Toppersnotes®
Unleash the topper in you

DDA – Junior Engineer

Civil Engineering

Delhi Development Authority (DDA)

Volume - 8

Junior Engineer (Civil)

Strength of Materials (SOM)

SIERRA INNOVATIONS

Table of Content

1 CHAPTER Introduction

THEORY

1.1 MATERIAL CLASSIFICATION

According to behaviour on loading, material can be classified as:

(1) Elastic : Undergoes deformation when subjected to the external loading and comes back to its original state after removal of load.

(2) Plastic : Material do not regain its original dimensions and the deformation is permanent.

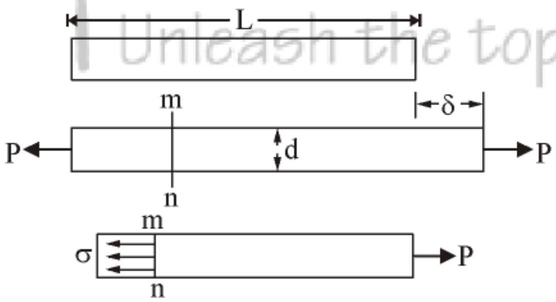
(3) Rigid : Does not undergo any deformation when loaded externally.

In statics and dynamics, we dealt with forces and motions associated with particles and rigid bodies. In strength of materials, we examine the stresses and strains that occur inside real bodies those deform under loads. here, you must appreciate the difference between rigid body and real body.

1.2 STRESS AND STRAIN

1.2.1 Normal Stress :

Consider a prismatic bar loaded by axial forces P at the ends. A prismatic bar is straight structural member having constant cross section throughout its length. The axial force produce a uniform stretching of bar. Here, bar is said to be in tension.



A section taken perpendicular to longitudinal axis of bar is cross-section. Considering free-body diagram, the tensile force P acts on right hand of free body at the other end is force representing the action of removed part of bar upon the part that remains. These forces are continuously distributed over the cross-section. The intensity of force (i.e. force per unit area) is called the stress and is denoted by, s

Hence, under equilibrium, $F = \sigma A$

$$\Rightarrow \qquad \sigma = \frac{F}{A}$$

The stress is the force of resistance per unit area offered by a body against the deformation.

When the bar is stretched by force P, as shown in figure, the resulting stresses are tensile stresses and if forces are reversed in direction, causing the bar to be compressed, the stresses are compressive stresses.

As stress acts in direction perpendicular to cut surface, it is referred as normal stress. The normal stresses may be tensile or compressive. The shear stress act parallel to the surface. Conventionally the tensile stresses are taken as positive and compressive stresses are negative.

The unit of stress is N/m^2 also referred as pascal.

$$1 \text{ Pa} = 1\text{N/m}^2$$

It can also be expressed as MPa. i.e. N/mm^2.

1.2.2 Normal Strain :

An axially loaded bar undergoes a change in length, becoming longer in tension and shorter when in compression, strain is defined as change is length per unit length.

$$(\text{strain}) \in = \frac{\delta}{L}$$

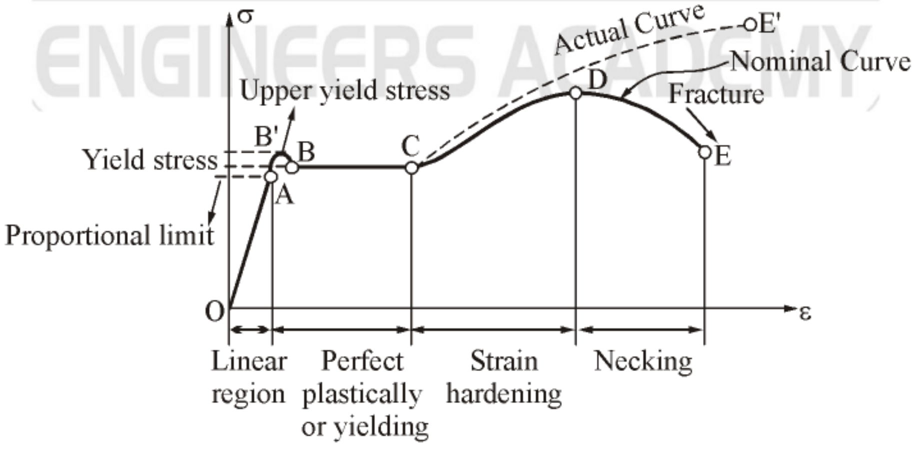
If bar is in tension, the strain is called a tensile strain, representing stretching of material. If the bar is in compression, the strain is compressive strain. The tensile strain is taken as positive and compressive strain is negative. The strain $\in$ is called normal strain because it is associated with normal stresses. As normal strain $\in$ is ratio of two lengths it is a dimensionless quantity i.e. it has no units.

Note: *The definition of normal stress and normal strain are based purely on statical and geometrical consideration. It can be used for load of any magnitude and for any material.*

1.3 STRESS STRAIN DIAGRAM OF MILD STEEL IN TENSION

The mechanical properties of material are determined by test performed on small specimen of the material. The most common material tension is tension test, in which tensile loads are applied on cylindrical specimen. The ASTM (American society for testing and materials) standard tension specimen has diameter of 0.5 in and a gauge length of 2.0 in. The machine used in test is UTM (universal testing machine).

In a static test, the load is applied very slowly and in dynamic test, the rate of loading may be very high. Here, we are analyzing properties based on static test.



The typical stress strain diagram of mild steel is shown in figure. Here, the stress is nominal stress or engineering stress and strain is nominal strain or engineering strain.

$$\text{Nominal (engineering stress)} = \frac{\text{Load}}{\text{Initial cross section area}} = \frac{P}{A_0}$$

$$\text{True stress} = \frac{\text{Load}}{\text{Actual Area}} = \frac{P}{A_a}$$

$$\text{Nominal strain} = \frac{\Delta L}{L_0}$$

$$\text{True strain} = \frac{\Delta L}{L_a}$$

The nominal stress is obtained by dividing the load P by initial cross sectional area A. the true stress is calculated by using the actual area of the bar.

Similarly, for calculation of strain, if initial gauge length is used nominal strain is obtained. If the actual length is used, true strain is obtained.

(1) The diagram begin in with straight line from O to A. In this region the stress and strain are directly propotional and behaviour of material is linearally elastic.

(2) Beyond point A, linear relationship between stress and strain no longer exists. A is called proportional limit.

(3) When load is increased beyond A, the slope of curve become smaller and smaller, unit at point B, the curve becomes horizontal.

(4) From B to C, considerable elongation ocurs with no increase in tensile force. The phenomenon is known as yielding, reigon BC is called as yield platue.

(5) In region CD, the material begin to strain hardening, the material undergoes change in its atomic and crystalline structure, resulting in increased resistance of material to further deformation.

(6) The load reach its maximum value and corresponding stress is called ultimate stress.

(7) The fracture finally occur at point E as shown in figure. Various properties of material can be deduced from stress-strain diagram, stress crresponding to E is called fracture / rupture stress.

1.4 SOME IMPORTANT PROPERTIES OF MATERIAL

1.4.1 Ductility :

The ductility of material by which it can be drawn as wire of small cross-section upon tensioning forces.

The percent elongation is defined as

$$\text{Percent elongation} = \frac{L_f - L_o}{L_o} \times 100$$

$$\text{Percent change in Area} = \frac{A_f - A_o}{A_o} \times 100$$

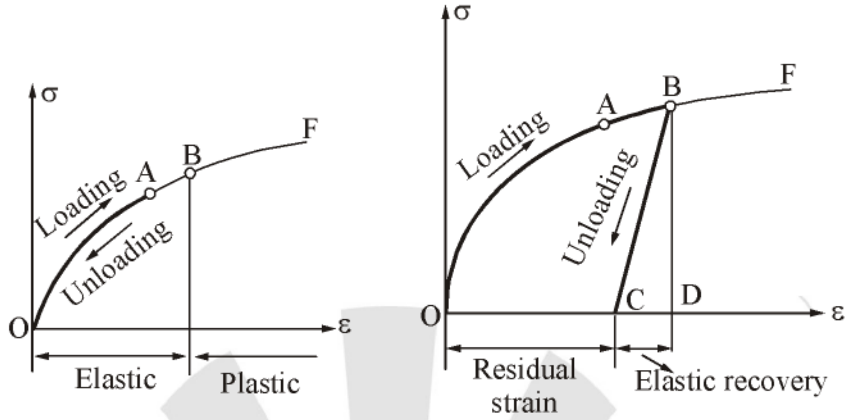
Where, A_0 = Original cross-sectiuon area

A_f = Final area at fracture

The material which fails in tension at lower values of strain are classified as brittle materials.

1.4.2 Elasticity and Plasticity :

When load is slowly removed, two different situation may occur as shown in figure.



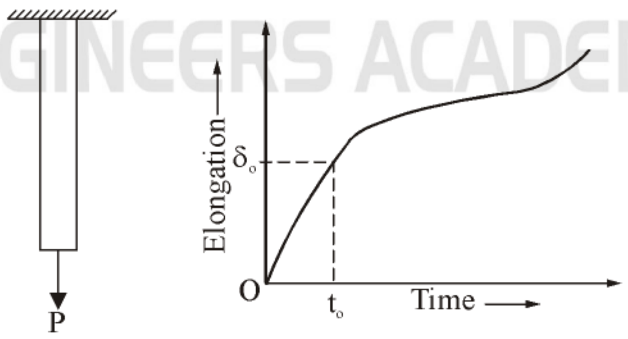
Suppose material is loaded and strain go from O to A. When load is removed, the material follow exactly the same curve back to origin O. This Property of material by which it returns to its original dimension during unloading is called elasticity and material is said to be elastic.

When material is loaded to higher level and when unloading occurs, the material follows line BC (which is parallel to initial slope of OA) on the diagram. When point C is reached, the load has entirely been removed but a residual strain or permanent strain, OC remain in the material. The corresponding residual elongation of bar is called permanent set. Thus, the bar returns partially to original shape, hence the material is said to be partially elastic. Then stress level above which all the strain is not recovered is called elastic limit of material.

The characieristic of material by which it undergoes inelastic strain beyond those at the elastic limit is known as plasticity.

1.4.3 Creep :

When a constant load is applied on a material, over a time, a permanent deformation occures. Which is called creep.



Consider a vertical bar loaded by constant force P. When load P is applied initially, the bar elongates by amount δ_0 during time t_0. Subsequent to time t_0, the load remains constant. However, due to creep, the bar elongatics even though load does not change. Creep is more at higher temperature than at ordinary temperature, i.e., creep is propotional to temperature.

1.5 HOOKE'S LAW

According to hooke's law with in the proportional limit, the stress is proportional directly the strain.

i.e. $\sigma \alpha \in$

or $$\frac{\sigma}{\in} = E$$

E = Young's modulus or modulus of elasticity

When a material behave elastically and also exhibits a linear relationship between stress and strain it is said to be linearly elastic. The linear relationship between stress and strain for bar in simple tension and compression is expressed as:

$$\sigma = E \in$$

Where E is constant of proportionality known as the modulus of elasticity for the material. The modulus of elasticity is slope of stress strain diagram in linearly elastic region. The unit of E is same as the unit of stress.

This equation is known as Hooke's law. The modulus of elasticity E has relatively large values for materials that are stiff. The elasticity of modulus of common material are.

(A) $E_{Steel} = 200$ GPa

(B) $E_{Aluminium} = 80$ GPa

(C) $E_{Wood} = 11$ GPa

The modulus of elasticity is often called Young modulus.

1.5.1 Poisson's Ratio :

When prismatic bar is loaded in tension, the axial elongation is accompanied by lateral contraction (direction normal to applied load). The lateral strain is proportional to axial strain in linear elastic range when material is both homogenous and isotropic.

The ratio of strain in lateral direction to strain in axial direction is known as Poisson's ratio.

$$\nu = \frac{\text{lateral strain}}{\text{axial strain}}$$

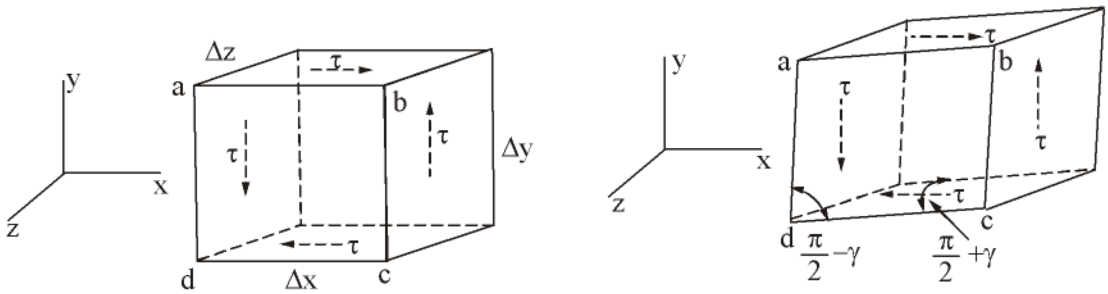
A material is homogenous if it has the same composition throughout the body and Isotropic material have same properties in all direction.

1.5.2 Shear Stress and Strain :

Shear stresses act parallel or tangential to the surface. The shear stress is given as

$$\tau = \frac{F}{A}$$

To understand nature of shear stress, let us consider stress element in form of rectangular parallel block having sides of $\Delta x, \Delta y$ and Δz



Considering force equilibrium total shear force on the top face is $\tau\Delta x\Delta z$ and this force is balanced by equal and opposite force on bottom face. These two force form a couple having moment about z-axis of magnitude $\tau\Delta x\Delta y$ Equilibrium of element requires this moment to be balanced by equal and opposite moment resulting from shear stress acting on side face of element.

This requires the magnitude of shear stress on opposite face of an element to be equal in magnitude and opposite in direction.

Under the action of these stresses, the material is deformed, The angle γ is measure of distortion or change in shape of element and is called shear strain the unit of shear strains are radians.

A shear stress acting on positive face of element is positive if it acts in positive direction of one of the coordinate axis and negative if it acts in the negative direction of the axis. A shear stress acting on a negative face of an element is positive if is acts in the negative direction of an axis and negative if it acts in the positive direction.

Shear strain in an element is positive when the angle between two positive (or negative) faces is reduced. The strain is negative when angle between two positive faces (or two negative) faces is increased.

The shear stress-strain diagram can be plotted in same way as in tension-test diagram. Hooke's law in shear is

$$\tau = G\gamma$$

Where, G = Shear modulus of elasticity or modulus of rigidity.

1.5.3 Bulk Modulus :

If material is subjected to simmilar and equal triaxial stresses, then ratio of stress to volumetrix strain is called bulk modulus:

$$K = \frac{\sigma}{\varepsilon_v}$$

$$= \frac{\sigma E}{3\sigma(1-2v)}$$

$$E = 3K(1-2v)$$

For material to be incompressible

Either $\sigma_x + \sigma_y + \sigma_z = 0$

or $v = 0.5 \Rightarrow$ Not possible

1.5.4 Relation Between Elastic Constants :

$$E = 2G(1 + \nu)$$

$$E = 3K(1 - 2\nu)$$

$$E = \frac{9KG}{3K + G}$$

$$\nu = \frac{3K - 2G}{6K + 2G}$$

Where, E = Modulus of elasticity

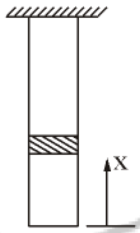
K = Bulk modulus

G = Modulus of rigidity

ν = Possion's ratio

Example: A long wire of specific mass m hang freely under its own weight. Derive formula for tensile stress in the wire.

Solution: Let the weight per unit volume of the rope be w.



Force on the cross section of the elemental part

$$w \times \text{volume below} = wAx$$

$$\therefore \quad \text{stress} = \frac{wAx}{A} = wx$$

Note: *$w = \frac{mg}{v}$ where m = total mass of the whole wire & v = total volume of the whole = Al*

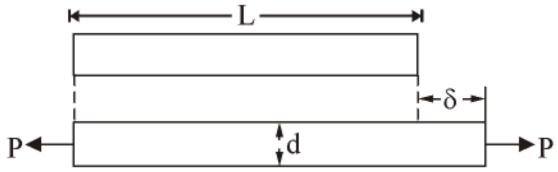
❍❍❍

2 CHAPTER Axially Loaded Members

THEORY

2.1 DEFLECTION OF BAR

Consider a prismatic bar of length L loaded in tension by axial forces P.



$$\sigma = \frac{P}{A}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P}{A}\frac{L}{\delta}$$

$$\Rightarrow \qquad \delta = \frac{PL}{AE}$$

The deflection in bar is $\delta = \frac{PL}{EA}$

$$\text{Energy stored in bar} = \frac{1}{2}P\delta$$

$$= \frac{1}{2}\times P \times \frac{PL}{AE} = \frac{P^2L}{2AE}$$

Where,
P = applied load
L = length
A = Cross-sectional area
E = Modulus of elasticity

The stiffness k of axial loaded bar is defined as force required to produce a unit deflection

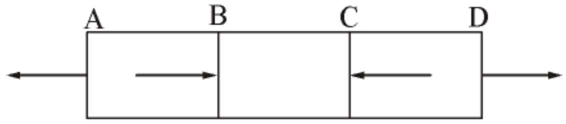
$$P = k = \frac{EA}{L}$$

The flexibility f is defined as deflection due to unit load.

$$f = \frac{L}{EA}$$

2.1.1 Principle of Super position :

According to principle of superposition the resulting strain will be equal to algebraic sum of strain caused by individual forces acting along the length of the member.

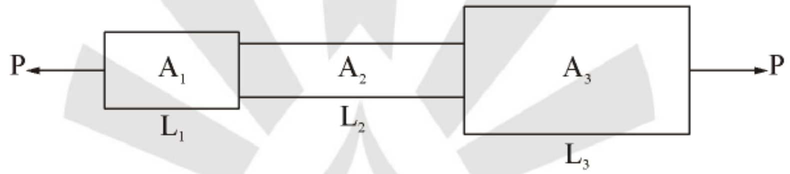


Considering the free body diagram of different section of bar, total deformation can be calculated.

$$\Delta = \Sigma \frac{PL}{AE} = \frac{1}{AE}\left(P_1L_1 + P_2L_2 + ...P_nL_n\right)$$

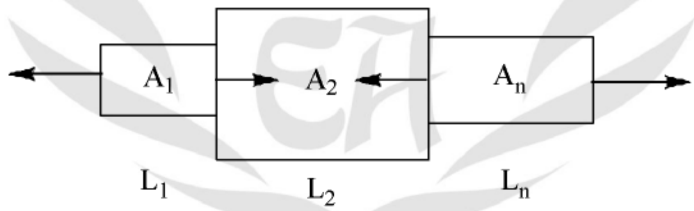
2.1.2 Bar of Varying Cross-section :

When a structural member having varying areas of cross-section along its length is subjected to axial force P, the total deformation is equal to sum of deformation of individual section under action of axial force P.



Total elongation is $\Delta = \Delta_1 + \Delta_2 + \Delta_3 = \dfrac{P}{E}\left(\dfrac{L_1}{A_1} + \dfrac{L_2}{A_2} + \dfrac{L_3}{A_3}\right)$

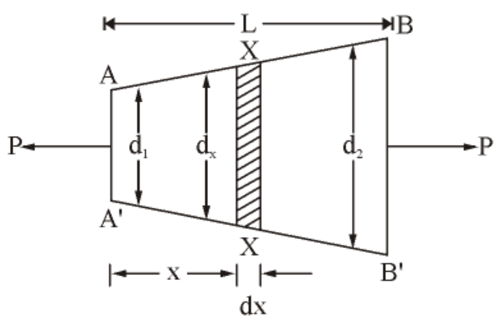
Similarly if bar of varying cross-section is subjected to various forces, both at the ends as well as intermediate points, principle of superposition is applied and total deformation can be computed by drawing the free body diagram of individual section.



$$\Delta = \Sigma\Delta = \frac{1}{E}\left(\frac{P_1L_1}{A_1} + \frac{P_2L_2}{L_2} + ...\frac{P_nL_n}{A_n}\right)$$

2.1.3 Uniformly Tapering Circular Bars :

Let us consider uniform tapering circular bar subjected to axial load P. The bar of length L has diameter d_1 at one end d_2 at other end ($d_1 > d_2$)



Consider an element of length dx and diameter d_x situated at distance x from end A

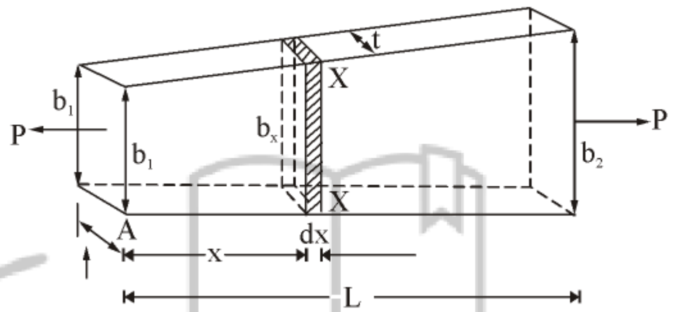
Diameter $$d_x = d_1 + \frac{d_2 - d_1}{L}x = d_1 + kx$$

Where, $$k = \frac{d_2 - d_1}{L}$$

Extension, $$\delta = \frac{Pdx}{\frac{\pi}{4}d_x^2 E}$$

$$\Delta = \int_0^L \frac{4Pdx}{\pi(d_1 + kx)^2 E} = \frac{4PL}{\pi E d_1 d_2}$$

2.1.4 Uniformly Tapering rectangular Bar :



Consider a very short section XX of length dx and width b_x, situated at distance x from end A.

Width $$b_x = b_1 + \left(\frac{b_2 - b_1}{L}\right)x = b_1 + kx$$

Where, $$k = \frac{b_2 - b_1}{L} \quad A' = t \times (b_1 + kx)$$

Area of section XX = $(b_1 + kx)t$

Extension in small element

$$\delta = \frac{Pdx}{(b_1 + kx)tE}$$

Extension in whole length of the rod is

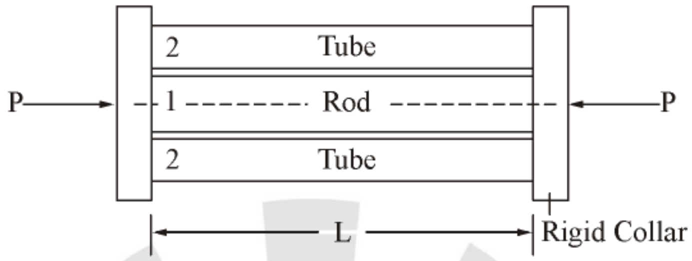
$$\Delta = \int_0^L \frac{Pdx}{(b_1 + kx)tE} = \frac{P}{tE}\frac{1}{k}\left[\log_e(b_1 + kx)\right]_0^L$$

$$= \frac{P}{ktE}\log_e\frac{b_1 + kL}{b_1} = \frac{P}{ktE}\log_e\frac{b_2}{b_1}$$

$$= \frac{PL}{(b_2 - b_1)tE}\log_e\frac{b_2}{b_1}$$

2.1.5 Compound Bars :

A structural member composed of two or more elements of different materials rigidly connected together to form a parallel arrangement and subjected to axial loading is termed as compound bar. These are statically indeterminate, since equation of statics alone provide only one equation for stresses in individual sections. Other equation can be obtained from consideration of deformation of whole structure.



From the condition of equilibrium,

$$P_1 + P_2 = P \qquad \text{...(i)}$$

Since both are shortened by same amount,

$$\Delta_1 = \Delta_2$$

$$\Rightarrow \qquad \frac{P_1L_1}{A_1E_1} = \frac{P_2L_2}{E_2A_2}\left(\because L_1 = L_2\right)$$

$$P_1 = \frac{A_1E_1}{A_2E_2}P_2 \qquad \text{...(ii)}$$

From (1) and (2)

$$P_2 + P_2\frac{A_1E_1}{A_2E_2} = P$$

$$\Rightarrow \qquad P_2\left[1+\frac{A_1E_1}{A_2E_2}\right] = P$$

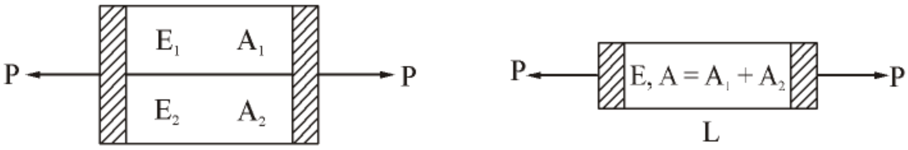
$$P_2 = \frac{P}{1+\frac{A_1E_1}{A_2E_2}}$$

$$P_2 = \frac{PA_2E_2}{A_1E_1 + A_2E_2}$$

Similarly, $$P_1 = \frac{PA_1E_1}{A_1E_1 + A_2E_2}$$

Equivalent Modulus of Compound Bar :

Figure shows a compound bar of length L consisting of two materials of modulus E_1 and E_2 and having area A_1 and A_2. The equivalent bar of length L and made up of material having modulus E and cross-sectional area $A = A_1 + A_2$. Such equivalent bar should have same deformation under the load as that of compound bar.



For equivalent bar $$\Delta = \frac{PL}{AE} = \frac{PL}{(A_1 + A_2)E}$$

$$\frac{PL}{\Delta} = (A_1 + A_2)E \qquad ...(1)$$

For the compound bar,

Since $$\Delta_1 = \Delta_2$$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$\Rightarrow \quad \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

Now, $$P_1 + P_2 = P$$

$$\sigma_2 \frac{E_1}{E_2} A_1 + \sigma_2 A_2 = P$$

$$\Rightarrow \quad \sigma_2 \left[\frac{E_1 A_1 + E_2 A_2}{E_2} \right] = P$$

$$\Rightarrow \quad \sigma_2 = \frac{PE_2}{E_1 A_1 + E_2 A_2}$$

$$A_1 E_1 + A_2 E_2 = P \frac{E_2}{\sigma_2}$$

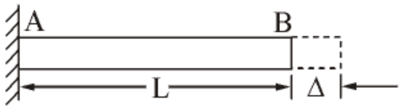
$$A_1 E_1 + A_2 E_2 = \frac{PL}{\Delta} \qquad ...(2)$$

From (1) and (2) $$(A_1 + A_2)E = A_1 E_1 + A_2 E_2$$

$$E = \frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}$$

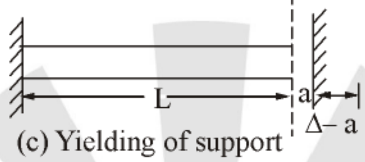
2.2 THERMAL STRESSES

Consider a bar of length L, subjected to uniform temperature increase ΔT, the bar being free to expand,



(a) Bar free to expand

(b) Bar constrained



(c) Yielding of support

(a) When bar is free to expand, there will be no thermal stress in the bar.

The increase in length of bar $\Delta L \; \alpha \; L\Delta T$

Where, α = coefficient of thermal expansion.

(b) When bar is fixed at both ends, the expansion is prevented which result in thermal stresses.

$$\sigma = E\varepsilon$$

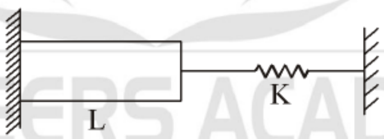
$$= E\frac{\Delta L}{L}$$

$$= \frac{E\alpha L\Delta T}{L} = E\alpha\Delta T$$

This stress will be compressive when change in temperature is positive and stress will be tensile when change in temperature is negative. (when temperature drops.)

Special Cases :

Case-I :



Initially there is no compression is spring. When there is temperature rise ΔT.

As forces in block and spring will be same for system to be in equilibirium.

$$\sigma A = Kx$$

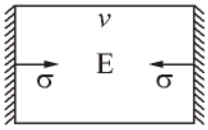
As total deflection of system is equal to zero.

$$-\frac{\sigma L}{E} - \frac{\sigma A}{K} + L\alpha\Delta T = 0$$

$$\sigma\left(\frac{L}{E} + \frac{A}{K}\right) = L\alpha\Delta T$$

So, stress developed in block $\sigma = \dfrac{L\alpha\Delta T}{\left(\frac{L}{E} + \frac{A}{K}\right)}$

Case-II : Consider a block subjected to temperature rise $\Delta T'$ prevented to expand in x direction.



Stress generated in x direction= $E\alpha\Delta T$

$$\text{Strain in y direction} = \alpha\Delta T + v\alpha\Delta T$$

$$= (1 + v)\,\alpha\Delta T$$

It is to be noted that thermal stresses depends on :

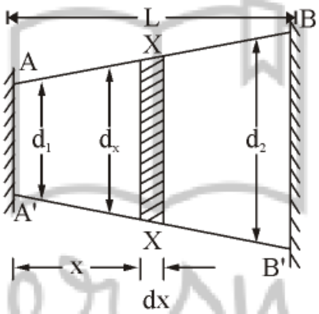
(i) Properties of metal α and E.

(ii) Temperature change ΔT

(iii) It does not depend on properties of cross-section such as L and A.

2.2.1 Temperature Stresses in Bars of Tapering Section :

Consider a bar of uniform tapering section. When the temperature is rasied by ΔT, compressive force P will be induced since the bar is not free to expand. The force is same for all cross section and hence maximum stress will be induced at section AA where diameter is d_1.



If bar were free to expand $\qquad \Delta = L\alpha\Delta T \qquad$ .... (i)

The force induced in bar will be compressive force P which is required to prevent free expansion of Δ.

For an element of length dx, the deformation due to P is

$$\delta = \frac{Pdx}{A_x E}$$

Total deformation
$$\Delta = \int_0^L \frac{pdx}{A_x E}$$

But,
$$A_x = \frac{\pi}{4}\left(d_1 + \frac{d_2 - d_1}{L}x\right)^2 = \frac{\pi}{4}\left(d_1 + kx\right)^2$$

Where,
$$k = \frac{d_2 - d_1}{L}$$

$\therefore$
$$\Delta = \frac{4P}{\pi E}\int_0^L \frac{dx}{\left(d_1 + kx\right)^2}$$

$$= \frac{4PL}{\pi E d_1 d_2} \qquad \text{...(ii)}$$

From (i) and (ii)

$$L\alpha\Delta T = \frac{4PL}{\pi E d_1 d_2}$$

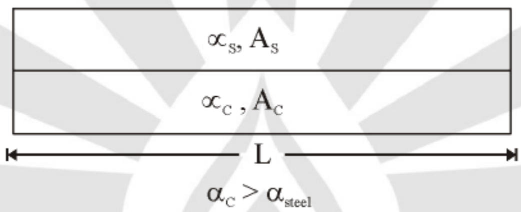
$$P = \frac{\pi E}{4} d_1 d_2 \alpha \Delta T$$

Maximum stress induced at section AA is (Smallest Area)

$$\sigma_{max} = \frac{\frac{\pi E}{4} d_1 d_2 \alpha \Delta T}{\frac{\pi}{4} d_1^2} = \frac{E d_2 \propto \Delta T}{d_1}$$

2.2.2 Composite Section :

Here we will consider the determination of temperature stresses in composite section due to increase in temperature.



Let there is no initial stress and force in composite section. Now, if temperature is increased by T°C, then the free expansion of copper will be greater than that free expansion of steel.

But since actual extension of copper is same as actual extension of steel because bond is not broken.Therefore, due to rise in temperature, copper will be in compression and steel will be in tension.

$$L\alpha_s T < \delta < L\alpha_C T$$

$$L \propto_S T + \frac{\sigma_S}{E_S} L = L\alpha_C T - \frac{\sigma_C}{E_C} L$$

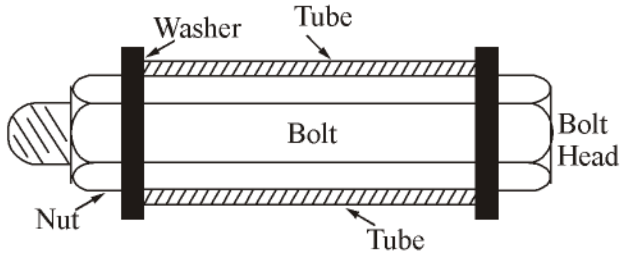
$$\Rightarrow \quad \left(\frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_c}\right) L = (\alpha_c - \sigma_s) T \qquad ...(1)$$

From equilbrium condition, total tension force in steel is equal to total compressive force is copper.

$$\sigma_s A_s = \sigma_c A_c \qquad ...(2)$$

From equation (1) and (2), σ_C and σ_S can be calculated.

2.3 STRESSES IN BOLTS AND NUTS :



If nut is tightened by one revolution, then axial change in length is equal to length of pitch of threads.

$$\Delta L = nP$$

Where, P = Pitch length

n = Number of revolutions

Let, σ_T = Compressive stress in tube

and σ_b = Tensile stress in bolt due to tightening of nut.

Change in length of tube $= \dfrac{\sigma_t}{E_t} L$

Change in length of bolt $= \dfrac{\sigma_b}{E_b} L$

Total change in length $\Delta L = nP$

Change in length of tube + change in length of bolt = nP

$$\left|\frac{\sigma_t}{E_t} L\right| + \left|\frac{\sigma_b}{E_b} L\right| = nP$$

and second equation can be obtained from equilibrium condition

$$\sigma_t A_t = \sigma_b A_b$$

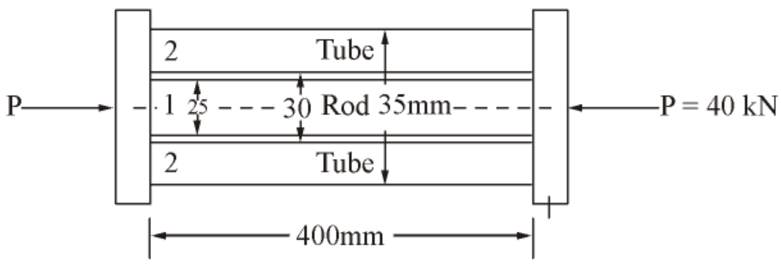
These two equation can be solved to obtain stresses in bolt and tube.

Note: *If nut-bolt assembly and tube undergoes change in temperature, both cases are analysed individualiy and finally algebraic sum is taken.*

Example: A mild steel rod of 25 mm diameter and 400 mm long is encased centrally inside a hollow copper tube of external diameter 35 mm and inside diameter 30 mm. The ends of the rod tube are rigidly attached, and the composite bar is subject to an axial pull of 40 kN.

If E for steel and copper is 200 GN/m^2 and 100 GN/m^2 respectively, find the stress developed into the rod and the tube. Find also the extension of the rod.

Solution: Let us-use suffix 1 for the steel rod and suffix 2 for the copper tube.



$$A_1 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4}\left(35^2 - 30^2\right) = 255.25 \text{ mm}^2$$

From the equilibrium of the bar

$$\sigma_1 A_1 + \sigma_2 A_2 = P = 40 \times 10^3 \quad ...(1)$$

Also, from compatibility, $\sigma_1 \dfrac{L}{E_1} = \sigma_2 \dfrac{L}{E_2}$

or $$\sigma_1 = \sigma_2 \frac{E_1}{E_2} = \sigma_2 \times \frac{200}{100} = 2\sigma_2 \quad ...(2)$$

Substituting the value of P_1 in (1), we get

$$2\sigma_2 A_1 + \sigma_2 A_2 = 40 \times 10^3$$

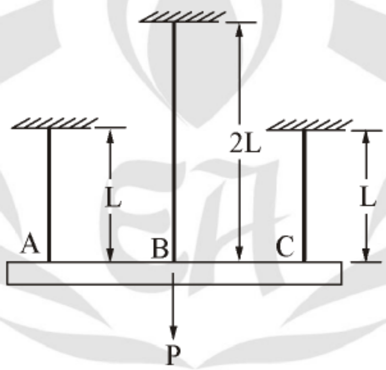
or $$2\sigma_2(490.87) + \sigma_2(255.25) = 40 \times 10^3$$

From which, $\sigma_2 = 32.34 \text{ N/mm}^2$

Hence, $\sigma_1 = 2\sigma_2 = 64.68 \text{ N/mm}^2$

Also, extension of rod $\Delta L = \sigma_1 \dfrac{L}{E} = \dfrac{64.88 \times 400}{2 \times 10^5} = 0.1294 \text{ mm}$

Example: A rigid bar ABC is supported by three rods of the same material and of equal diameter as shown in Fig. Calculate the forces in the bars due to an applied force P on the bar ABC remains horizontal after the load has been applied. Neglect the weight of the rigid bar.



Solution: Let us use suffix 1 for outer bars and suffix 2 for the inner bar.

From statical equilibrium, $2P_1 + P_2 = P$ ....(1)

From compatibility, $\Delta_1 = \Delta_2$

or $$\frac{P_1 L}{AE} = \frac{P_2(2L)}{AE}$$

From which $P_1 = 2P_2$ ...(2)

Substrtuting in (1), we get

$$2(2P_2) + P_2 = P,$$

from which $P_2 = 0.2\ P$

Hence, $P_1 = 2 \times 0.2\ P = 0.4\ P$

Example: A compound bar is made by fastening one flat bar of steel between two similar bars of aluminum alloy. The dimensions of each bar are 40 mm wide × 8 mm thick, so that the cross-section of the composite bar measure 40 mm × 24 mm. If E for steel = 2.04×10^5 N/mm^2 and E for alloy = 0.612×10^5 N/mm^2, find the apparent value of E for the composite bar loaded in tension. If the respective elastic limits are 230 N/mm^2 and 50 N/mm^2 for mild steel and alloy, find the elastic limit of the compound bar.

Solution: Let us use suffix s for steel and a for aluminium alloy.

$\therefore$ $$A_s = (40 \times 8) = 320 \text{ mm}^2$$

and $$A_a = 2 \times 40 \times 8 = 640 \text{ mm}^2$$

$\therefore$ Total area $$A = A_s + A_a$$

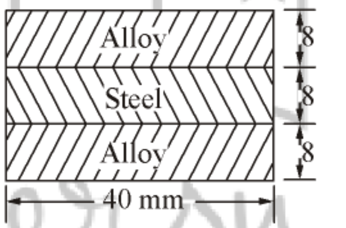
$$= 320 + 640 = 960 \text{ mm}^2$$

$$E = \frac{A_a E_a + A_s E_s}{A_a + A_s}$$

$$= \frac{640 \times 0.612 \times 10^5 + 320 \times 2.04 \times 10^5}{640 + 320}$$

$$= 1.088 \times 10^5 \text{ N/mm}^2$$

The elastic limit of the compound bar is the stress at which one of the members, either steel or aluminium alloy, is stressed to its elastic limit.



Let P be the load at elastic limit of the compound bar, and σ_a and σ_5 be the corresponding stresses in the two materials at the elastic limit of the compound bar.

Since the strains in the two materials are equal,

$$\frac{\sigma_S}{E_S} = \frac{\sigma_a}{E_a}$$

$\Rightarrow$ $$\frac{\sigma_S}{\sigma_a} = \frac{E_s}{E_a} = \frac{2.04 \times 10^5}{0.612 \times 10^5}$$

$$\sigma_s = 3.333\ \sigma_a \qquad \text{...(1)}$$

Also, $$\sigma_S\ A_S + \sigma_a\ A_a = P$$

$\therefore$ $$3.333\ \sigma_a A_s + \sigma_a A_a = P$$

$\therefore$ $$[3.333\sigma_a \times 320 + \sigma_a 640] = P$$

$$\sigma_a = \frac{P}{1706.7}$$

Hence, $$\sigma_S = \frac{P}{512} \qquad \text{...(2)}$$