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Preliminary

Delhi Development Authority (DDA)

Volume - 2

Numerical Ability & Reasoning

SIERRA INNOVATIONS

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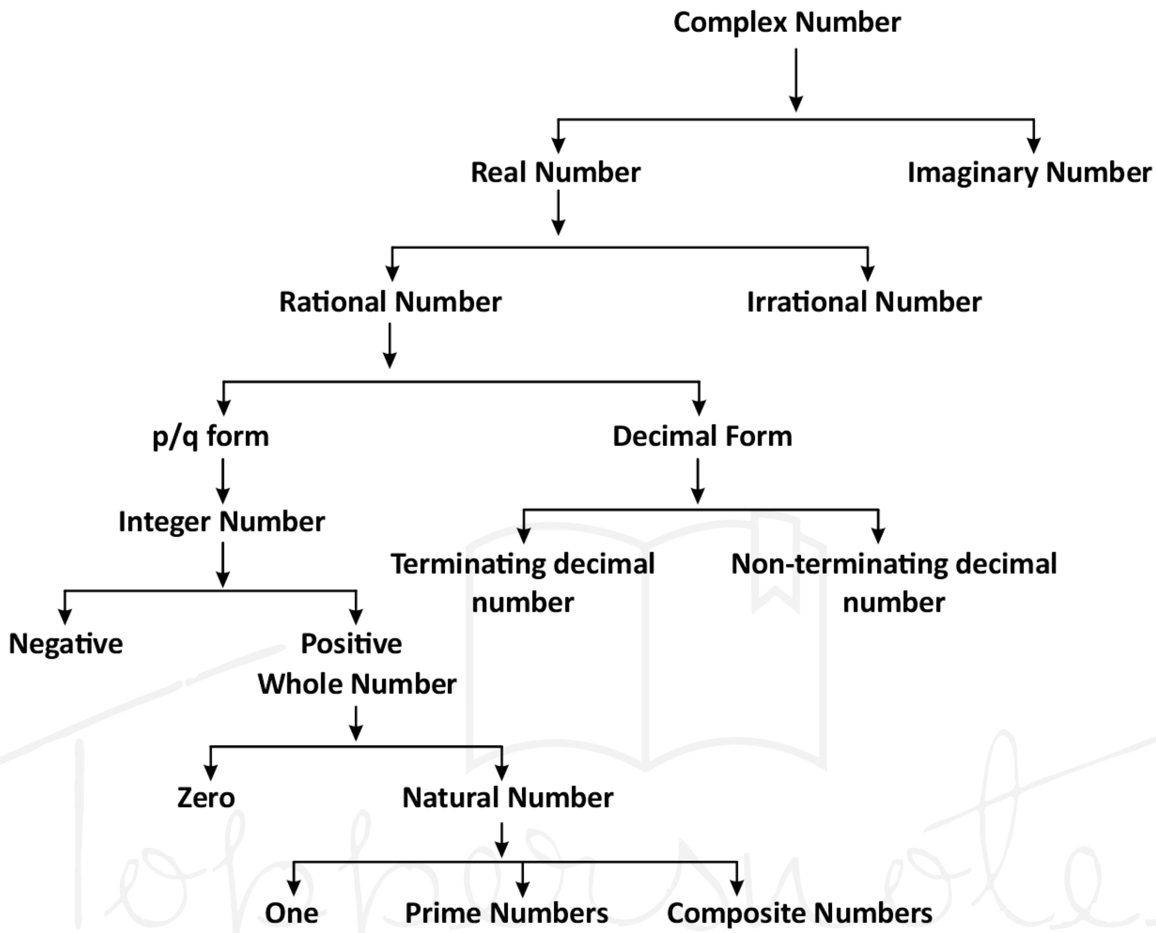
1 CHAPTER

Number System



Complex Number (Z)

Z = Real numbers + Imaginary numbers

Z = a + ib

Where, a = Real numbers.
b = Imaginary numbers.

Real Numbers

Rational and irrational numbers together are called real numbers. These can be represented on the number line.

Imaginary Numbers

Numbers that can not be represented on the number line.

Integer Numbers

A set of numbers which includes whole numbers as well as negative numbers, is called integer numbers, it is denoted by I.

I = {–4, –3, –2, –1, 0, 1, 2, 3, 4,.....}

Natural Numbers

The numbers which are used to count things are called natural numbers.

N = {1, 2, 3, 4, 5, ...}

Whole Numbers

When 0 is also included in the family of natural numbers, then they are called whole numbers.

W = {0, 1, 2, 3, 4, 5, …..}

The product of four consecutive natural numbers is always exactly divisible by 24.

Even Numbers

Numbers which are completely divisible by 2 are called even numbers.

n^{th} term = 2n

Sum of first n even natural numbers = n(n+1)

Sum of square of first n even natural numbers = $\frac{2n(n+1)(2n+1)}{3}$

$$\left\{n = \frac{\text{Last term}}{2}\right\}$$

Odd Numbers

The numbers which are not divisible by 2 are odd numbers.

Sum of first n odd numbers = n^2

$$\left\{n = \frac{\text{Last term} + 1}{2}\right\}$$

Natural Numbers

Sum of first n natural numbers $= \frac{n(n+1)}{2}$

Sum of square of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$

Sum of cube of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

The difference of the squares of two consecutive natural numbers is equal to their sum.

Example - $11^2 = 121$

$12^2 = 144$

11 + 12 → 23

Difference 144 – 121 = 23

Prime Numbers – Which have only two forms - 1 × numbers

E.g. - {2, 3, 5, 7, 11, 13, 17, 19……..}

Where, 1 isn't a Prime Number.

- The digit 2 is only even prime number.
- 3, 5, 7 is the only pair of consecutive odd prime numbers.
- Total prime numbers between 1 to 25 = 9
- Total prime numbers between 25 to 50 = 6
- There are total of 15 prime numbers between 1-50.
- There are total of 10 prime numbers between 51 – 100.
  So there are total 25 prime numbers from 1-100.
- Total prime numbers from 1 to 200 = 46
- Total prime numbers from 1 to 300 = 62
- Total prime numbers from 1 to 400 = 78
- Total prime numbers from 1 to 500 = 95

Co-prime Numbers

Numbers whose HCF is only 1.

E.g. - (4,9), (15, 22), (39, 40)

HCF = 1

Perfect Number

A number whose sum of its factors is equal to that number (except the number itself in the factors)

E.g. - 6 → 1, 2, 3 → Here 1 + 2 + 3 → 6

28 → 1, 2, 4, 7, 14 → 1 + 2 + 4 + 7 + 14 → 28

Rational Numbers

Numbers that can be written in the form of P/Q, but where Q must not be zero and P and Q must be integers.

E.g. - 2/3, 4/5, $\frac{10}{-11}, \frac{7}{8}$

Irrational Numbers

These cannot be displayed in P/Q form.

E.g. - $\sqrt{2}, \sqrt{3}, \sqrt{11}, \sqrt{19}, \sqrt{26}$...

Perfect square numbers

↓

Unit Digit which can be of square
0
1
4
5 or 25
6
9

Which can't be square
2 ——
3 ——
7 ——
8 ——

- The last two digits of the square of any number will be the same as the last two digits of the square of numbers 1-24.

Note: Therefore, everyone must remember the squares of 1-25.

Convert to Binary and Decimal –

1. **Convert Decimal Number to Binary Number**

   To find the binary number equivalent to a decimal number, we continuously divide the given decimal number by 2 until we get 1 as the final quotient.

   E.g.

2	89	2 × 44 = 88 ; 89 – 88 = 1
2	44	2 × 22 = 44; 44 – 44 = 0
2	22	2 × 11 = 22 ; 22 – 22 = 0
2	11	2 × 5 = 10 ; 11 – 10 = 1
2	5	2 × 2 = 4 ; 5 – 4 = 1
2	2	2 × 1 = 2 ; 2 – 2 = 0
	1	Final quotient

   Hence, binary number equivalent to 89 = $(1011001)_2$

2. **Convert Binary to Decimal Nubmer**

   In binary system the value of 1 when it moves one place to its left every time it doubles itself and wherever 0 comes its value is 0.

   E.g.

1	0	1	1	0	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

   Now

   $(1011001)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 \times 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

   $= 64 + 0 + 16 + 8 + 8 + 0 + 1 \quad \{2^0 = 1\}$

   $= 89$

Finding the Number of Divisors or Number of Factors

First we will do the prime factorization of the number and write it as Power and multiply by adding

One to each power, then the number of divisors will be obtained.

Ex: By how many total numbers can 2280 be completely divided?

Sol. $2280 = 2^3 \times 3^1 \times 5^1 \times 19^1$

Number of divisors = (3 + 1) (1 + 1) (1 + 1) (1 + 1)

= 4 × 2 × 2 × 2 = 32

Find the unit's digit

1. **When the number is in the form of power –**

   When the unit digit of Base is 0, 1, 5 or 6, the unit digit of the result remains the same for any natural power. When the unit digit of base is 2, 3, 4, 7, 8, or 9, divide the power by 4 and put the same power on the unit digit of the base as the remainder. When the power is rounded off to 4, then the 4th power will be placed on the unit digit of the base.

2. **In the form of simplification –**

   Write the unit digit of each number and simplify it according to the symbol, the result that will come will be its unit digit answer.

Divide by Power of Numbers (Finding the Divisor)

1. If $a^n + b^n$ is given –

   If n is odd, then (a+b) will be its divisor.

2. If $a^n - b^n$ is given –

   Divisor (when n is odd) $\rightarrow$ (a–b)

   Divisor (when n is even) $\rightarrow$ (a – b) or (a + b) or both.

1. If $a^n \div (a-1)$ then the remainder always be 1.
2. $a^n \div (a+1)$ { If n is an even then the remainder always be 1. If n is an odd then the remainder always be a.
3. If $(a^n + a) \div (a-1)$ then the remainder always be 2 .
4. $(a^n + a) \div (a+1)$ { If n is an even then the remainder always be zero (0). If n is an odd then the remainder always be (a – 1)

Terminating Decimal

Those numbers which end after a few digits after the decimal like - 0.25, 0.15, 0.375 can be written in a fraction number.

Non-Terminating Decimal

Those numbers which continue after the decimal and can be of two types.

0.3333, 0.7777, 0.183183183...........

Repeating	Numbers that never end after the decimal, but repeat, till infinity. It can be written in fractions.
Non Repeating Decimal	Numbers that never end after the decimal point, but they do not repeat their numbers.

Recurring Decimal Fraction

That decimal fraction is the repetition of one or more digits after the decimal point, then one or more digits are repeated after the dot.

Eg. $\frac{1}{3} = 0.333...,\ \frac{22}{7} = 3.14285714.....$ To represent such fractions, a line is drawn over the repeating digit.

$$0.35\overline{24} = \frac{3524-35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$$

$$\frac{22}{7} = 3.14285714.... = 3.\overline{142857}$$

It is called bar.

- **Convert pure recurring decimal fraction to simple fraction as follows –**

$$0.\overline{P} = \frac{P}{9} \qquad 0.\overline{pq} = \frac{pq}{99} \qquad 0.\overline{pqr} = \frac{pqr}{999}$$

- **Convert a mixed recurring decimal fraction to an ordinary fraction as follows –**

$$0.p\overline{q} = \frac{pq-p}{90} \qquad 0.pq\overline{r} = \frac{pqr-pq}{900}$$

$$0.p\overline{qr} = \frac{pqr-p}{990} \qquad 0.pq\overline{rs} = \frac{pqrs-pq}{9900}$$

Example -

(i) $0.\overline{39} = \frac{39}{99} = \frac{13}{33}$

(ii) $0.6\overline{25} = \frac{625-6}{990} = \frac{619}{990}$

(iii) $0.35\overline{24} = \frac{3524-35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$

Symbol of the Roman Method

1	→	I
2	→	II
3	→	III
4	→	IV
5	→	V
6	→	VI
7	→	VII
8	→	VIII
9	→	IX
10	→	X
20	→	XX
30	→	XXX
40	→	XL
50	→	L
100	→	C
500	→	D
1000	→	M

Rule of Divisibility

Rule of 2	The last digit is an even number or zero (0) as - 236, 150, 1000004
Rule of 3	If the sum of the digits of a number is divisible by 3, then the whole number will be divisible by 3. E.g. 729, 12342, 5631
Rule of 4	Last two digits are zero or divisible by 4. E.g. 1024, 58764, 567800
Rule of 5	The last digit is zero or 5. E.g. 3125, 625, 1250
Rule of 6	If a number is divisible by both 2 and 3 then it is also divisible by 6. E.g. 3060, 42462, 10242
Rule of 7	After multiplying the last digit of a number by 2 and subtracting it from the remaining number, if the number is a multiple of 0 or 7 or if any digit is repeated in a multiple of 6, then the number will be divisible by 7. E.g. 222222, 44444444444, 7854
Rule of 8	If the last three digits of a number are divisible by 8 or the last three digits are '000' (zero). E.g. 9872, 347000
Rule of 9	If the sum of the digits of a number is divisible by 9, then the whole number will be divisible by 9.
Rule of 10	The last digit should be zero (0).
Rule of 11	If the difference between the sum of digits at odd places and sum of digits at even places is zero (0) or 11 or a multiple of 11. E.g. 1331, 5643, 8172659
Rule of 12	Composite form of divisible by 3 and 4.
Rule of 13	Repeating the digit 6 times, or multiplying the last digit by 4 and adding it to the remaining number, if the number is divisible by 13, then the whole number will be divisible by 13. E.g. 222222, 17784

Practice Questions

Q.1 If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of that number, then what will be $\frac{5}{3}$ of that number?

(a) 12 (b) 18
(c) 15 (d) 20

Q.2 If the sum of two numbers is a and their product is a then their reciprocals will be –

(a) $\frac{1}{a}+\frac{1}{b}$ (b) $\frac{b}{a}$
(c) $\frac{a}{b}$ (d) $\frac{a}{ab}$

Q.3 The sum of two numbers is 75 and their difference is 25, then what will be the product of those two numbers?

(a) 1350 (b) 1250
(c) 1000 (d) 125

Q.4 Divide 150 into two parts such that the sum of their reciprocal is $\frac{3}{112}$. Calculate both parts.

(a) 50, 90 (b) 70, 80
(c) 60, 90 (d) 50, 100

Q.5 If the sum of any three consecutive odd natural numbers is 147, then the middle number will be –

(a) 47 (b) 48
(c) 49 (d) 51

Q.6 If the product of first three and last three of 4 consecutive prime numbers is 385 and 1001, then find the greatest prime number.

Q.7 What will be the sum of the even numbers between 50 and 100?

Q.8 What will be the sum of odd numbers between 50 and 100?

Q.9 In a division method, the divisor is 12 times the quotient and 5 times the remainder. Accordingly, if the remainder is 36, then what will be the dividend?

(a) 2706 (b) 2796
(c) 2736 (d) 2826

Q.10 What is the unit digits of $(3694)^{1739} \times (615)^{317} \times (841)^{491}$

(a) 0 (b) 2
(c) 3 (d) 5

Q.11 What will be written in the form of $\frac{p}{q}$ of 18.484848....?

(a) $\frac{462}{25}$ (b) $\frac{610}{33}$
(c) $\frac{200}{11}$ (d) $\frac{609}{33}$

Q.12 Put $\frac{0.\overline{936}-0.\overline{568}}{0.\overline{45}+2.\overline{67}}$ in the form of rational number.

Q.13 What will be the common factor of $\left\{(127)^{127}+(97)^{127}\right\}$ and $\left\{(127)^{97}+(97)^{97}\right\}$?

(a) 127 (b) 97
(c) 30 (d) 224

Answer Key

Q.1 (d)	**Q.2 (c)**	**Q.3 (b)**	**Q.4 (b)**
Q.5 (c)	**Q.6 13**	**Q.7 1800**	**Q.8 1875**
Q.9 (c)	**Q.10 (a)**	**Q.11 (b)**	**Q.12** $\frac{2024}{17205}$
Q.13 (d)			

2 CHAPTER

Simplification

- In simplification, we represent the given data in a simple form, such as the data is done in fraction, in decimal, in division, in power and by solving or changing the mathematical operation.
- If different types of operations are given on some number, then how can we solve it so that the answer to the question is correct, for that there is a rule which we call the rule of VBODMAS.
- Which operation we should do first, it decides the rule of VBODMAS.

V → Vinculum
B → Bracket[{()}]
O → OF
D → Divide (/)
M → Multiplication (×)
A → (+) Addition
S → Subtraction (–)

- The first of all these mathematical operations is V which means Vinculum (line bracket). If there is a line bracket in the question, then first we will solve it and then (BODMAS) Rule will work in it.
- B (Bracket) in the second place means brackets which can be –
  1. Small bracket ( )
  2. Middle/curly bracket { }
  3. Big bracket/ [ ]
- First the small brackets, then the curly bracket, and then the big brackets are solved.
- In the third place is "O" which is formed from "of" or "order", which means "multiply" or "of".
- In the fourth place is "D" which means "Division", in the given expression do the first division in different actions if given.
- There is "M" in the fifth place which means "Multiplication", in the given expression after "Division" we will do "Multiplication".
- Sixth position is held by "A" which is related to "Addition". Addition action takes place after division and multiplication.
- There is "S" in the seventh place which is made of "Subtraction".

Q. Simplify –

$$\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\,4\frac{1}{3}\right)$$

Sol: Step 1 – Convert the mixed fraction into simple fraction

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\frac{13}{3}\right)$$

Now, according to VBODMAS –

Step 2 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{3-2}{12}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\frac{13}{3}\right)$$

Step 3 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{1}{12}\right)\right\}\right]\div\frac{13}{6}$$

Step 4 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\left(\frac{30-1}{12}\right)\right\}\right]\div\frac{13}{6}$$

Step 5 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\frac{29}{12}\right\}\right]\div\frac{13}{6}$$

Step 6 –

$$\left[\frac{13}{4}\div\left\{\frac{30-29}{24}\right\}\right]\div\frac{13}{6}$$

Step 7 –

$$\left[\frac{13}{4}\div\frac{1}{24}\right]\div\frac{13}{6}$$

Step 8 –

$$\left[\frac{13}{4}\times 24\right]\div\frac{13}{6}$$

Step 9 –

$$13\times 6\times\frac{6}{13}$$

= 36 Ans.

Algebraic Formulas –

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
4. $(a^2 - b^2) = (a + b)(a - b)$
5. $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$
6. $a^2+\frac{1}{a^2}=\left(a+\frac{1}{a}\right)^2-2$
7. $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}\left[(a-b)^2+(b+c)^2+(c-a)^2\right]$
8. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + ab + b^2)$
10. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

    $=\frac{1}{2}(a+b+c)\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}$

    If $a + b + c = 0$, then

    $a^3 + b^3 + c^3 = 3abc$
11. $a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$
12. $a^3-\frac{1}{a^3}=\left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)$

Square and Square Root Table

Square	Square Root	Square	Square Root
$1^2 = 1$	$\sqrt{1}=1$	$16^2 = 256$	$\sqrt{256}=16$
$2^2 = 4$	$\sqrt{4}=2$	$17^2 = 289$	$\sqrt{289}=17$
$3^2 = 9$	$\sqrt{9}=3$	$18^2 = 324$	$\sqrt{324}=18$

$4^2 = 16$	$\sqrt{16} = 4$	$19^2 = 361$	$\sqrt{361} = 19$
$5^2 = 25$	$\sqrt{25} = 5$	$20^2 = 400$	$\sqrt{400} = 20$
$6^2 = 36$	$\sqrt{36} = 6$	$21^2 = 441$	$\sqrt{441} = 21$
$7^2 = 49$	$\sqrt{49} = 7$	$22^2 = 484$	$\sqrt{484} = 22$
$8^2 = 64$	$\sqrt{64} = 8$	$23^2 = 529$	$\sqrt{529} = 23$
$9^2 = 81$	$\sqrt{81} = 9$	$24^2 = 576$	$\sqrt{576} = 24$
$10^2 = 100$	$\sqrt{100} = 10$	$25^2 = 625$	$\sqrt{625} = 25$
$11^2 = 121$	$\sqrt{121} = 11$	$26^2 = 676$	$\sqrt{676} = 26$
$12^2 = 144$	$\sqrt{144} = 12$	$27^2 = 729$	$\sqrt{729} = 27$
$13^2 = 169$	$\sqrt{169} = 13$	$28^2 = 784$	$\sqrt{784} = 28$
$14^2 = 196$	$\sqrt{196} = 14$	$29^2 = 841$	$\sqrt{841} = 29$
$15^2 = 225$	$\sqrt{225} = 15$	$30^2 = 900$	$\sqrt{900} = 30$

Cube and Cube Root Table

Cube	Cube Root	Cube	Cube Root
$1^3 = 1$	$\sqrt[3]{1} = 1$	$16^3 = 4096$	$\sqrt[3]{4096} = 16$
$2^3 = 8$	$\sqrt[3]{8} = 2$	$17^3 = 4913$	$\sqrt[3]{4913} = 17$
$3^3 = 27$	$\sqrt[3]{27} = 3$	$18^3 = 5832$	$\sqrt[3]{5832} = 18$
$4^3 = 64$	$\sqrt[3]{64} = 4$	$19^3 = 6859$	$\sqrt[3]{6859} = 19$
$5^3 = 125$	$\sqrt[3]{125} = 5$	$20^3 = 8000$	$\sqrt[3]{8000} = 20$
$6^3 = 216$	$\sqrt[3]{216} = 6$	$21^3 = 9261$	$\sqrt[3]{9261} = 21$
$7^3 = 343$	$\sqrt[3]{343} = 7$	$22^3 = 10648$	$\sqrt[3]{10648} = 22$
$8^3 = 512$	$\sqrt[3]{512} = 8$	$23^3 = 12167$	$\sqrt[3]{12167} = 23$
$9^3 = 729$	$\sqrt[3]{729} = 9$	$24^3 = 13824$	$\sqrt[3]{13824} = 24$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$	$25^3 = 15625$	$\sqrt[3]{15625} = 25$
$11^3 = 1331$	$\sqrt[3]{1331} = 11$	$26^3 = 17576$	$\sqrt[3]{17576} = 26$
$12^3 = 1728$	$\sqrt[3]{1728} = 12$	$27^3 = 19683$	$\sqrt[3]{19683} = 27$
$13^3 = 2197$	$\sqrt[3]{2197} = 13$	$28^3 = 21952$	$\sqrt[3]{21952} = 28$
$14^3 = 2744$	$\sqrt[3]{2744} = 14$	$29^3 = 24389$	$\sqrt[3]{24389} = 29$
$15^3 = 3375$	$\sqrt[3]{3375} = 15$	$30^3 = 27000$	$\sqrt[3]{27000} = 30$

Arithmetic Progression

The series in which each term can be found by adding or subtracting with its preceding term is
called the arithmetic progression.

E.g. 2, 5, 8, 11, …….

n^{th} term of an Arithmetic Progression

$T_n = a + (n – 1) d$

Where, a = First term

d = Common difference (2^{nd} term – 1^{st} term)

n = Number of all terms.

Addition of n^{th} terms of an Arithmetic Progression –

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

If the first and last term is known –

$$S_n = \frac{n}{2}[a+\ell]$$

Where, $\ell =$ Last term

Arithmetic progression between the two variables

$A = \frac{a+b}{2}$ [The arithmetic progression of a & b is A]

Geometric Progression

If the ratio of each term of the series to its preceding term is a certain variable, then it is called a geometric series. This fixed variable is called the common ratio.

n^{th} term of Geometric Series –

$T_n = a.r^{n-1}$

Where, a = First term
r = Common ratio
n = Number of terms

Addition of n^{th} terms of Geometric Series –

$$S_n = a\left(\frac{1-r^n}{1-r}\right); \text{ When } r<1$$

$$S_n = a\left(\frac{r^n-1}{r-1}\right); \text{ when } r>1$$

1. Geometric series between two variables $G=\sqrt{ab}$
2. If the arithmetic mean and geometric mean between two positive quantities a and b are A and G, then A > G, $\frac{a+b}{2} > \sqrt{ab}$

Harmonic Progression

If the reciprocals of the terms of a series are written in the same order and it is in arithmetic progression, then this is known as harmonic series.

n^{th} term of a Harmonic Progression –

$$T_n = \frac{1}{a+(n-1)d}$$

Harmonic series (H) = $\frac{2ab}{a+b}$

Relation between Arithmetic Mean, Geometric Mean and Harmonic Mean

Let A, G and H be the arithmetic mean, geometric mean and harmonic mean between two
quantities a and b respectively, then

$\boxed{G^2 = AH}$ and $\boxed{A>G>H}$

Practice Question

Q.1 The value of 24 × 2 ÷ 12 + 12 ÷ 6 of 2 ÷ (15 ÷ 8 × 4) of (28 ÷ 7 of 5) is –

(a) $4\frac{32}{75}$ (b) $4\frac{8}{75}$
(c) $4\frac{2}{3}$ (d) $4\frac{1}{6}$

Q.2 Simplify –

$$\left[3\frac{1}{4} \div \left\{1\frac{1}{4} - \frac{1}{2}\left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6}\right)\right\}\right] \div \left(\frac{1}{2} \text{ of } 4\frac{1}{3}\right)$$

Q.3 Evaluate –

$$2\frac{3}{4} \div 1\frac{5}{6} \div \frac{7}{8} \times \left(\frac{1}{3}+\frac{1}{4}\right) + \frac{5}{7} \div \frac{3}{4} \text{ of } \frac{3}{7}$$

(a) $\frac{56}{77}$ (b) $\frac{49}{80}$
(c) $\frac{2}{3}$ (d) $3\frac{2}{9}$

Q.4 If $(102)^2$ = 10404 then the value of $\sqrt{104.04}+\sqrt{1.0404}+\sqrt{0.010404}$ is equals to?

(a) 0.306 (b) 0.0306
(c) 11.122 (d) 11.322

Q.5 If a = 64 & b = 289 then find the value of $\left(\sqrt{\sqrt{a}+\sqrt{b}} - \sqrt{\sqrt{\sqrt{b}-\sqrt{a}}}\right)^{\frac{1}{2}}$

(a) $2^{1/2}$ (b) 2
(c) 4 (d) -2

Q.6 The cube root of 175616 is 56 then find the value of

$\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$?

(a) 0.168 (b) 62.16

(c) 6.216 (d) 6.116

Q.7 What is the smallest number to be added to 710 so that the sum becomes a perfect cube?

(a) 29 (b) 19

(c) 11 (d) 21

Q.8 Find the value of the following –

$4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$

(c) $\frac{1}{16}$ (d) $\frac{1}{32}$

Q.9 If $2=x+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}$ then find the value of x ?

(a) $\frac{18}{17}$ (b) $\frac{21}{17}$

(c) $\frac{13}{17}$ (d) $\frac{12}{17}$

Q.10 $999\frac{998}{999}\times 999$ equals to ?

(a) 998999 (b) 999899

(c) 989999 (d) 999989

Q.11 Find the value of $\frac{(0.03)^2-(0.01)^2}{0.03-0.01}$?

(a) 0.02 (b) 0.004

(c) 0.4 (d) 0.04

Q.12 $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2$ equals to ?

(a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$

(c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$

Q.13 Find the value of

$\frac{0.051\times0.051\times0.051+0.041\times0.041\times0.041}{0.051\times0.051-0.051\times0.041+0.041\times0.041}$

(a) 0.92 (b) 0.092

(c) 0.0092 (d) 0.00092

Q.14 Find the sum of all the multiples of 3 less than 50 ?

(a) 400 (b) 408

(c) 404 (d) 412

Q.5 How many terms are there in the following arithmetic series?

7, 13, 19, ... , 205

Q.16 If the sum of two numbers is 22, and the sum of their squares is 404, then find the product of those numbers?

(a) 40 (b) 44

(c) 80 (d) 89

Q.17 When a two digit number is multiplied by the sum of its digits, the product is 424. When the number obtained by interchanging its digits is multiplied by the sum of the digits, the result is 280. What is the sum of the digits of the number?

(a) 7 (b) 9

(c) 6 (d) 8

Answer Key

Q.1	**(d)**	**Q.2**	$7\frac{1}{5}$	**Q.3**	**(d)**	**Q.4**	**(d)**
Q.5	**(a)**	**Q.6**	**(c)**	**Q.7**	**(b)**	**Q.8**	**(a)**
Q.9	**(b)**	**Q.10**	**(a)**	**Q.11**	**(d)**	**Q.12**	**(c)**
Q.13	**(b)**	**Q.14**	**(b)**	**Q.15**	**34**	**Q.16**	**(a)**
Q.17	**(d)**						

3 CHAPTER

Least Common Multiple and Highest Common Factor (LCM & HCF)

Factor: A number is said to be a factor of another if it completely divides the other. Like 3 and 4 are factors of 12.

Common Factor: The number which completely divides two or more given numbers is called the common factor of those numbers. Thus, one common factor of 9, 18, 21 and 33 is 3.

LCM (Least common multiple)

- The smallest number which is completely divisible by the given numbers is called LCM.
- **Finding the LCM of the number having power** - After factoring the prime, we will write it in the form of quotient and the number of primes that will be used will be written as multiplication and will keep the maximum power on it.

Ex-1: Find LCM of $(12)^{16}$, $(18)^{15}$, $(30)^{18}$

Sol. $(12)^{16} = (2 \times 2 \times 3)^{16} = (2^2 \times 3)^{16} = 2^{32} \times 3^{16}$

$(18)^{15} = (2 \times 3 \times 3)^{15} = (2 \times 3^2)^{15} = 2^{15} \times 3^{30}$

$(30)^{18} = (2 \times 3 \times 5)^{18} = 2^{18} \times 3^{18} \times 5^{18}$

Therefore, LCM = $2^{32} \times 3^{30} \times 5^{18}$ Ans.

LCM of fractions

$$\text{LCM} = \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$$

Ex-2: Find LCM of $\frac{1}{2}$ and $\frac{5}{8}$?

Sol. - $\text{LCM} = \frac{\text{LCM of 1 and 5}}{\text{HCF of 2 and 8}} \Rightarrow \frac{5}{2}$

HCF (Highest Common Factor)

- The greatest number by which all the given numbers are completely divisible is called HCF.
- Like – H.C.F. of 18 and 24 is 6.

Ex.1: If the H C F of two numbers is found by the division method, then the quotient is 3, 4, and 5 respectively. If the mean of two numbers is 18, then find the numbers.

Sol. - There are two numbers a and b

```
a) b (3
   c) a (4
      d) c (5
         x x
```

The last denominator is HCF.

$d = 18$

$c = 5 \times d = 5 \times 18 = 90$

$a = (4 \times c) + d$

$= (4 \times 90) + 18 = 378$

$b = 3a + c$

$= (3 \times 378) + 90 = 1134 + 90 = 1224$

So, the numbers are 1224 and 378

To find the HCF of a number with powers-

- First factor it into the base and write it as a power, and write it as a multiplication of all prime numbers in the base and put the lowest power on it.

Ex:1 Find HCF of $(24)^8$, $(36)^{12}$, $(18)^{16}$

Sol. $24 = (2^3 \times 3)^8 = 2^{24} \times 3^8$

$36 = (2^2 \times 3^2)^{12} = 2^{24} \times 3^{24}$

$18 = (2 \times 3^2)^{16} = 2^{16} \times 3^{32}$

So, HCF $= 2^{16} \times 3^8$

Finding the HCF of a Fraction –

$$\text{HCF} = \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$$

Ex: $\frac{18}{25}, \frac{12}{7}, \frac{6}{35}$

Sol. $\frac{\text{HCF of 18, 12, 16}}{\text{LCM of 25, 7, 35}}$

- HCF of Addition of any two numbers and their L.C.M is equal to the HCF of given two numbers.

  Let the two numbers be x and y, and their H.C.F is H.

  Therefore, x = Ha

  y = Hb

  Where a and b are mutually prime.

  LCM of x, y = Hab

  and $x + y = H(a + b)$

  Now 'a' and 'b' are mutually prime numbers, then (a + b) and ab will also be primes with each other. So we can conclude that the HCF of H(a + b) and Hab is H which is also the H.C.F of x and y.

Relation between LCM and HCF: -

LCM × HCF = Product of both numbers

Ex.1 The LCM and HCF of two numbers are 420 and 28. If one number is 84, find the other number –

Sol. Second Number $= \frac{420 \times 28}{84} = 140$

- The smallest number for x, y, z in which the remainder r is left after dividing, The answer for this will be (LCM of x, y, z + r).

Practice Questions

Q.1 What is the greatest common factor of 84, 126, 140 ?

Q.2 Find HCF of $x^6 - 1$ and $x^4 + 2x^3 - 2x^1 - 1$

(a) $x^2 + 1$ (b) $x - 1$

(c) $x^2 - 1$ (d) $x + 1$

Q.3 What will be the LCM of 15, 18, 24, 27, 36?

Q.4 Six bells started ringing simultaneously, if these bells rang respectively at an interval of 2, 4, 6, 8, 10, 12 seconds, then how many times in 30 minutes will they ring together?

(a) 4 times

(b) 10 times

(c) 16 times

(d) None of the above

Q.5 Three persons start walking together on a 11 km long circular path in the same direction. Their speed is 4, 5.5 and 8 km/hr respectively. After how much time will they meet together at the starting point ?

Q.6 Find the greatest common factor of 1.75, 5.6 and 7.

Q.7 The sum of two numbers is 36, their highest common factor is 3 and the least common factor is 105, what will be the sum of the reciprocals of these numbers ?

(a) $\frac{2}{35}$ (b) $\frac{3}{25}$

(c) $\frac{4}{35}$ (d) $\frac{2}{25}$

Q.8 Find two such three-digit numbers whose L.C.M. is 5760 and H.C.F. is 80?

Answer Key

1. 14	**2. (c)**	**3. 1080**
4. (c)	**5. 22 Hours**	**6. 0.35**
7. $\frac{4}{35}$	**8. 640 and 720**	

4 CHAPTER

Surds and Indices

- **Surds** - Those quantities whose definite value cannot be determined precisely, they are called surds.
- If 'a' is a rational number and 'm' is an integer then the m^{th} root of a will be an irrational number which would be $a^{\frac{1}{m}}$ or $\sqrt[m]{a}$ where $\sqrt[m]{a}$ called a surds.

  E.g. - $\sqrt{2}, \sqrt{3}$ etc.
- There are many forms of surds, for example - $\sqrt{\ }, \sqrt[3]{\ }, \sqrt[4]{\ }, \sqrt[5]{\ }$ ....
- $a^{\frac{1}{m}}$ is called a surds containing the m^{th} power.
- When both the power and the surds number of the root (i.e., the number written under the root) are equal, then the surds are said to be homogeneous.

  E.g.- $\sqrt{x}, 3\sqrt{x}, 7\sqrt{x}$
- When the powers of the surds are different or the surds numbers are different or both the powers and the surd numbers are different, then they are called heterogeneous.

  E.g. - $\sqrt[3]{xy}$, $\sqrt[3]{x}$, $\sqrt[2]{5y}$ etc.

When the all variables are in the form of surds

- If the number written in the surds cannot have two consecutive factors, then assuming the whole sum equal to x, square the two sides and convert the quadratic equation into the form ($ax^2 + bx + c = 0$).
- Then, according to Shri Dharacharya formula – $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Operations on Surds

(1) Addition and subtraction of surds

Only homogeneous surds can be added or subtracted.

E.g. $\sqrt{75} + \sqrt{48}$

Sol.- $\sqrt{25 \times 3} + \sqrt{16 \times 3}$

$= 5\sqrt{3} + 4\sqrt{3}$

$= 9\sqrt{3}$

E.g. $\sqrt{27} - \sqrt{12}$

Sol. $\sqrt{9 \times 3} - \sqrt{4 \times 3}$

$= 3\sqrt{3} - 2\sqrt{3}$

$= \sqrt{3}$

(2) Multiplication of surds

Multiplication of surds is possible only when their powers are equal.

E.g. $\sqrt[3]{2} \times \sqrt[3]{5} \times \sqrt[3]{4}$

Sol. $\sqrt[3]{2 \times 5 \times 4}$

$= \sqrt[3]{40}$

E.g. Divide $12 \times 4^{1/3}$ by $3\sqrt{2}$

Sol. $\dfrac{12 \times 4^{1/3}}{3\sqrt{2}} = \dfrac{4 \times 4^{1/3}}{2^{1/2}} = \dfrac{4 \times 4^{2/6}}{2^{3/6}}$

$= 4 \times \left[\dfrac{4^2}{2^3}\right]^{1/6} = 4 \times \left[\dfrac{16}{8}\right]^{1/6}$

$= 4 \times 2^{1/6}$

Some Important Rules for Surds

(1) $\sqrt{a}\times\sqrt{a}=a$

(2) $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

(3) $\sqrt{a^2\times b}=a\sqrt{b}$

(4) $\left(\sqrt{a}+\sqrt{b}\right)^2=a+b+2\sqrt{ab}$

(5) $\left(\sqrt{a}-\sqrt{b}\right)^2=a+b-2\sqrt{ab}$

(6) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b$

(7) $\sqrt{2}=1.41421$

(8) $\sqrt{3}=1.73205$

(9) $\sqrt{5}=2.23607$

(10) $\sqrt{6}=2.44949$

Conjugate

- **Surds with two terms which are the same but the symbols used between those two terms are different, then such surds are called conjugate surds.**
- **To find the value of such quantities, multiply both the numerator and the denominator by the conjugate of the denominator.**

E.g. Find the value of $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Sol.

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\left(\sqrt{3}-1\right)}{\left(\sqrt{3}-1\right)}=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{3+1-2\sqrt{3}}{3-1}$$

$$=\frac{4-2\sqrt{3}}{2}$$

$$=\frac{2(2-\sqrt{3})}{2}$$

$$=2-\sqrt{3}$$

Comparison of Surds (Greatest and Smallest)

- To find the greater or smallest of the given surds, we equalize the exponents and compare the bases.

E.g. Which one is greater among $\sqrt[3]{5},\sqrt{4},\sqrt[3]{6}$?

Sol. Here, power of $\sqrt[3]{5},\sqrt{4},\sqrt[3]{6}$ is 3, 2, 3 and their LCM = 6

$\sqrt[3]{5}=\sqrt[6]{5^2}=\sqrt[6]{25}$

$\sqrt{4}=\sqrt[6]{4^3}=\sqrt[6]{64}$

$\sqrt[3]{6^2}=\sqrt[6]{36}$

Hence, the greatest number = $\sqrt[6]{64}=\sqrt{4}$

Indices – The number which is multiplied by itself multiple times is called the power of that number and that number is called the base.

Some Important Rules for Indices

(i) $a^m = a \times a \times a \times$ …….. m times

(ii) $(a^m)^n = a^{mn}$

(iii) $a^m \times a^n = a^{(m+n)}$

(iv) $a^m \div a^n = a^{(m-n)}$

(v) $[(a^m)^n]^l = a^{mnl}$

(vi) $a^{-m}=\frac{1}{a^m}$

(vii) a° = 1 {If the power of any number is zero, then the value of that whole sum is 1.}

(viii) $(a/b)^{-m} = (b/a)^m$

(ix) $a^m = b^n$

$a = (b)^{n/m}$ or $b = (a)^{m/n}$

(x) $a^m = b$ then $a = b^{1/m}$

- If the power is in a different form, then to find the greatest or smallest value, we will take the LCM of the denominator of the power and multiply each power by the LCM and the one whose larger value comes will be greatest and the one whose smaller value comes will be smallest.

E.g. $(2)^{1/4}, (3)^{1/6}, (4)^{1/8}, (8)^{1/12}$

Sol. $(2)^{\frac{1}{4}\times24} = 2^6 = 64$

$(3)^{\frac{1}{6}\times24} = 3^4 = 81$

$(4)^{\frac{1}{8}\times24} = 4^3 = 64$

$(8)^{\frac{1}{12}\times24} = 8^2 = 64$

Hence, the greatest number is $3^{1/6}$ (Note – Here, the LCM of 4, 6, 8, 12 is 24)

Practice Question

Q.1 The value of $\sqrt{214+\sqrt{107+\sqrt{196}}}$

(a) 23 (b) 15
(c) 24 (d) 18

Q.2 What is the value of the following?

$\sqrt{6+\sqrt{6+\sqrt{6+\cdots\ldots}}}$

(a) 5 (b) 3
(c) 2 (d) 30

Q.3 Find the value of the following?

$\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

(a) 2 (b) 4
(c) ±2 (d) -2

Q.4 The value of $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ is

(a) $2\sqrt{10}$ (b) 0
(c) $2\sqrt{6}$ (d) $2\sqrt{15}$

Q.5 The value of $\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\ldots\ldots\ldots\ldots}}}}$ is

(a) 2 (b) 2^2
(c) 2^3 (d) 2^5

Q.6 Which one is the greatest among them $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}$ **?**

(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$
(c) $\sqrt[4]{4}$ (d) $\sqrt[6]{6}$

Q.7 Arrange the following in descending order (from greatest to smallest).

$\sqrt[3]{4}, \sqrt{2}, \sqrt[6]{3}, \sqrt[4]{5}$

(a) $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$
(b) $\sqrt[3]{4} > \sqrt{2} > \sqrt[6]{3} > \sqrt[4]{5}$
(c) $\sqrt{2} > \sqrt[3]{4} > \sqrt[6]{3} > \sqrt[4]{5}$
(d) $\sqrt[6]{3} > \sqrt[4]{5} > \sqrt[3]{4} > \sqrt{2}$

Q.8 The smallest number among the following

$\sqrt[6]{12}, \sqrt[3]{4}, \sqrt[4]{5}, \sqrt{3}$

(a) $\sqrt[6]{12}$ (b) $\sqrt[3]{4}$
(c) $\sqrt[4]{5}$ (d) $\sqrt{3}$

Q.9 What is the square root of $\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$?

(a) $\sqrt{3}+\sqrt{2}$ (b) $\sqrt{3}-\sqrt{2}$
(c) $\sqrt{2} \pm \sqrt{3}$ (d) $\sqrt{2}-\sqrt{3}$

Q.10 If x,y is a rational number and the value of $(\mathbf{5}+\sqrt{\mathbf{11}})/(\mathbf{3}-\mathbf{2}\sqrt{\mathbf{11}}) = x$

$y\sqrt{11}$ then the value of x and y is ?

(a) $x=\frac{-14}{17}, y=\frac{-13}{26}$
(b) $x=\frac{4}{13}, y=\frac{11}{17}$
(c) $x=\frac{-27}{25}, y=\frac{-11}{37}$
(d) $x=\frac{-37}{35}, y=\frac{-13}{35}$

Q.11 If $\sqrt{50}+\sqrt{128}=\sqrt{N}$, then the value of N ?

(a) 26 (b) 390
(c) 338 (d) 182

Q.12 $\sqrt{2+\sqrt{2+\sqrt{2+\ \ldots\ldots}}}$ equals to ?

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) 2 (d) 3

Q.13 If $\left(4+\sqrt{7}\right)$ is written as a perfect square, it will be equal to which of the following?

(a) $(2+\sqrt{7})^2$ (b) $\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)^2$

(c) $\left\{\frac{1}{\sqrt{2}}\left(\sqrt{7}+1\right)\right\}^2$ (d) $\left(\sqrt{3}+\sqrt{4}\right)^2$

Q.14 If $\sqrt{7} = 2.6457$ and $\sqrt{3} = 1.732$ then the value of $\frac{1}{\sqrt{7}-\sqrt{3}}$?

(a) 1.0944 (b) 1.944

(c) 1.009 (d) 1.0844

Q.15 If $10^{0.48} = X, 10^{0.70} = y$ and $X^z = y^2$, then the approximate value of z will be?

(a) 1.45 (b) 1.88

(c) 2.9 (d) 3.7

Q.16 If $5^a = 3125$, then the value of $5^{(a-3)}$?

(a) 25 (b) 125

(c) 625 (d) 1625

Q.17 $\frac{(243)^{n/5}\times 3^{2n+1}}{g^n\times 3^{n-1}}$ =?

(a) 1 (b) 2

(c) 9 (c) 9^n

Q.18 If $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ equals to?

(a) 0 (b) 1

(c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

Answer Key

Q.1	**(b)**	**Q.2**	**(b)**	**Q.3**	**(c)**	**Q.4**	**(b)**
Q.5	**(a)**	**Q.6**	**(b)**	**Q.7**	**(a)**	**Q.8**	**(c)**
Q.9	**(a)**	**Q.10**	**(d)**	**Q.11**	**(c)**	**Q.12**	**(c)**
Q.13	**(c)**	**Q.14**	**(a)**	**Q.15**	**(c)**	**Q.16**	**(a)**
Q.17	**(c)**	**Q.18**	**(a)**				