left(\frac{1}{2} \text{ of } 4\frac{1}{3}\right)$$

Q.3 Evaluate –

$$2\frac{3}{4} \div 1\frac{5}{6} \div \frac{7}{8} \times \left(\frac{1}{3} + \frac{1}{4}\right) + \frac{5}{7} \div \frac{3}{4} \text{ of } \frac{3}{7}$$

(a) $\frac{56}{77}$ (b) $\frac{49}{80}$
(c) $\frac{2}{3}$ (d) $3\frac{2}{9}$

Q.4 If $(102)^2 = 10404$ then the value of $\sqrt{104.04} + \sqrt{1.0404} + \sqrt{0.010404}$ is equals to?

(a) 0.306 (b) 0.0306
(c) 11.122 (d) 11.322

Q.5 If a = 64 & b = 289 then find the value of $\left(\sqrt{\sqrt{a}+\sqrt{b}} - \sqrt{\sqrt{\sqrt{b}-\sqrt{a}}}\right)^{\frac{1}{2}}$

(a) $2^{1/2}$ (b) 2
(c) 4 (d) -2

Q.6 The cube root of 175616 is 56 then find the value of $\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$?

(a) 0.168 (b) 62.16

(c) 6.216 (d) 6.116

Q.7 What is the smallest number to be added to 710 so that the sum becomes a perfect cube?

(a) 29 (b) 19

(c) 11 (d) 21

Q.8 Find the value of the following –

$$4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}$$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$

(c) $\frac{1}{16}$ (d) $\frac{1}{32}$

Q.9 If $2=x+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}$ then find the value of x ?

(a) $\frac{18}{17}$ (b) $\frac{21}{17}$

(c) $\frac{13}{17}$ (d) $\frac{12}{17}$

Q.10 $999\frac{998}{999}\times 999$ equals to ?

(a) 998999 (b) 999899

(c) 989999 (d) 999989

Q.11 Find the value of $\frac{(0.03)^2-(0.01)^2}{0.03-0.01}$?

(a) 0.02 (b) 0.004

(c) 0.4 (d) 0.04

Q.12 $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2$ equals to ?

(a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$

(c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$

Q.13 Find the value of

$$\frac{0.051\times 0.051\times 0.051+0.041\times 0.041\times 0.041}{0.051\times 0.051-0.051\times 0.041+0.041\times 0.041}$$

(a) 0.92 (b) 0.092

(c) 0.0092 (d) 0.00092

Q.14 Find the sum of all the multiples of 3 less than 50 ?

(a) 400 (b) 408

(c) 404 (d) 412

Q.5 How many terms are there in the following arithmetic series?

7, 13, 19, ... , 205

Q.16 If the sum of two numbers is 22, and the sum of their squares is 404, then find the product of those numbers?

(a) 40 (b) 44

(c) 80 (d) 89

Q.17 When a two digit number is multiplied by the sum of its digits, the product is 424. When the number obtained by interchanging its digits is multiplied by the sum of the digits, the result is 280. What is the sum of the digits of the number?

(a) 7 (b) 9

(c) 6 (d) 8

Answer Key

Q.1 (d)	**Q.2** $7\frac{1}{5}$	**Q.3 (d)**	**Q.4 (d)**
Q.5 (a)	**Q.6 (c)**	**Q.7 (b)**	**Q.8 (a)**
Q.9 (b)	**Q.10 (a)**	**Q.11 (d)**	**Q.12 (c)**
Q.13 (b)	**Q.14 (b)**	**Q.15 34**	**Q.16 (a)**
Q.17 (d)			