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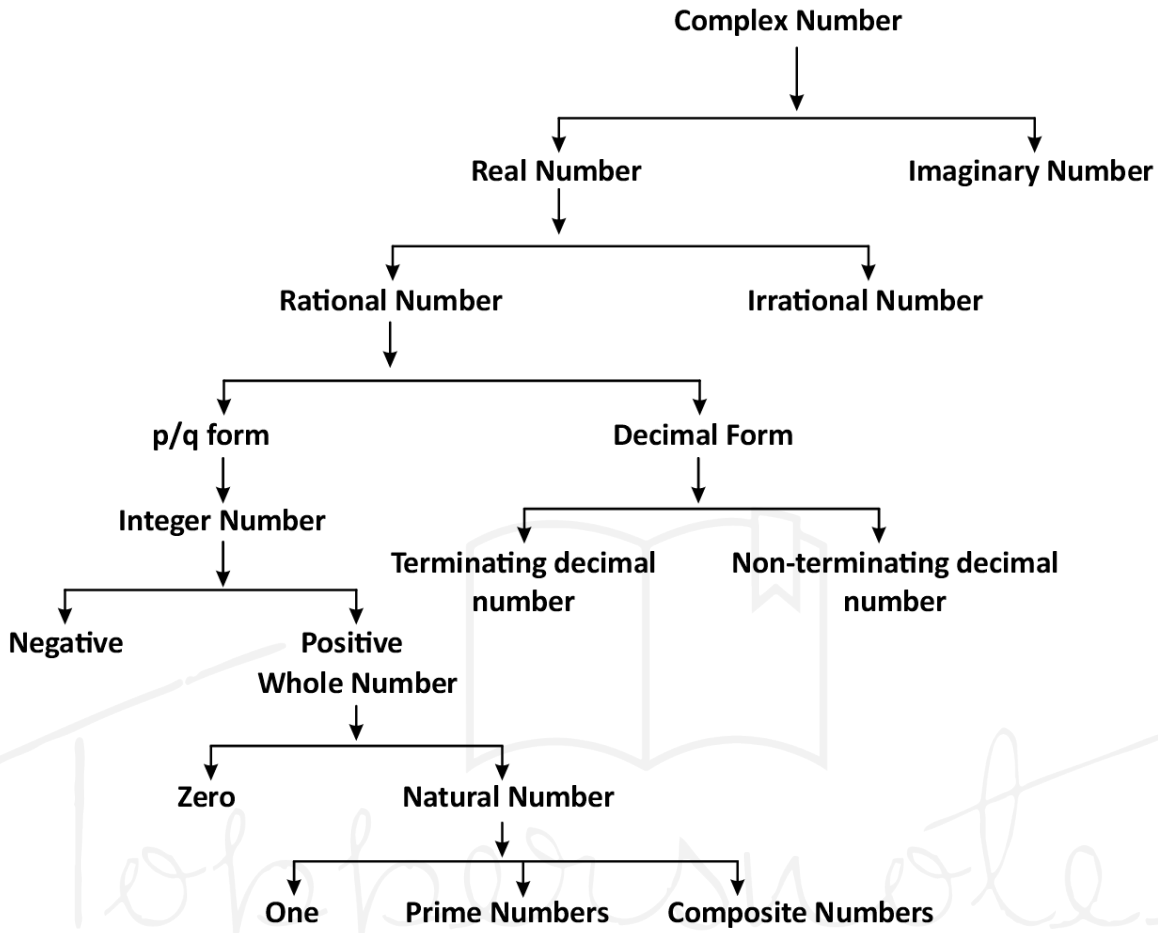
Volume - 2

Quantitative Aptitude

SIERRA INNOVATIONS

INDEX

1 CHAPTER Number System



Complex Number (Z)

Z = Real numbers + Imaginary numbers

$$Z = a + ib$$

Where, a = Real numbers.
b = Imaginary numbers.

Real Numbers

Rational and irrational numbers together are called real numbers. These can be represented on the number line.

Imaginary Numbers

Numbers that can not be represented on the number line.

Integer Numbers

A set of numbers which includes whole numbers as well as negative numbers, is called integer numbers, it is denoted by I.

I = {–4, –3, –2, –1, 0, 1, 2, 3, 4,.....}

Natural Numbers

The numbers which are used to count things are called natural numbers.

N = {1, 2, 3, 4, 5, ……}

Whole Numbers

When 0 is also included in the family of natural numbers, then they are called whole numbers.

W = {0, 1, 2, 3, 4, 5, …..}

The product of four consecutive natural numbers is always exactly divisible by 24.

Even Numbers

Numbers which are completely divisible by 2 are called even numbers.

n^{th} term = 2n

Sum of first n even natural numbers = n(n+1)

Sum of square of first n even natural numbers = $\frac{2n(n+1)(2n+1)}{3}$

$$\left\{n = \frac{\text{Last term}}{2}\right\}$$

Odd Numbers

The numbers which are not divisible by 2 are odd numbers.

Sum of first n odd numbers = n^2

$$\left\{n = \frac{\text{Last term} + 1}{2}\right\}$$

Natural Numbers

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

Sum of square of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

Sum of cube of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

The difference of the squares of two consecutive natural numbers is equal to their sum.

Example - $11^2 = 121$

$12^2 = 144$

$11 + 12 \rightarrow 23$

Difference 144 – 121 = 23

Prime Numbers – Which have only two forms - $1 \times$ numbers

E.g. - {2, 3, 5, 7, 11, 13, 17, 19……..}

Where, 1 isn't a Prime Number.

- The digit 2 is only even prime number.
- 3, 5, 7 is the only pair of consecutive odd prime numbers.
- Total prime numbers between 1 to 25 = 9
- Total prime numbers between 25 to 50 = 6
- There are total of 15 prime numbers between 1-50.
- There are total of 10 prime numbers between 51 – 100.
  So there are total 25 prime numbers from 1-100.
- Total prime numbers from 1 to 200 = 46
- Total prime numbers from 1 to 300 = 62
- Total prime numbers from 1 to 400 = 78
- Total prime numbers from 1 to 500 = 95

Co-prime Numbers

Numbers whose HCF is only 1.

E.g. - (4,9), (15, 22), (39, 40)

HCF = 1

Perfect Number

A number whose sum of its factors is equal to that number (except the number itself in the factors)

E.g. - $6 \rightarrow 1, 2, 3 \rightarrow$ Here $1 + 2 + 3 \rightarrow 6$

$28 \rightarrow 1, 2, 4, 7, 14 \rightarrow 1 + 2 + 4 + 7 + 14 \rightarrow 28$

Rational Numbers

Numbers that can be written in the form of P/Q, but where Q must not be zero and P and Q must be integers.

E.g. - 2/3, 4/5, $\frac{10}{-11}, \frac{7}{8}$

Irrational Numbers

These cannot be displayed in P/Q form.

E.g. - $\sqrt{2}, \sqrt{3}, \sqrt{11}, \sqrt{19}, \sqrt{26}$...

Perfect square numbers

↓

Unit Digit which can be of square	Which can't be square
0	2 ——
1	3 ——
4	7 ——
5 or 25	8 ——
6	
9	

- The last two digits of the square of any number will be the same as the last two digits of the square of numbers 1-24.

Note: Therefore, everyone must remember the squares of 1-25.

Convert to Binary and Decimal –

1. **Convert Decimal Number to Binary Number**

   To find the binary number equivalent to a decimal number, we continuously divide the given decimal number by 2 until we get 1 as the final quotient.

   E.g.

2	89	2 × 44 = 88 ; 89 – 88 = 1
2	44	2 × 22 = 44; 44 – 44 = 0
2	22	2 × 11 = 22 ; 22 – 22 = 0
2	11	2 × 5 = 10 ; 11 – 10 = 1
2	5	2 × 2 = 4 ; 5 – 4 = 1
2	2	2 × 1 = 2 ; 2 – 2 = 0
	1	Final quotient

   Hence, binary number equivalent to 89 = $(1011001)_2$

2. **Convert Binary to Decimal Nubmer**

   In binary system the value of 1 when it moves one place to its left every time it doubles itself and wherever 0 comes its value is 0.

   E.g.

1	0	1	1	0	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

   Now

   $(1011001)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 \times 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

   $= 64 + 0 + 16 + 8 + 8 + 0 + 1 \quad \{2^0 = 1\}$

   $= 89$

Finding the Number of Divisors or Number of Factors

First we will do the prime factorization of the number and write it as Power and multiply by adding

One to each power, then the number of divisors will be obtained.

Ex: By how many total numbers can 2280 be completely divided?

Sol. $2280 = 2^3 \times 3^1 \times 5^1 \times 19^1$

Number of divisors = (3 + 1) (1 + 1) (1 + 1) (1 + 1)

$= 4 \times 2 \times 2 \times 2 = 32$

Find the unit's digit

1. **When the number is in the form of power –**

   When the unit digit of Base is 0, 1, 5 or 6, the unit digit of the result remains the same for any natural power. When the unit digit of base is 2, 3, 4, 7, 8, or 9, divide the power by 4 and put the same power on the unit digit of the base as the remainder. When the power is rounded off to 4, then the 4^{th} power will be placed on the unit digit of the base.

2. **In the form of simplification –**

   Write the unit digit of each number and simplify it according to the symbol, the result that will come will be its unit digit answer.

Divide by Power of Numbers (Finding the Divisor)

1. If $a^n + b^n$ is given –
   If n is odd, then (a+b) will be its divisor.
2. If $a^n - b^n$ is given –
   Divisor (when n is odd) $\rightarrow$ (a–b)
   Divisor (when n is even) $\rightarrow$ (a – b) or (a + b) or both.

1. If $a^n \div (a-1)$ then the remainder always be 1.
2. $a^n \div (a+1)$ { If n is an even then the remainder always be 1. / If n is an odd then the remainder always be a.
3. If $(a^n + a) \div (a-1)$ then the remainder always be 2 .
4. $(a^n + a) \div (a+1)$ { If n is an even then the remainder always be zero (0). / If n is an odd then the remainder always be (a – 1)

Terminating Decimal

Those numbers which end after a few digits after the decimal like - 0.25, 0.15, 0.375 can be written in a fraction number.

Non-Terminating Decimal

Those numbers which continue after the decimal and can be of two types.

0.3333, 0.7777, 0.183183183…….

Repeating	Numbers that never end after the decimal, but repeat, till infinity. It can be written in fractions.
Non Repeating Decimal	Numbers that never end after the decimal point, but they do not repeat their numbers.

Recurring Decimal Fraction

That decimal fraction is the repetition of one or more digits after the decimal point, then one or more digits are repeated after the dot.

Eg. $\frac{1}{3} = 0.333..., \frac{22}{7} = 3.14285714.....$ To represent such fractions, a line is drawn over the repeating digit.

$$0.3\overline{524} = \frac{3524-35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$$

$$\frac{22}{7} = 3.14285714.... = 3.14\overline{2857}$$

It is called bar.

- **Convert pure recurring decimal fraction to simple fraction as follows –**

$$0.\overline{P} = \frac{P}{9} \qquad 0.\overline{pq} = \frac{pq}{99} \qquad 0.\overline{pqr} = \frac{pqr}{999}$$

- **Convert a mixed recurring decimal fraction to an ordinary fraction as follows –**

$$0.p\overline{q} = \frac{pq-p}{90} \qquad 0.pq\overline{r} = \frac{pqr-pq}{900}$$

$$0.p\overline{qr} = \frac{pqr-p}{990} \qquad 0.pq\overline{rs} = \frac{pqrs-pq}{9900}$$

Example -

(i) $0.\overline{39} = \frac{39}{99} = \frac{13}{33}$

(ii) $0.6\overline{25} = \frac{625-6}{990} = \frac{619}{990}$

(iii) $0.3\overline{524} = \frac{3524-35}{9900} = \frac{3489}{9900} = \frac{1163}{3300}$

Symbol of the Roman Method

1	→	I
2	→	II
3	→	III
4	→	IV
5	→	V
6	→	VI
7	→	VII
8	→	VIII
9	→	IX
10	→	X
20	→	XX
30	→	XXX
40	→	XL
50	→	L
100	→	C
500	→	D
1000	→	M

Rule of Divisibility

Rule of 2	The last digit is an even number or zero (0) as - 236, 150, 1000004
Rule of 3	If the sum of the digits of a number is divisible by 3, then the whole number will be divisible by 3. E.g. 729, 12342, 5631
Rule of 4	Last two digits are zero or divisible by 4. E.g. 1024, 58764, 567800
Rule of 5	The last digit is zero or 5. E.g. 3125, 625, 1250
Rule of 6	If a number is divisible by both 2 and 3 then it is also divisible by 6. E.g. 3060, 42462, 10242
Rule of 7	After multiplying the last digit of a number by 2 and subtracting it from the remaining number, if the number is a multiple of 0 or 7 or if any digit is repeated in a multiple of 6**,** then the number will be divisible by 7**.** E.g. 222222, 44444444444, 7854
Rule of 8	If the last three digits of a number are divisible by 8 or the last three digits are '000' (zero). E.g. 9872, 347000
Rule of 9	If the sum of the digits of a number is divisible by 9, then the whole number will be divisible by 9.
Rule of 10	The last digit should be zero (0).
Rule of 11	If the difference between the sum of digits at odd places and sum of digits at even places is zero (0) or 11 or a multiple of 11. E.g. 1331, 5643, 8172659
Rule of 12	Composite form of divisible by 3 and 4.
Rule of 13	Repeating the digit 6 times, or multiplying the last digit by 4 and adding it to the remaining number, if the number is divisible by 13, then the whole number will be divisible by 13**.** E.g. 222222, 17784

Practice Questions

Q.1 If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of that number, then what will be $\frac{5}{3}$ of that number?

(a) 12 (b) 18

(c) 15 (d) 20

Q.2 If the sum of two numbers is a and their product is a then their reciprocals will be –

(a) $\frac{1}{a}+\frac{1}{b}$ (b) $\frac{b}{a}$

(c) $\frac{a}{b}$ (d) $\frac{a}{ab}$

Q.3 The sum of two numbers is 75 and their difference is 25, then what will be the product of those two numbers?

(a) 1350 (b) 1250

(c) 1000 (d) 125

Q.4 Divide 150 into two parts such that the sum of their reciprocal is $\frac{3}{112}$. Calculate both parts.

(a) 50, 90 (b) 70, 80

(c) 60, 90 (d) 50, 100

Q.5 If the sum of any three consecutive odd natural numbers is 147, then the middle number will be –

(a) 47 (b) 48

(c) 49 (d) 51

Q.6 If the product of first three and last three of 4 consecutive prime numbers is 385 and 1001, then find the greatest prime number.

Q.7 What will be the sum of the even numbers between 50 and 100?

Q.8 What will be the sum of odd numbers between 50 and 100?

Q.9 In a division method, the divisor is 12 times the quotient and 5 times the remainder. Accordingly, if the remainder is 36, then what will be the dividend?

(a) 2706 (b) 2796

(c) 2736 (d) 2826

Q.10 What is the unit digits of $(3694)^{1739} \times (615)^{317} \times (841)^{491}$

(a) 0 (b) 2

(c) 3 (d) 5

Q.11 What will be written in the form of $\frac{p}{q}$ of 18.484848....?

(a) $\frac{462}{25}$ (b) $\frac{610}{33}$

(c) $\frac{200}{11}$ (d) $\frac{609}{33}$

Q.12 Put $\frac{0.\overline{936}-0.\overline{568}}{0.\overline{45}+2.\overline{67}}$ in the form of rational number.

Q.13 What will be the common factor of $\left\{(127)^{127}+(97)^{127}\right\}$ and $\left\{(127)^{97}+(97)^{97}\right\}$?

(a) 127 (b) 97

(c) 30 (d) 224

Answer Key

Q.1 (d) **Q.2 (c)** **Q.3 (b)** **Q.4 (b)**

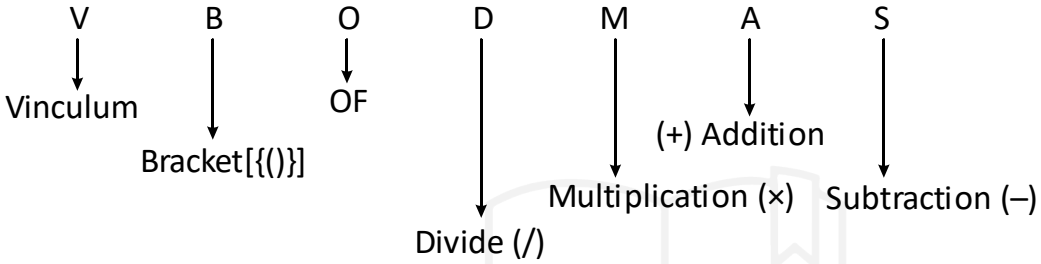
Q.5 (c) **Q.6 13** **Q.7 1800** **Q.8 1875**

Q.9 (c) **Q.10 (a)** **Q.11 (b)** **Q.12** $\frac{2024}{17205}$

Q.13 (d)

2 CHAPTER Simplification

- In simplification, we represent the given data in a simple form, such as the data is done in fraction, in decimal, in division, in power and by solving or changing the mathematical operation.
- If different types of operations are given on some number, then how can we solve it so that the answer to the question is correct, for that there is a rule which we call the rule of VBODMAS.
- Which operation we should do first, it decides the rule of VBODMAS.



- The first of all these mathematical operations is V which means Vinculum (line bracket). If there is a line bracket in the question, then first we will solve it and then (BODMAS) Rule will work in it.
- B (Bracket) in the second place means brackets which can be –
  1. Small bracket ( )
  2. Middle/curly bracket { }
  3. Big bracket/ [ ]
- First the small brackets, then the curly bracket, and then the big brackets are solved.
- In the third place is "O" which is formed from "of" or "order", which means "multiply" or "of".
- In the fourth place is "D" which means "Division", in the given expression do the first division in different actions if given.
- There is "M" in the fifth place which means "Multiplication", in the given expression after "Division" we will do "Multiplication".
- Sixth position is held by "A" which is related to "Addition". Addition action takes place after division and multiplication.
- There is "S" in the seventh place which is made of "Subtraction".

Q. Simplify –

$$\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\,4\frac{1}{3}\right)$$

Sol: Step 1 – Convert the mixed fraction into simple fraction

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\,\frac{13}{3}\right)$$

Now, according to VBODMAS –

Step 2 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{3\text{-}2}{12}\right)\right\}\right]\div\left(\frac{1}{2}\text{of}\,\frac{13}{3}\right)$$

Step 3 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{1}{12}\right)\right\}\right]\div\frac{13}{6}$$

Step 4 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\left(\frac{30-1}{12}\right)\right\}\right]\div\frac{13}{6}$$

Step 5 –

$$\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\frac{29}{12}\right\}\right]\div\frac{13}{6}$$

Step 6 –

$$\left[\frac{13}{4}\div\left\{\frac{30-29}{24}\right\}\right]\div\frac{13}{6}$$

Step 7 –

$$\left[\frac{13}{4}\div\frac{1}{24}\right]\div\frac{13}{6}$$

Step 8 –

$$\left[\frac{13}{4}\times 24\right]\div\frac{13}{6}$$

Step 9 –

$$13\times 6\times\frac{6}{13}$$

= 36 Ans.

Algebraic Formulas –

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a – b)^2 = a^2 – 2ab + b^2$
3. $(a + b)^2 + (a – b)^2 = 2(a^2 + b^2)$
4. $(a^2 – b^2) = (a + b)(a – b)$
5. $a^2 + b^2 + c^2 = (a + b + c)^2 – 2(ab + bc + ca)$
6. $a^2+\frac{1}{a^2}=\left(a+\frac{1}{a}\right)^2-2$
7. $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}\left[(a-b)^2+(b+c)^2+(c-a)^2\right]$
8. $a^3 + b^3 = (a + b)^3 – 3ab(a + b) = (a + b)(a^2 – ab + b^2)$
9. $a^3 – b^3 = (a – b)^3 + 3ab(a – b) = (a – b)(a^2 + ab + b^2)$
10. $a^3 + b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ca)$

$$=\frac{1}{2}(a+b+c)\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}$$

If $a + b + c = 0$, then

$a^3 + b^3 + c^3 = 3abc$

11. $a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$
12. $a^3-\frac{1}{a^3}=\left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)$

Square and Square Root Table

Square	Square Root	Square	Square Root
$1^2 = 1$	$\sqrt{1}=1$	$16^2 = 256$	$\sqrt{256}=16$
$2^2 = 4$	$\sqrt{4}=2$	$17^2 = 289$	$\sqrt{289}=17$
$3^2 = 9$	$\sqrt{9}=3$	$18^2 = 324$	$\sqrt{324}=18$

$4^2 = 16$	$\sqrt{16} = 4$	$19^2 = 361$	$\sqrt{361} = 19$
$5^2 = 25$	$\sqrt{25} = 5$	$20^2 = 400$	$\sqrt{400} = 20$
$6^2 = 36$	$\sqrt{36} = 6$	$21^2 = 441$	$\sqrt{441} = 21$
$7^2 = 49$	$\sqrt{49} = 7$	$22^2 = 484$	$\sqrt{484} = 22$
$8^2 = 64$	$\sqrt{64} = 8$	$23^2 = 529$	$\sqrt{529} = 23$
$9^2 = 81$	$\sqrt{81} = 9$	$24^2 = 576$	$\sqrt{576} = 24$
$10^2 = 100$	$\sqrt{100} = 10$	$25^2 = 625$	$\sqrt{625} = 25$
$11^2 = 121$	$\sqrt{121} = 11$	$26^2 = 676$	$\sqrt{676} = 26$
$12^2 = 144$	$\sqrt{144} = 12$	$27^2 = 729$	$\sqrt{729} = 27$
$13^2 = 169$	$\sqrt{169} = 13$	$28^2 = 784$	$\sqrt{784} = 28$
$14^2 = 196$	$\sqrt{196} = 14$	$29^2 = 841$	$\sqrt{841} = 29$
$15^2 = 225$	$\sqrt{225} = 15$	$30^2 = 900$	$\sqrt{900} = 30$

Cube and Cube Root Table

Cube	Cube Root	Cube	Cube Root
$1^3 = 1$	$\sqrt[3]{1} = 1$	$16^3 = 4096$	$\sqrt[3]{4096} = 16$
$2^3 = 8$	$\sqrt[3]{8} = 2$	$17^3 = 4913$	$\sqrt[3]{4913} = 17$
$3^3 = 27$	$\sqrt[3]{27} = 3$	$18^3 = 5832$	$\sqrt[3]{5832} = 18$
$4^3 = 64$	$\sqrt[3]{64} = 4$	$19^3 = 6859$	$\sqrt[3]{6859} = 19$
$5^3 = 125$	$\sqrt[3]{125} = 5$	$20^3 = 8000$	$\sqrt[3]{8000} = 20$
$6^3 = 216$	$\sqrt[3]{216} = 6$	$21^3 = 9261$	$\sqrt[3]{9261} = 21$
$7^3 = 343$	$\sqrt[3]{343} = 7$	$22^3 = 10648$	$\sqrt[3]{10648} = 22$
$8^3 = 512$	$\sqrt[3]{512} = 8$	$23^3 = 12167$	$\sqrt[3]{12167} = 23$
$9^3 = 729$	$\sqrt[3]{729} = 9$	$24^3 = 13824$	$\sqrt[3]{13824} = 24$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$	$25^3 = 15625$	$\sqrt[3]{15625} = 25$
$11^3 = 1331$	$\sqrt[3]{1331} = 11$	$26^3 = 17576$	$\sqrt[3]{17576} = 26$
$12^3 = 1728$	$\sqrt[3]{1728} = 12$	$27^3 = 19683$	$\sqrt[3]{19683} = 27$
$13^3 = 2197$	$\sqrt[3]{2197} = 13$	$28^3 = 21952$	$\sqrt[3]{21952} = 28$
$14^3 = 2744$	$\sqrt[3]{2744} = 14$	$29^3 = 24389$	$\sqrt[3]{24389} = 29$
$15^3 = 3375$	$\sqrt[3]{3375} = 15$	$30^3 = 27000$	$\sqrt[3]{27000} = 30$

Arithmetic Progression

The series in which each term can be found by adding or subtracting with its preceding term is

called the arithmetic progression.

E.g. 2, 5, 8, 11, …….

n^{th} term of an Arithmetic Progression

$T_n = a + (n - 1)\, d$

Where, a = First term

d = Common difference (2^{nd} term – 1^{st} term)

n = Number of all terms.

Addition of n^{th} terms of an Arithmetic Progression –

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

If the first and last term is known –

$$S_n = \frac{n}{2}[a+\ell]$$

Where, $\ell =$ Last term

Arithmetic progression between the two variables

$A = \frac{a+b}{2}$ [The arithmetic progression of a & b is A]

Geometric Progression

If the ratio of each term of the series to its preceding term is a certain variable, then it is called a geometric series. This fixed variable is called the common ratio.

n^{th} term of Geometric Series –

$T_n = a.r^{n-1}$

Where, a = First term
r = Common ratio
n = Number of terms

Addition of n^{th} terms of Geometric Series –

$$S_n = a\left(\frac{1-r^n}{1-r}\right); \text{ When } r<1$$

$$S_n = a\left(\frac{r^n-1}{r-1}\right); \text{ when } r>1$$

1. Geometric series between two variables $G = \sqrt{ab}$
2. If the arithmetic mean and geometric mean between two positive quantities a and b are A and G, then A > G, $\frac{a+b}{2} > \sqrt{ab}$

Harmonic Progression

If the reciprocals of the terms of a series are written in the same order and it is in arithmetic progression, then this is known as harmonic series.

n^{th} term of a Harmonic Progression –

$$T_n = \frac{1}{a+(n-1)d}$$

Harmonic series (H) = $\frac{2ab}{a+b}$

Relation between Arithmetic Mean, Geometric Mean and Harmonic Mean

Let A, G and H be the arithmetic mean, geometric mean and harmonic mean between two
quantities a and b respectively, then

$\boxed{G^2 = AH}$ and $\boxed{A > G > H}$

Practice Question

Q.1 The value of $24 \times 2 \div 12 + 12 \div 6$ of $2 \div (15 \div 8 \times 4)$ of $(28 \div 7$ of $5)$ is –

(a) $4\frac{32}{75}$ (b) $4\frac{8}{75}$
(c) $4\frac{2}{3}$ (d) $4\frac{1}{6}$

Q.2 Simplify –

$$\left[3\frac{1}{4} \div \left\{1\frac{1}{4} - \frac{1}{2}\left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6}\right)\right\}\right] \div \left(\frac{1}{2} \text{ of } 4\frac{1}{3}\right)$$

Q.3 Evaluate –

$$2\frac{3}{4} \div 1\frac{5}{6} \div \frac{7}{8} \times \left(\frac{1}{3} + \frac{1}{4}\right) + \frac{5}{7} \div \frac{3}{4} \text{ of } \frac{3}{7}$$

(a) $\frac{56}{77}$ (b) $\frac{49}{80}$
(c) $\frac{2}{3}$ (d) $3\frac{2}{9}$

Q.4 If $(102)^2 = 10404$ then the value of $\sqrt{104.04} + \sqrt{1.0404} + \sqrt{0.010404}$ is equals to?

(a) 0.306 (b) 0.0306
(c) 11.122 (d) 11.322

Q.5 If a = 64 & b = 289 then find the value of $\left(\sqrt{\sqrt{a}+\sqrt{b}} - \sqrt{\sqrt{\sqrt{b}-\sqrt{a}}}\right)^{\frac{1}{2}}$

(a) $2^{1/2}$ (b) 2
(c) 4 (d) -2

Q.6 The cube root of 175616 is 56 then find the value of $\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$?

(a) 0.168 (b) 62.16

(c) 6.216 (d) 6.116

Q.7 What is the smallest number to be added to 710 so that the sum becomes a perfect cube?

(a) 29 (b) 19

(c) 11 (d) 21

Q.8 Find the value of the following –

$$4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}$$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$

(c) $\frac{1}{16}$ (d) $\frac{1}{32}$

Q.9 If $2=x+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}$ then find the value of x ?

(a) $\frac{18}{17}$ (b) $\frac{21}{17}$

(c) $\frac{13}{17}$ (d) $\frac{12}{17}$

Q.10 $999\frac{998}{999}\times 999$ equals to ?

(a) 998999 (b) 999899

(c) 989999 (d) 999989

Q.11 Find the value of $\frac{(0.03)^2-(0.01)^2}{0.03-0.01}$?

(a) 0.02 (b) 0.004

(c) 0.4 (d) 0.04

Q.12 $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2$ equals to ?

(a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$

(c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$

Q.13 Find the value of

$$\frac{0.051\times 0.051\times 0.051+0.041\times 0.041\times 0.041}{0.051\times 0.051-0.051\times 0.041+0.041\times 0.041}$$

(a) 0.92 (b) 0.092

(c) 0.0092 (d) 0.00092

Q.14 Find the sum of all the multiples of 3 less than 50 ?

(a) 400 (b) 408

(c) 404 (d) 412

Q.5 How many terms are there in the following arithmetic series?

7, 13, 19, ... , 205

Q.16 If the sum of two numbers is 22, and the sum of their squares is 404, then find the product of those numbers?

(a) 40 (b) 44

(c) 80 (d) 89

Q.17 When a two digit number is multiplied by the sum of its digits, the product is 424. When the number obtained by interchanging its digits is multiplied by the sum of the digits, the result is 280. What is the sum of the digits of the number?

(a) 7 (b) 9

(c) 6 (d) 8

Answer Key

Q.1 (d)	**Q.2** $7\frac{1}{5}$	**Q.3 (d)**	**Q.4 (d)**
Q.5 (a)	**Q.6 (c)**	**Q.7 (b)**	**Q.8 (a)**
Q.9 (b)	**Q.10 (a)**	**Q.11 (d)**	**Q.12 (c)**
Q.13 (b)	**Q.14 (b)**	**Q.15 34**	**Q.16 (a)**
Q.17 (d)			

3 CHAPTER Percentage

- Percentage means 'Per hundred'.
- The fraction whose denominator is 100**,** is called percentage and the numerator of that fraction is called percentage rate.
- 5 out of 100 $= \frac{5}{100} = 5\%$
- 10 out of 100 $= \frac{10}{100} = 10\%$
- That is, when an amount is compared with 100**,** it is called percentage. The basis with which the comparison is made. In the fraction, the base will be called the denominator.

Conversion From Percentage to Fraction

$100\% = 1$	$10\% = \frac{1}{10}$	$5\frac{5}{19}\% = \frac{1}{19}$
$50\% = \frac{1}{2}$	$9\frac{1}{11}\% = \frac{1}{11}$	$5\% = \frac{1}{20}$
$33\frac{1}{3}\% = \frac{1}{3}$	$8\frac{1}{3}\% = \frac{1}{12}$	$4\frac{1}{6}\% = \frac{1}{24}$
$25\% = \frac{1}{4}$	$7\frac{9}{13}\% = \frac{1}{13}$	$4\% = \frac{1}{25}$
$20\% = \frac{1}{5}$	$7\frac{1}{7}\% = \frac{1}{14}$	$2\frac{1}{2}\% = \frac{1}{40}$
$16\frac{2}{3}\% = \frac{1}{6}$	$6\frac{2}{3}\% = \frac{1}{15}$	$37\frac{1}{2}\% = \frac{3}{8}$
$14\frac{2}{7}\% = \frac{1}{7}$	$6\frac{1}{4}\% = \frac{1}{16}$	$62\frac{1}{2}\% = \frac{5}{8}$
$12\frac{1}{2}\% = \frac{1}{8}$	$5\frac{15}{17}\% = \frac{1}{17}$	$57\frac{1}{7}\% = \frac{4}{7}$
$11\frac{1}{9}\% = \frac{1}{9}$	$5\frac{5}{9}\% = \frac{1}{18}$	$66\frac{2}{3}\% = \frac{2}{3}$
$80\% = \frac{4}{5}$	$75\% = \frac{3}{4}$	

Note -

(i) To convert a fraction or a decimal fraction or an integer into a percentage, multiply it by 100.

(ii) To convert a percentage to a fraction, divide it by 100.

- One number is what percent of another number –

$$\%\ \text{Quantity} = \frac{\text{Given number}}{\text{Basic}\left(\text{another number}\right)} \times 100$$

Ex.1 What percent of 48 is 6 ?

Sol. Let x % of 48 = 6

Now, $48 \times \frac{x}{100} = 6$

$\frac{12x}{25} = 6$

$x = \frac{6 \times 25}{12} = 12\frac{1}{2}$

The percent $12\frac{1}{2}\%$ of 48 is 6.

Ex.2 Convert $\frac{9}{16}$ into percentage.

Sol. $\frac{9}{16} = \left(\frac{9}{16} \times 100\right)\% = \frac{225}{4}\% = 56\frac{1}{4}\%$

Ex.3 When 60 is subtracted from 60% of a number, the result is 60. The number is-

(a) 120 (b) 150

(c) 180 (d) 200

Sol. **(b)**

Let the number be x, then –

$X \times 60\% - 60 = 60$

$\Rightarrow 0.60x - 60 = 60$

$\Rightarrow 0.60x = 120$

$\Rightarrow x = \frac{120}{0.60}$

$X = 200$

$\therefore$ The number is 200.

If there is a percentage change

Case I – If there is increase of x_1 % & x_2 % then –

Percentage increase = $x_1 + x_2 + \frac{x_1 . x_2}{100}$

Case II – If there is decrease of x_1 % & x_2 % then –

Percent decrease = $x_1 + x_2 - \frac{x_1 . x_2}{100}$

Case III – If there is increase of x_1 % and decrease of x_2 % then –

Percentage change = $x_1 - x_2 - \frac{x_1 . x_2}{100}$ $\begin{bmatrix} x_1 = \text{ Always percentage increase} \\ x_2 = \text{ Always percentage decrease.} \end{bmatrix}$

Note- If you want to calculate the percentage change in expenses, sales income, revenue, area, etc., then the same rule will be used.

Ex.1 Two successive price increases of 10% and 10% of an article are equal to which one single price increase?

(a) 19% (b) 20%

(c) 21% (d) 22%

Sol. **(c)**

Let the initial price of any item = 100

New price = 110 % of 100 of 110%.

$\left(100 \times \frac{110}{100} \times \frac{110}{100}\right) = 121 = 21\%$

Hence, the single price increase is equal to 21 percent.

Ex.2 The price of laptop is increased by 25%. Now by what percent is the price increased for the second time so that the total increase becomes 35%?

(a) 7.5 (b) 9

(c) 8 (d) 10

Sol. **(c)**

Given that –

Price increased by 25% = x

The overall increase was 35%.

Formula:

Total Increment = x + y +xy/100

Calculation:

$\Rightarrow$ 35 = 25 + y + (25 × y/100)

$\Rightarrow$ 35 = 25 + y + y/4

$\Rightarrow$ 140 = 100 + 4y + y

$\Rightarrow$ 140 – 100 = 5y

$\Rightarrow$ 40 = 5y

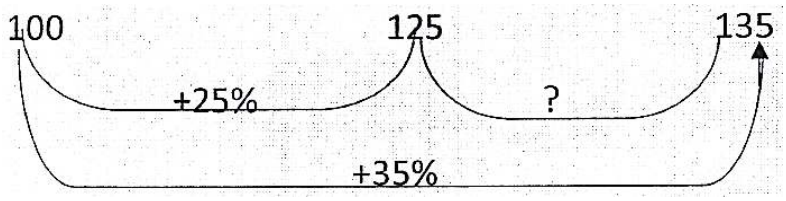
$\Rightarrow$ y = 8

Hence, the second time increase is 8%.

Method II

Let the price of laptop = 100

Price First Increase Second Increase



Percent increase $= \frac{135-125}{125} \times 100$

$= \frac{10}{125} \times 100 = 8\%$

Ex.3 The value of a tool decreases by 20% every year. What will be the cost of that tool after 3 years?

(a) 48.8% (b) 51.2% (c) 54% (d) 60%

Sol. **(a)**

20% = 1/5

Initial	Final
5	4
5	4
5	4
125	64

Required Percentage = $\frac{61}{125} \times 100 = 48.8\%$

Method II

Let the price of tool = x

According to question,

Price after 8 years $= x \times \frac{80}{100} \times \frac{80}{100} \times \frac{80}{100}$

$= \frac{64}{125} x$

Decrease in price $= x - \frac{64}{125}x = \frac{61}{125}x$

Percentage decrease $= \frac{\frac{61}{125}x}{x} \times 100 = 48.8\%$

Formula Related to Population

- If the population of a city is P and it is increasing at the rate of x% per annum, then the population after n years –

$$= P\left(1 + \frac{x}{100}\right)^n$$

- If it is decreasing, then the population

$$= P\left(1 - \frac{x}{100}\right)^n$$

Ex.1 If the population of a city is increasing at the rate of 4% per annum and the present number of people is 15625, then what will be the population after 3 years?

Sol. Required Population $= 15625\left(1 + \frac{4}{100}\right)^3$

$$= 15625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} = 17576$$

Ex.2 The population of a city is 8000. If the number of males increases at the rate of 6 percent and the rate of increase in the number of females is 10 percent, then the population will become 8600. Find the number of females in the city.

Sol. Let the number of females = x

Then, 110 % of x + 106% of (8000 + x) = 8600

$$\frac{110x}{100} + \frac{106(8000 - x)}{100} = 8600$$

x (110 – 106) = 8600 × 100 – 8000 × 106

$$\therefore x = \frac{8600 \times 100 - 8000 \times 106}{110 - 106}$$

$$= \frac{12000}{4} = 3000$$

Ex.3 In 1988 the population of a city decreased by 12%. In 1989 it increased by 15%. What was the overall impact on the city's population in the early 1990s?

Sol. % Impact = % Increase – % Decrease – $\frac{(\%\text{Increase})(\%\text{ Decrease})}{100}$

$$= 15 - 12 - \frac{15 \times 12}{100} = 15 - 13.8 = 12$$

$\therefore$ The population increased by 1.2%.

- If x percent of any amount is taken by the first person, y percent of the balance is taken by the second person and after taking z percent of the remaining amount by the third person, if A is left, then the total amount initially –

$$= \frac{A \times 100 \times 100 \times 100}{(100 - x)(100 - y)(100 - z)}$$

- An increase or decrease in the price of a commodity, a decrease or increase in it consumption –

(a) % Increase in consumption $= \frac{100 \times \text{decrease}}{100 - \text{decrease}}$

(b) % Decrease in consumption $= \frac{100 \times \text{increase}}{100 + \text{increase}}$

Ex.1 If the price of sugar is increased by 40%, by what percent should a family reduce it annual consumption of sugar so that the expenditure of the family does not increase?

(a) $24\frac{4}{7}\%$ (b) $28\frac{4}{7}\%$

(c) $29\frac{4}{7}\%$ (d) $30\frac{4}{7}\%$

Sol. **(b)**

Let , the consumption initially was = 100 units and price per unit = ₹100.

Initially total expenditure = ₹ (100 × 100) = ₹ 10000

New price = 140 per units and let the new consumption ¾ (100 – X) units.

Now, expenditure = 140 × (100 – x) × (14000 – 140 x)

$14000 - 140x = 10000$

$\Rightarrow 140x = 4000$

$\Rightarrow x = \frac{4000}{140} = \frac{200}{7}\% = 28\frac{4}{7}\%$

Decrease in consumption = $28\frac{4}{7}\%$

Method II

Decrease in comsumption

$$\%\,\text{Decrease} = \frac{100 \times \text{Increase}}{100 + \text{Increase}}$$

$$\%\,\text{Decrease} = \frac{100 \times 40}{100 + 40}$$

$$= \frac{4000}{140} = \frac{200}{7}$$

$$= 28\frac{4}{7}\%$$

Ex.2 A 10% reduction in the price of sugar allows a housewife to buy 6.2 kg more sugar for ₹ 1116. What is the reduced price of sugar per kg?

(a) ₹ 12 (b) ₹ 14

(c) ₹ 16 (d) ₹ 18

Sol. **(d)**

Let the price initially = X per kg.

New price $= \frac{90x}{100}$ Rs./kg. $= \frac{9x}{10} =$ Per kg.

$$\frac{116}{(9x/10)} - \frac{1116}{x} = 6.2$$

$$\Rightarrow \frac{1240}{x} - \frac{1116}{x} = 6.2$$

$6.2x = (1240 - 1116) = 124$

$\Rightarrow x = \frac{124}{6.2} = \frac{1240}{62} = 20$

Reduced price = $\left(\frac{90}{100} \times 20\right)$ per kg. = ₹ 18 per kg.

Method II

For the current price –

Rs. × % change = Weight (Less/More)

$1116 \times \frac{10}{100} = 6.2$

$\Rightarrow \frac{1162}{62} = 18$ ₹ $\Rightarrow$ Current price.

Ex.3 If the price of petrol is reduced by 10%, by how much will a consumer have to increase the consumption of petrol so that his expenditure on petrol does not decrease?

(a) $11\frac{1}{9}\%$ (b) $12\frac{1}{3}\%$

(c) $10\frac{1}{2}\%$ (d) 14%

Sol. **(a)**

Let the initial consumption of petrol = 100 Units and it's price = ₹100 per unit.

Total price = ₹ (100 x 100) =₹ 10000

New price = ₹90 per unit.

Let, new consumption = (100 + X) units.

Now, the total price = (₹100 + x) × 90]
= ₹ (9000 + 90x)
∴ 9000 + 90x = 10000 = 90x = 1000

$\Rightarrow x = \frac{1000}{90} = \frac{100}{9} = 11\frac{1}{9}$

Increase in consumption = $11\frac{1}{9}\%$

Method II

Increase in consumption

$= \frac{100 \times \text{Decrease}}{100 - \text{Decrease}}$

$\%\ \text{Increase} = \frac{100 \times 10}{100 - 10}$

$= \frac{1000}{90} = \frac{100}{9}$

$= 11\frac{1}{9}\%$

- If each side or vertices of an equilateral triangle, each side or diagonal or perimeter of a square, the radius, diameter or circumference of a circle, each side of a cube or a cuboid, the radius or diameter of a sphere or a hemisphere, etc., increase or decrease by x percent then the percentage decrease or increase in it's area is –

$= 2x \pm \frac{x^2}{100}$ $\quad \begin{bmatrix} +\text{Increase} \\ -\text{Decrease} \end{bmatrix}$

Ex.1 When the radius is increased by 25%, then find the percentage increase in the area of the circle.

(a) 50% (b) 56.25%
(c) 56% (d) 56.15%

Sol. Gradual increase = A + B + (AB/100)
= 25 + 25 + (25 × 25/100)
= 50 + 6.25 = 56.25%
∴ % increase in the area = 56.25%

Ex.2 By what percent will the surface area of a cube increase when each of it's time is doubled?

(a) 200% (b) 300%
(c) 150% (d) 50%

Sol. **(b)**

We know that -
Total surface area of the cube = $6a^2$
If both the sides are multiplied, then –

$1 \rightarrow 2$
$1 \rightarrow 2$
Total surface area $\rightarrow \underbrace{1}_{3}$ $\quad 4 \leftarrow$ New surface area

∴ The surface area of the cube will increase = $\left(\frac{3}{1} \times 100\%\right)$ = 300%

Ex.3 If the radius and height of the base of a right circular cylinder are increased by 20% each, then by how much will the volume of the cylinder increase?

(a) 40% (b) 60%
(c) 72.80% (d) 96%

Sol. **(c)**

Let the radius and height of the cylinder be r and h respectively.
It's volume, $v = \pi r^2 h$

New radius = $r + \frac{20}{100}r = \frac{6}{5}r$

New height = $h + \frac{20}{100}h = \frac{6}{5}h$

So, new volume

$V_1 = \pi\left(\frac{6}{5}r\right)^2 \times \left(\frac{6}{5}h\right) = \frac{216}{125}\pi r^2 h$

Hence, increase in volume = $\frac{216}{125}\pi r^2 h - \pi r^2 h = \frac{91}{125}\pi r^2 h$

Hence, % increase = $\frac{\frac{91}{125}\pi r^2 h}{\pi r^2 h} \times 100$

$= \frac{91}{125} \times 100$

$= \frac{364}{5} = 72.8$

Method II

Volume of cylinder $= \pi r^2 h$

$= \pi.r.rh$

Here we will use permutation formula twice

First time $= 20 + 20 + \frac{20 \times 20}{100}$

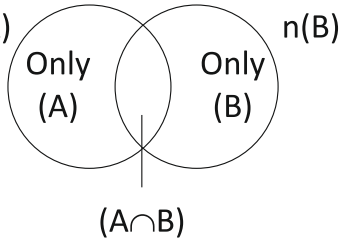
$= 44\%$

Second time $= 44 + 20 + \frac{44 \times 20}{100}$

$= 64 + 8.8 = 72.8\%$

Question Based On Sets

- $n(A \cup B)$ = A and B together or at least one.
- $n(A \cap B)$ = Includes both.



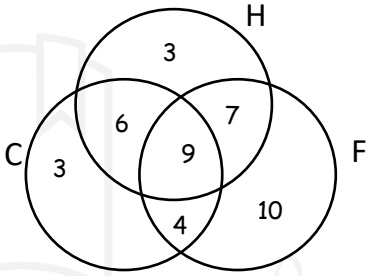
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- Only $A = n(A) - n(A \cap B)$
- Only $B = n(B) - n(A \cap B)$

Ex.1 In a school there are 22 boys in cricket team, 25 in hockey team and 30 in football team. Now if 15 boys play hockey and cricket, 16 boys play hockey and football, 13 boys play football and cricket and 9 boys play hockey, football and cricket, then find the total number of boys who play?

Sol. Let C, H and F be the set of boys playing Cricket, Hockey and Football respectively.

Given that = n(C) = 22, n(H) = 25, n(F) = 30, $n(C \cap H \cap F) = 9$

$n(C \cap H) = 15$, $n(H \cap F) = 16$, $n(C \cap F) = 13$



Now, the number of players playing only C and H = $n(C \cap H) - n(C \cap H \cap F)$

$= 15 - 9 = 6$

Number of players playing only H and F = $n(H \cap F) - n(C \cap H \cap F)$

Number of players playing only C and F = $n(C \cap F) - n(C \cap H \cap F)$

$= 13 - 9 = 4$

Number of players playing only C = 22 – 6 – 9 – 4 = 3

Number of players playing only H = 25 – 6 – 9 – 4 = 6

And, number of players playing only F = 30 – 7 – 9 – 4 = 10

Hence, the total number of players = 3 + 6 + 3 + 9 + 7 + 10 + 4 = 42

Ex.2 In an office 72% of employees like to drink tea and 44% like to drink coffee. If each employee must like one of the two and 40 like both, then what is the total number of employees in the office?

(a) 200 (b) 240

(c) 250 (d) 320

Sol. **(c)**

Let, the total number of employees = x

Now,

$$n(A) = \left(\frac{72}{100} \times x\right) = \frac{18x}{25}, n(B) = \left(\frac{44}{100} \times x\right) = \frac{11x}{25}$$

And, $n(A \cap B) = 40$

$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$x = \left(\frac{18x}{25} + \frac{11x}{25} - 40\right) \Rightarrow \left(\frac{29x}{25} - x\right) = 40$$

$4x = (40 \times 25) \Rightarrow x = 250$

Hence, the total number of employees in the office = 250

Method II

Tea = 72%

Coffee = 44%

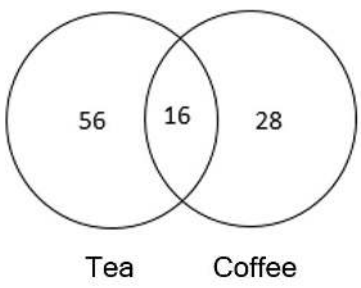
Both (Tea + Coffee) = 72 + 44 + 100

= 116 – 100

= 16%

Total employees $= \frac{40}{16} \times 100$

= 250



Ex.3 In an examination 34% of the students failed in Mathematics and 41% failed in English. If 20% of the students failed in both the subjects, then what is the percentage of students passed in both the subjects?

(a) 44% (b) 50%

(c) 54% (d) 56%

Sol. **(a)**

Let A = set of failed students of Mathematics and B = set of failed students of English

Then, n(A) = 34, n(B) = 42, $n(A \cap B) = 20$

$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

= (34 + 42 - 20) = 56

Hence, the number of people who failed in one or both the subjects = 56

Percentage of students who passed = (100 – 56)% = 44%