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UPPSC – Polytechnic Lecturer

Electrical Engineering

Uttar Pradesh Public Service Commission (UPPSC)

Volume - 4

Electromagnetic & Network Theory

SIERRA INNOVATIONS

INDEX

1 CHAPTER Basics of Electromagnetic Theory and Maxwell's Equations

THEORY

1.1 VECTOR ALGEBRA :

There are 3-types of product

(i) Dot Product

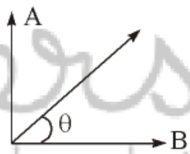
(ii) Cross Product

(iii) Triple Product

1.11 VECTOR PRODUCT :

(i) Dot Product :

The Dot Product of two Vectors $\overline{A}$ and $\overline{B}$ is given by,



$$\overline{A}\,\overline{B} = A \cdot B \cdot \cos\theta$$

Let,
$$\overline{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$
$$\overline{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$$

Thus, Dot product is given by :

$$\overline{A}\,\overline{B} = A_xB_x + A_yB_y + A_zB_z$$

Dot product is a Scalar quantity.

(ii) Cross Product

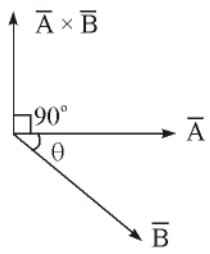
The Cross product of two Vectors $\overline{A}$ and $\overline{B}$ is given by :

$$\vec{A}\times\vec{B} = |\vec{A}|\bullet|\vec{B}|\sin\theta\,\hat{a}_n$$

where, $\hat{a}_n$ = Normal unit vector. (Normal unit vector to AB plane)

$$\overline{A}\times\overline{B} = \hat{a}_x\left(A_yB_z - A_z\bullet B_y\right) - \hat{a}_y\left(A_xB_z - A_zB_x\right) + \hat{a}_z\left(A_xB_y - B_xA_y\right)$$

It is represented in the determinant form as given below:



i.e. $$\overline{A}\times\overline{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cross product is a Vector quantity.

(iii) Triple Product :

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{C}.\vec{A}) - \vec{C}(\vec{A}.\vec{B})$$

1.12 VECTOR OPERATORS :

1.12 Operators :

(1) Gradient Operator is ∇V of scaler v

(2) Divergence operator is $\nabla.\vec{V}$ of vector $\vec{V}$

(3) Curl $\nabla\times A$ of vector $\vec{A}$

(4) Laplacian $\nabla^2 V$ of scalar V

(1) Gradient, Divergence and Curl

Gradient ($\overline{\nabla}$ operator) : operator is given by ∇V

Here del operator $$\overline{\nabla} = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

Gradient is applicable for Scalar fields only.

It given the Rate of change of Scalar field along the different co-ordinate axes.

Example: Gradient of potential field V is given by-

$$\overline{\nabla}V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

where, V is a Scalar field.

Note : *Gradient of a potential field gives the electric field.*

i.e. $\overline{E} = -\overline{\nabla}\cdot V$, where, E is the electric field intensity.

Note : *Gradient of a scalar field is a Vector quantity.*

(2) Divergence :

It is applicable for a Vector field. Divergence of a Vector field gives the flux coming out of a closed surface, when volume of the surface shrinks to zero.

Let, $\overline{D} = D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z$ = Electric flux density

$$\overline{\nabla}\cdot\overline{D} = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\cdot\left(D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z\right)$$

$$\therefore \quad \overline{\nabla}\cdot\overline{D} = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z$$

The above equation represent the divergence of a Vector quantity $(\overline{D})$.

Note : *Divergence of a Vector field is a Scalar field.*

Example : $\overline{\nabla}\cdot\overline{D} = \rho_v$ = Charge density

(3) Curl of a Vector field : The curl of a Vector field.

$\overline{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$ is given by-

$$\overline{\nabla}\times\overline{A} = \left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right)\hat{a}_x - \left(\frac{\partial}{\partial x}A_z - \frac{\partial}{\partial z}A_x\right)\hat{a}_y + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{a}_z$$

For example, The curl of Magnetic field intensity $(\overline{H})$ represened as

$$\overline{H} = H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z$$

can be given by determinant form as shown below-

$$\overline{\nabla}\times\overline{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(\frac{\partial}{\partial y}H_z - \frac{\partial}{\partial z}H_y\right)\hat{a}_x - \left(\frac{\partial}{\partial x}H_z - \frac{\partial}{\partial z}H_x\right)\hat{a}_y + \left(\frac{\partial}{\partial x}H_y - \frac{\partial}{\partial y}H_x\right)\hat{a}_z$$

Note : *Curl of a Vector field is a Vector quantity.*

Example : $\overline{\nabla}\times\overline{H} = J$ = Current density.

(4) Laplacian (∇^2) :

Laplacian of scalar V is divergence of gradient laplacian

$$\nabla^2 V = \nabla.\nabla V = \text{Divergence (Gradient V)}$$

For cartesion Cordinate :

Laplacian $$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}$$

Note : *A vector* $\vec{A}$ *is said to be solenoidal of its divergence is zero*

$$\nabla.\vec{A} = 0$$

Example : Magnetic field is solenoidal

$$\nabla.\vec{B} = 0$$

- *A vector* $\vec{A}$ *is said to be urrotational if its curl is zero.*

$$\nabla \times \vec{A} = 0$$

Example : In static environment Electric field is urrotational or conservative

$$\nabla \times E = 0$$

- *A scalar field is said to be harmonic in given region if its laplacion is zero.*

$$\nabla^2 V = 0$$

- *Divergence of curl is always zero* $\nabla.(\nabla \times A) = 0$
- *Curl of gradient is always zero* $\nabla \times (\nabla.A) = 0$

1.13 Divergence Theorem

According the divergence theorem, "The surface integral of a vector field over a closed surface S is equal to the Volume integral of divergence of the Vector field".

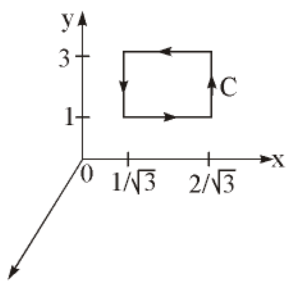
$$\oint_s \overline{D} \cdot \overline{ds} = \int_v \overline{\nabla} \cdot \overline{D}\, dv$$

1.14 Stokes Theorem

According to this theorem, "The Line integral of a Vector field over a closed path is equal to the Surface integral of curl of the Vector field".

$$\oint_\ell \overline{H} \cdot \overline{d\ell} = \int_s (\overline{\nabla} \times \overline{H}) \cdot \overline{ds}$$

Example : Given vector field $\vec{A} = xy\hat{a}_x + x^2.\hat{a}_y$. Find $\oint_c A dl$ circulation by stoke's theorem over path given below.



Solution : Stoke's theorem

$$\oint_c A dl = \iint (\nabla \times A) ds$$

Curl $\nabla\times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & x^2 & 0 \end{vmatrix}$

$$= \left[\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(x^2)\right]a_x + \left[\frac{\partial}{\partial z}(xy) - 0\right]a_y + \left[\frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial y}(xy)\right]a_z$$

$$\nabla\times A = x.\hat{a}_z$$

area element $ds = dx\ .\ dy\ .\ \hat{a}_z$

using stokes' theorem

$$\oint A.dl = \iint(\nabla\times A).ds$$

$$\boxed{= \int_1^3\int_{1/\sqrt{3}}^{2/\sqrt{3}} x.dx\,dy = 1}$$

Example : Given the vector field $A = y^2 a_x + (2xy + x^2 + z^2)\ a_y + (4x + 2yz)\ a_z$.

Find duvergence of vector field.

Solution : Divergence is given by $\nabla.\vec{A}$

$$= \frac{\partial}{\partial x}.A_x + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}A_z$$

$$= \frac{\partial}{\partial x}[y^2] + \frac{\partial}{\partial y}[2xy + x^2 + z^2] + \frac{\partial}{\partial z}[4x + 2yz]$$

$$= 0 + 2x + 2y$$

$$= 2(x + y)$$

Example : A scalar field $g = (1 + 2k)x^2y + xyz$ will be harmonic at all point for which value of k.

Solution : Condition for harmonic field $\nabla^2 g = 0$

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} = 0$$

$$= \frac{\partial}{\partial x}[2x(1+2k)y + yz] + \frac{\partial}{\partial y}[(1+2k)x^2 + xz] + \frac{\partial}{\partial z}[xy]$$

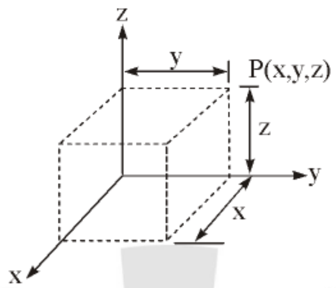
$$= 2(1+2k) + 0 + 0 = 0$$

Therefore $\boxed{k = -\frac{1}{2}}$

1.2 *CO-ORDINATE SYSTEMS :*

1.21 CARTESIAN CO-ORDINATE SYSTEM

The co-ordinates of a point P in the cartesian co-ordinate system is x, y & z along th x, y & z axes. It can be represented as P (x, y, z) as shown below-



- Differential length in cartesian coordinate is

$$\overline{dl} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

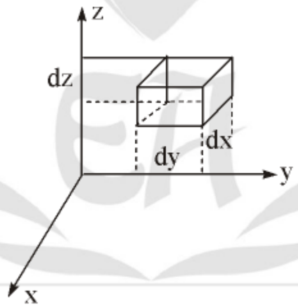
- Differential area in cartesian co-ordinates

$$\overline{ds_1} = dy\,dz \cdot \hat{a}_x$$

$$\overline{ds_2} = dx\,dz \cdot \hat{a}_y$$

$$\overline{ds_3} = dx\,dy \cdot \hat{a}_z$$

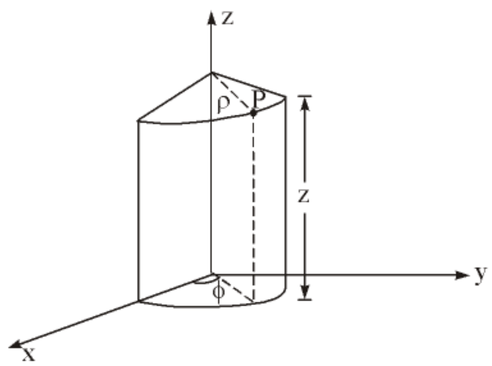
- Differential volume in cartesian co-ordinates is



$$dv = dx\ dy\ dz$$

1.22 CYLINDRICAL CO-ORDINATE SYSTEM

The cylindrical co-ordinates of a point is represented in terms of ρ, ϕ & z along the cylinder as given below



Here, ρ = Radius of cylinder.

ϕ = Angle between x-axis and perpendicular on x-axis of the point.

- *Differential volume in cylindrical co-ordinates is given by*

$$dv = \rho d\rho . d\phi . dz$$

- *Differential Length in cylindrical co-ordinates is given by*

$$d\ell = d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z$$

- *Differential Area in cylindrical co-ordinates is given by*

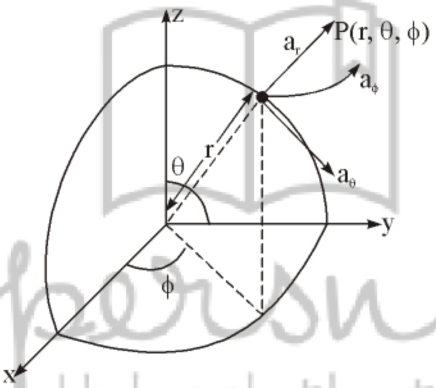
$$\overline{ds_\rho} = \rho d\phi dz \cdot \hat{a}_\rho$$

$$\overline{ds_\phi} = d\rho \cdot dz \cdot \hat{a}_\phi$$

$$\overline{ds_z} = (d\rho)(\rho d\phi) \cdot \hat{a}_z$$

1.23 SPHERICAL CO-ORDINATE SYSTEM

The spherical co-ordinates of a point is represented interms of r, θ & ϕ along the spherical surface as shown below :



Differential length

$$dl = dr\ \hat{a}_r + rd\theta\ \hat{a}_\theta + r\sin\theta . d\phi\ \hat{a}_\phi$$

Differential area

$$ds = \begin{cases} r^2 \sin\theta\ d\theta\ d\phi\ \hat{a}_r \\ r\sin\theta\ dr d\phi\ \hat{a}_\theta \\ r\ dr\ d\theta\ \hat{a}_\phi \end{cases}$$

Defferential volume $dv = (dr)\ (rd\theta)\ (r\ \sin\ \theta\ d\phi)$

1.24 GENERAL CO-ORDINATE SYSTEM : (U, V, W)

	U	V	W	h_1	h_2	h_3
Rectangular	x	y	z	1	1	1
Cylindrical	ρ	ϕ	z	1	ρ	1
Spherical	r	θ	ϕ	1	r	$r \sin\theta$

Mathematical Expressions of Operators :

(i) Gradiant $$\nabla V = \frac{1}{h_1}\frac{\partial v}{\partial u}\hat{a}_u + \frac{1}{h_2}\frac{\partial v}{\partial v} + \frac{1}{h_3}\frac{\partial v}{\partial w}$$

(ii) Divergence of Vector $$\vec{A} = A_u\hat{a}_u + A_v\hat{a}_v + A_w\hat{a}_w$$

$$\nabla.\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}[h_2h_3A_u] + \frac{\partial}{\partial v}(h_3h_1A_v) + \frac{\partial}{\partial w}(h_1h_2A_w)\right]$$

(iii) Laplacian $$\nabla^2 V = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2h_3}{h_1}\frac{\partial v}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3h_1}{h_2}\frac{\partial v}{\partial u}\right) + \frac{\partial}{\partial w}\left(\frac{h_1h_2}{h_3}\frac{\partial v}{\partial w}\right)\right]$$

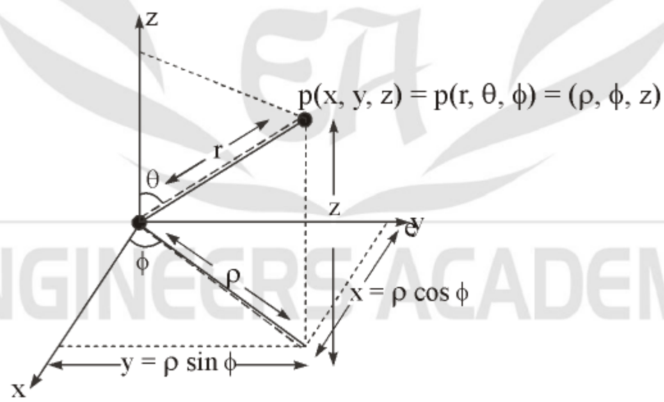
(iv) Curl $$\nabla\times\vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{a}_u & h_2\hat{a}_v & h_3\hat{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1A_u & h_2A_v & h_3A_w \end{vmatrix}$$

(v) Area $$ds = \begin{cases} h_2h_3 & \partial v\partial w & \hat{a}_u \\ h_1h_3 & \partial u\partial w & \hat{a}_v \\ h_1h_2 & \partial v\partial w & \hat{a}_w \end{cases}$$

(vi) Volume $$dv = h_1\,h_2\,h_3\,\partial u\ \partial v\ \partial w$$

(vii) Length $$dl = h_1du\ \hat{a}_u + h_2dv\ \hat{a}_v + h_3dw\ \hat{a}_w$$

1.25 CO-ORDINATE TRANSFEORMATION :



Relation between Cylindrical and cartesian co-ordenates

Cylindrical	***Cartesian***
$\rho = \sqrt{x^2 + y^2}$	$x = \rho \cos \phi$
$\phi = \tan^{-1}\left[\frac{y}{x}\right]$	$y = \rho \sin \phi$
$z = z$	

Relation between spherical and other co-ordinates

Spherical	***Other Co-ordinates***
$r = \sqrt{x^2 + y^2 + z^2}$	$\rho = r \sin \theta$
$\theta = \tan^{-1}\left(\frac{\rho}{z}\right) = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right]$	$x = r \sin \theta . \cos \phi$
$\phi = \tan^{-1}\left(\frac{y}{x}\right)$	$y = r \sin \theta \sin \phi$

Example : Determine divergence of vector fields

(a) $\vec{A} = \rho \sin \phi \ \hat{a}_e + e^2 z \ \hat{a}_\phi + z \cos\phi \ \hat{a}_z$

(b) $\vec{B} = \frac{1}{r^2} \cos \theta \ \hat{a}_r + r \sin\theta \cos\phi \ \hat{a}_\theta + \cos\theta \hat{a}_\phi$

Solution : (a) $\nabla . \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial}{r_\phi}(A_\phi) + \frac{\partial}{\partial z} A_z$

$$= \frac{1}{\rho}\frac{1}{\partial\rho}(\rho^2 \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho^2 z) + \frac{\partial}{\partial z}[z\cos\phi]$$

$$= 2\sin\phi + \cos\phi$$

(b) $\nabla . \vec{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 B_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(B_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(B_\phi)$

$$= \frac{1}{r^2}\frac{\partial}{\partial^r}(\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\cos\theta)$$

$$= 0 + 2 \cos \theta \cos \phi + 0$$

$$= 2 \cos \theta \cos \phi$$

Example : For above vector field $\vec{A}$ find curl $\nabla \times \vec{A}$

Solution : $\nabla \times A = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$

$$= \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ \rho\sin\phi & \rho^2 z & z\cos\phi \end{vmatrix}$$

$$= \left[\frac{z}{\rho}\sin\phi - \rho^2\right]\hat{a}_\rho + 0 + \frac{1}{\rho}[3\rho^2 z - \rho\cos\phi]\hat{a}_z$$

$$= -\frac{1}{\rho}(z\sin\phi + \rho^3).\hat{a}_\rho + (3\rho z - \cos\phi)a_z$$

1.3 *ELECTROSTATICS:*

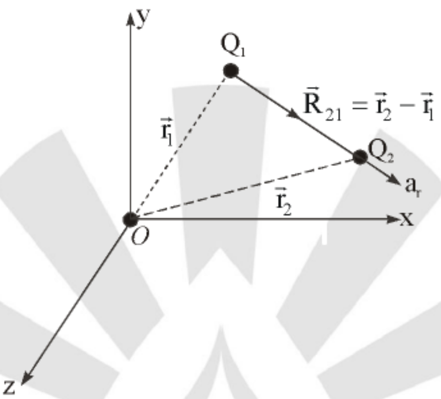
Stationary charge produces electric field $\vec{E}$.

A charge may be point charge, line charge, surface charge or volume charge distributed.

There are two laws in electrostatics **coulomb's law and gauss law.**

1.31 COULOMB'S LAW :

Statement : The force between two point charge Q_1 and Q_2 is inversely propotional to square of distance between two charges and directed along the vector connecting two charges.



force $$F = \frac{kQ_1Q_2}{|\vec{R}_{21}|}.\hat{a}_r$$

$$\boxed{F_{21} = \frac{Q_1Q_2.(\vec{r}_2 - \vec{r}_1)}{4\pi \in_0 |\vec{r}_2 - \vec{r}_1|^3}}$$

Electric field $\vec{E}$ untensity is defined as force per unit charge

$$\boxed{\vec{E} = \frac{\vec{F}}{Q}}$$

- Electric field due to point change

$$\vec{E} = \frac{Q}{4\pi \in_0 r^2}.\hat{a}_r$$

- Electric field due to line charge

$$\vec{E} = \frac{\int e_L dl}{4\pi \in_0 r^2}.\hat{a}_r$$

- Electric field due to surface charge

$$\vec{E} = \frac{\iint e_s ds}{4\pi \in_0 r^2}.\hat{a}_r$$

- Electric field due to volume charge

$$\vec{E} = \frac{\iiint e_v dv}{4\pi \in_0 r^2}.\hat{a}_r$$

Electrostatic potential is defined as work done per unit charge and it is scalar potential due to point charge.

$$\boxed{V = \frac{Q}{4\pi \in_0 r}}$$

Gradient of potential is electric field.

$$\boxed{\vec{E} = -\nabla V}$$

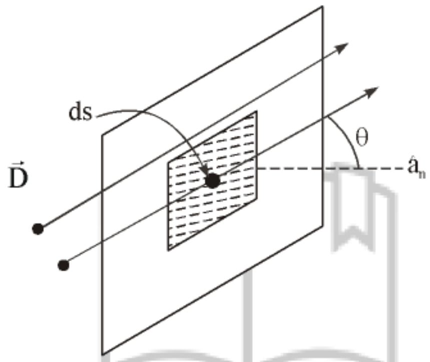
For close loop 'C' work done is zero

$$V = -\oint_C E.dl = 0$$

by stokes theorem for static field.

$$\boxed{\nabla \times \vec{E} = 0}$$

Electric flux passing through any surface areas



Electric flux

$$\boxed{\Psi = \iint D.ds}$$

where, D = Electric field density c/m^2.

1.32 GAUSS'S LAW :

Statement: The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

i.e., $$\boxed{\Psi = \oint_S D.ds = Q_{enclosed}}$$

Integral form $$\oint_S D.ds = \int_v e_v dv \qquad (e \neq \rho_v)$$

e_v = Volume charge density

Differential form $\boxed{\nabla.D = e_v}$

Example : Charge density inside a hollow spherical shell of radius r = 4 cm. centered at origion definad as

$$e_v = \begin{cases} 0 & \text{for} \quad r \le 2 \\ \frac{4}{r^2} c/m^3 & \text{for} \quad 2 < r \le 4 \end{cases}$$

Find Electric field intensity at r = 3

Solution: From Gauss law $\oint E.ds = \frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0}\int e_v dv$

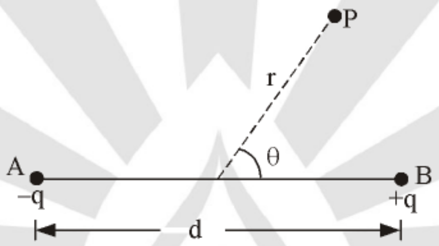
$$= \frac{1}{\epsilon_0}\int (0)dv + \frac{1}{\epsilon_0}\int \frac{4}{r^2}dv \qquad [0 < r \le 3]$$

$$E(4\pi R^2) = \frac{1}{\epsilon_0}\int_{r=2}^{3}\int_0^{\pi}\int_0^{2\pi}\frac{4}{r^2}\ [r^2 \sin\theta\ dr\ d\theta\ .d\phi]$$

$$E\ (4\pi \times 3^2) = \frac{4\pi \times 4}{\epsilon_0}(3-2)$$

$$\boxed{E = \frac{4}{9\epsilon_0}a_r}$$

1.33 Electric Dipole :



- Electric dipole consist of two point charge, separated by small destance d having opposite polarity.
- Dipole moment $p = qd$
- Electric potential the to depole is given by

$$\boxed{V = \frac{P\cos\theta}{4\pi\epsilon_c .r^2}}$$

Note : *Potential is maximum along depole and it is inversaly propotional to square of distance.*

- Electric field due to depole is given by

$$\boxed{\vec{E} = \frac{P}{4\pi\epsilon_0 r^3}[2\cos\theta\ a_r + \sin\theta\ a_\theta]}$$

Note: *For monopele* $\vec{E}\alpha\frac{1}{r^2}$

For Dipole $\vec{E}\alpha\frac{1}{r^3}$

1.34 ELECTROSTATIC ENERGY :

- Energy stored in the system with electric field E and electric flox density $\vec{D}$ is given by

$$W_e = \frac{1}{2}\int_v \vec{D}.\ \vec{E}.dv$$

$$= \frac{1}{2}\int_v \epsilon_0 E^2 dv$$

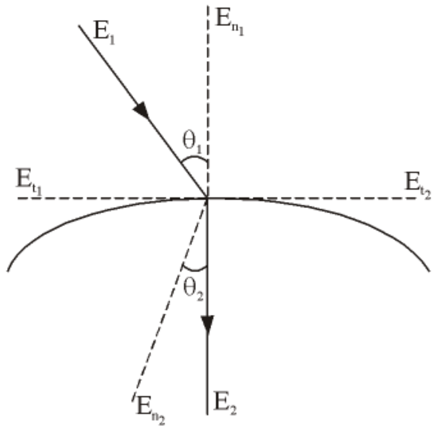
1.35 ELECTRIC BOUNDARY CONDITIONS :

Boundary conditions are defined when region consist of two different media.

Electric field composed of two orthogonal component, tangential compenent E_t and normal component E_n.

$$E = E_t + E_n$$

Consider the two defferent dielectric media (1) and (2) with permittivities $\in_1$ and $\in_2$ respectively as shown below



According to boundary condition, tangential component of electric field is continuous at boundary,

i.e., $\boxed{E_{t_1} = E_{t_2}}$ or $\frac{D_{t_1}}{\in_1} = \frac{D_{t_2}}{\in_2}$

If the surface charge density at boundary is e_s then boundary condition becomes.

$$\boxed{D_{n_1} - D_{n_2} = e_s}$$

1.36 POISSION'S AND LAPLACE EQUATION :

Electric potential V and volume charge density E_V in certain region is related by poissions equation

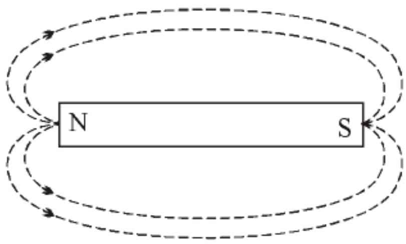
$$\boxed{\nabla^2 V = \frac{\rho_V}{\in}}$$

for charge free region $\boxed{\nabla^2 V = 0}$

Uniqueness theorem states that if solution of laplace or possion equation. Satisfies the boundary condetion, then solution is 'Unique'.

1.4 MAGNETOSTATIC FIELDS :

Magnetic field is produced by moving charges or constant current flow.



Magnetic flux is concentration of magnetic flux line outward from north pole to wards south pole of magnet.

Magnetic flux density is defined as magnetic flux perunit area and it is vector quantity. Its unit is Tesla (T) or weber per squared meter (1 vb /m^2)

$$\vec{B} = \frac{d\Psi}{ds} a_n$$

flux $$\boxed{\Psi = \int_s B.ds}$$

➢ Relation between magnetic flux density $\vec{B}$ and magnetic field untensity $\vec{H}$ is given by

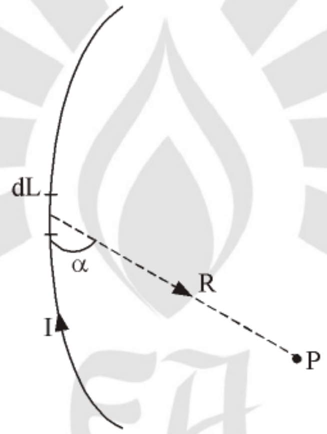
$$\boxed{B = \mu H = \mu_0 \mu_r H}$$

where, μ = Permeability of medium

$$\mu_0 = 4\pi \times 10^{-7} H/m$$

1.41 BIO-SAVART'S LAW:

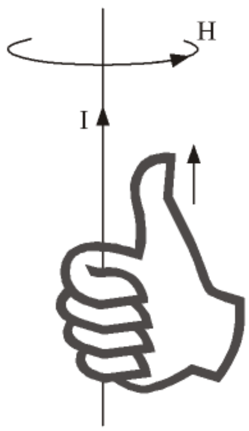
➢ ***Statement :*** The magnetic field intensity dH produces at point P due to current element Idl is given by



$$dH = \frac{IdL \sin\theta}{4\pi R^2}$$

or $$dH = \frac{IdL \times a_R}{4\pi R^2} = \frac{Idl \times \vec{R}}{4\pi R^3}$$

➢ Direction of magnetic field is given by right-hand rule where fingers shows magnetic field line and direction of thumb show current I.



1.42 AMPERE'S LAW :

➢ ***Statements :*** Line integral of magnetic field intensity arround any closed path is equal to current enclosed by the path

$$\boxed{\oint_L H.dL = I_{enclosed}}$$

By stoke's theorem

$$\oint_L H.dl = \iint_s (\nabla \times H).ds = \iint J.ds$$

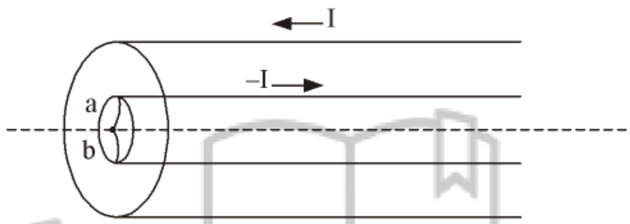
or $$\boxed{\nabla \times H = J}$$

Curl of magnetic field intensity $\vec{H}$ is equal to current density J.

Example : Consider hollow concentric cylindre carring I and –I current in opposite direction. Find magnetic field intensity inside and outside cylindre.

Solution : Case-I : If $r < a$

using ampere's Law



$$\oint H.dl = I_{enc.}$$

inside inner cylindre current enclosed is zero

$$\oint H.dl = 0$$

H = 0 inside inner cylinder.

Case-II : If $a < r < b$

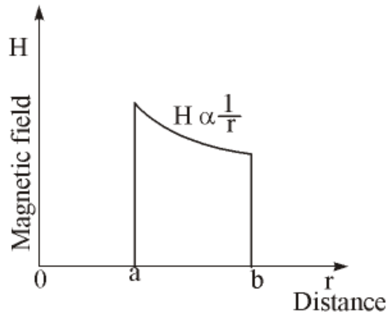
$$\oint H.dl = I$$

$$H = \frac{I}{2\pi r^2} a_\phi$$

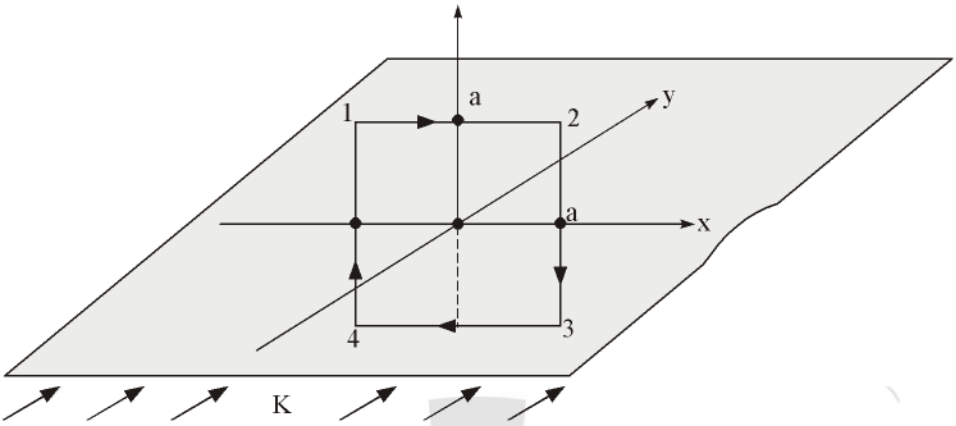
Case-III : If r > b

$$\oint H.dl = I - I = 0$$

H = 0 outside outer cyclinder



Example : An infinite current sheet lies in the z = 0 plane with $K = ka_y$, as shown in figure. Find H.



Solution : The Biot-Savart law and conderations of symmetry shown that H has only an x component, and is not a function of x or y.

Apply Ampere's Law to the square contour 2341, and using the afct that H must be antisymmetric in z,

$$\oint H.dl = (H)(2a) + 0 + (H)(2a) + 0$$

$$= (K)(2a) \quad \text{or} \quad H = \frac{K}{2}$$

Thus for all z > 0, $H = \left(\frac{K}{2}\right)a_x$.

More generally, for an arbitary orientation of the current sheet,

$$H = \frac{1}{2} K \times a_n$$

a_n is the unit vector perpendicular to the plane of the sheet.

Observe that H is independent of the distance from the sheet. Further, the directions of H above and below the sheet can be found by applying the right-hand rule to a few of the current elements in the sheet.

1.43 MAGNETIC POTENTIAL

There are two type of magnetic potentials :

(1) Magnetic scalar potential (V_m)

$$H = -\nabla V_m$$

or $$(V_m) = \int_y^x H.dL$$

For source free region (J = 0), then magnetic scalar potential satis fies then Laplace's equation

i.e., $$\nabla^2 V_m = 0$$

It is only defined for current free region.

(2) Magnetic Vector Potential $(\vec{A})$ *:*

magnetic field density $\vec{B}$ can be expressed as curl of magnetic vector potential $\vec{A}$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

Magnetic vector potential satisfies the poission's equation

i.e., $\boxed{\nabla^2 A = -\mu_0 J}$

Example : Find current density that would produce magnetic vector potential $A = 2a_\phi$ in cylindrical co-ordinate.

Solution : Magnetic flux density is given by

$$B = \nabla \times A$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\phi)a_z$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(2\rho)a_z$$

$$B = \frac{2}{\rho}a_z$$

Cirremt demsotu K om ter,s pf ,agemtoc f;ix demsotu N os defomed as

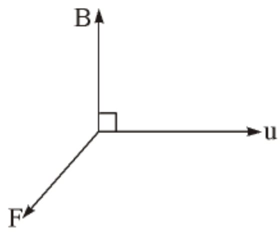
$$J = \frac{1}{\mu_0}(\nabla \times B)$$

$$= \frac{1}{\mu_0}\left[-\frac{\partial}{\partial \rho}\left(\frac{2}{\rho}\right)\right]a_\phi$$

$$= \frac{2}{\mu_0 \rho^2}a_\phi$$

1.44 FORCE IN MAGNETIC FIELD :

If a charge partical 'Q' is in motion with velocity 'u' in presence of magnetic field density 'B' the magnetic force experince by charge is



$$\boxed{F = Q(u \times B)}$$

If both magnetic field and electric field are present force on charged partical is given by 'Lorentz force.'

$$\boxed{F = Q(E + u \times B)}$$

Magnetic force can not perform work while electric force can perform work.

1.45 MAGNETIC DIPOLE :

Magnetic depole moment is product of current and area A, and it is normal to plane of Loop.

$$\boxed{m = I.A\ a_n}$$

1.46 MAGNETIZATION IN MAGNETIC MATERIAL :

Magnetization is directly propotional to magneticfield intensity.

i.e., $M \propto H$

$$\boxed{M = X_m H}$$

where, X_m is magnetic suceptibility and it is given by

$$\boxed{X_m = \mu_r - 1}$$

where, μ_r = Relative permeabity of medium.

1.47 MAGNETIC ENERGY :

In a magnetic field density B, stored magnetic energy density is given by

$$\boxed{W_m = \frac{1}{2}(B.H) = \frac{1}{2}\mu H^2 \quad (\because\ \vec{B} = \mu\vec{H})}$$

1.48 MAGNETIC BOUNDARY CONDETION :

In two defferent magnetic media with per meabilities μ_1 and μ_2 respectively, at boundary field components are given by boundary condition.

From the boundary condition, the normal a components of magnetic field are related.

$$\boxed{B_{1_n} = B_{2_n}}$$

or normal component of magnetic field density are equal at boundary.

Tangential components of magnetic field intensity are related as

$$\boxed{H_{1_t} - H_{2_t} = K \text{ or } (H_1 - H_2) \times a_{n12} = K}$$

where, K is surface charge density.

If K = 0

$$\boxed{H_{1_t} = H_{2_t}}$$

<u>1.5 *MAXWELL'S EQUATION :*</u>

1.51 FARADAY LAW :

➢ ***Faraday's Law of Electro-magnetic Induction***

The –ve sign indicates that the polarity of voltage induced opposes the cause of induction of the voltage.

This is as per the Lenz's Law.

$$e = \oint E \cdot d\ell.$$

also, $$e = \frac{d\phi}{dt}$$