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CBSE – XIIth

Maths

Central Board of Secondary Education (CBSE)

Previous Year Questions + PYQ Solution

SIERRA INNOVATIONS

INDEX

Previous Year Questions CBSE

Types of Relations

MCQ

1. A relation R defined on set A = {x : x ∈ Z and 0 ≤ x ≤ 10} as R = {(x, y) : x = y} is given to be an equivalence relation. The number of equivalence classes is : **(2024)**
(A) 1 (B) 2 (C) 10 (D) 11

2. **Asseration (A) :** The relation R = {(x,y) : (x + y) is a prime number and x,y ∈ N} is not a reflexive relation.
Reason (R) : The number '2n' is composite for all natural numbers n. **(2024)**
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

3. Let A = {3,5}.Then number of reflexive relations on A is **(2023)**
(a) 2 (b) 4 (c) 0 (d) 8

4. Let R be a relation in the set N given by **(2023)**
R = {(a, b) : a = b – 2, b > 6}. Then
(a) (8, 7) ∈ R (b) (6, 8) ∈ R
(c) (3, 8) ∈ R (d) (2, 4) ∈ R

5. A relation R is defined on N. Which of the following is the reflexive relation? **(Term I, 2021-22)**
(a) R = {(x, y): x > y, x, y ∈N}
(b) R = {(x, y): x + y = 10, x, y ∈N}
(c) R = {(x, y): xy is the square number, x, y ∈N}
(d) R = {(x, y): x + 4y = 10, x, y ∈N}

6. The number of equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) is **(Term I, 2021-22)**
(a) 0 (c) 2 (b) 1 (d) 3

7. A relation R is defined on Z as aRb if and only if $a^2 - 7ab + 6b^2 = 0$. Then, R is **(Term I, 2021-22)**
(a) reflexive and symmetric
(b) symmetric but not reflexive
(c) transitive but not reflexive
(d) reflexive but not symmetric

8. Let A = {1, 3, 5}. Then the number of equivalence relations in A containing (1, 3) is **(2020)**
(a) 1 (b) 2 (c) 3 (d) 4

9. The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is **(2020)**
(a) symmetric and transitive, but not reflexive
(b) reflexive and symmetric, but not transitive
(c) symmetric, but neither reflexive nor transitive
(d) an equivalence relation

VSA

10. Write the smallest reflexive relation on set A = {a,b,c} **(2021 C)**

11. A relation R in a set A is called_______, if (a_1,a_2) ∈R implies (a_1,a_2) ∈R, for all a_1,a_2 ∈A. **(2020 C)**

12. A relation in set A is called_____ relation, if each element of A is related to itself. **(2020 C)**

13. If R = {(x.y) : x + 2y = 8} is a relation on N, write the range of R. **(Al 2014)**

14. Let R = {(a,a^3) : a is a prime number less than 5} be a relation. Find the range of R. **(Foreign 2014)**

15. Let R be the equivalence relation in the set A = {0,1,2,3,4,5} given by R = {(a,b) : 2 divides (a – b)}. Write the equivalence class [0]. **(Delhi 2014 C)**

SAI

16. Let R be a relation on set of real numbers IR defined as {(x, y) : $x - y + \sqrt{3}$ is an irrational number, x, Y ∈ IR} Verify R for reflexivity, symmetry and transitivity. **(2025)**

17. A relation R on set A = {1, 2, 3, 4, 5} is defined as R = {(x, y) : $|x^2 - y^2| < 8$}. Check whether the relation R is reflexive, symmetric and transitive. **(2024)**

18. Check if the relation R in the set **?** of real numbers defined as R = {(a,b) : a < b} is (i) symmetric, (ii) transitive. **(2020)**

19. Let W denote the set of words in the English dictionary. Define the relation R by
R ={(x,y) ∈ W × W such that x and y have at least one letter in common}.
Show that this relation R is reflexive and symmetric, but not transitive. **(2020)**

LAI

20. Check whether the relation S in the set of real numbers R defined by S = {(a, b): where $a - b + \sqrt{2}$ is an irrational number) is reflexive, symmetric or transitive. **(2024)**

21. Show that the relation R in the set A = {1,2,3,4,5,6} given by R = {(a,b) : |a – b| is divisible by 2} is an equivalence relation. **(2020)**

22. Check whether the relation R defined R defined on the set A = {1,2,3,4,5,6} as
R = {(a,b) : b = a + 1} is reflexive, symmetric or transitive. **(2019)**

23. Show that the relation R on the set Z of all integers, given by
R = {(a,b) : 2 divides (a – b)} is an equivalence relation. **(2019)**

24. Show that the relation R on **?** defined as R = {(a,b) : a ≤ b}, is reflexive and transitive but not symmetric. **(NCERT, Delhi 2019)**

25. Show that the relation S in the set A = {x∈Z : 0 ≤ x ≤ 12} given by S = {(a,b) : a ∈ Z, |a – b| is divisible by 3} is an equivalence relation. **(AI 2019)**

26. Let A = {1,2,3,......9} and R be the relation in A × A Prove that R is an equivalence relation. Also obtain the equivalence the equivalence class [(2,5)]. **(Delhi 2014)**

27. Let R be a relation defined on the set of natural numbers N as follow : **(Delhi 2014 C)**
R = {(x, y) x ∈ N, y ∈ and 2x + y = 24
Find the domain and range of the relation R.
Also, find if R is an equivalence relation or not.

LA–II

28. If N denotes the set of all natural numbers and R is the relation on N × N defined by (a,b) R (c,d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation. **(2023, Delhi 2015)**

29. Let A = {x∈ Z : 0 ≤ x ≤ 12} Show that R = {(a,b) : a,b ∈A, |a – b| is divisible by 4}, is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. **(2018)**

30. Show that the relation R in the set A = {1,2,3,4,5} given by R = {(a,b) : |a – b| is divisible by 2} is an equivalence relation. Writhe all the equivalence classes of R. **(AI 2015)**

CSBQ :

31. A school is organizing a debate competition with participants as speakers S = {S_1, S_2, S_3, S} and these are judged by judges J = {J_1, J_2, J_3}. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as R = {(x, y) : speaker x is judged by judge y, x ∈ S, y ∈ J}.

Based on the above, answer the following : **(2025)**
(i) How many relations can be there from S to J?
(ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)} Check if it is bijective.
(iii) (a) How many one-one functions can be there from set S to set J ?

OR

(iii) (b) Another student considers a relation R_1 = {(S_1, S_2), {S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

32. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A= {l, 2, 3}: **(2025)**
R_1 = {(2, 3), (3, 2)}
R_2 = {(l, 2), (1, 3), (3, 2)}
R_3 = {(1, 2), (2, 1), (1, 1)}
R_4 = {(l, 1), (1, 2), (3, 3), (2, 2)}
R_5 = {(l, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)}
The students are asked to answer the following questions about the above relations:
(i) Identify the relation which is reflexive, transitive but not symmetric.
(ii) Identify the relation which is reflexive and symmetric but not transitive.
(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?

33. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history. **(2024)**

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R=\{(l_1, l_2): l_1$ is parallel to $l_2\}$

On the basis of the above information, answer the following questions:

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

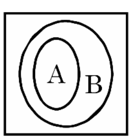
(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

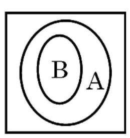
OR

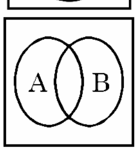
(b) Let S be the relation defined by $S = \{(l_1, l_2): l_1$ is perpendicular to $l_2\}$ check whether the relation S is symmetric and transitive.

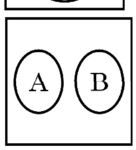
Types of Functions :

34. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ? **(2025)**

(A) 

(B) 

(C) 

(D) 

35. Which of the following is not a homogeneous function of x and y ? **(2025)**

(A) $y^2 - xy$

(B) $x - 3y$

(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$

(D) $\tan x - \sec y$

36. Assertion (A) Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x- 5, \forall x \in Z$ is a bijective.

Reason (R) A function is a bijective if it is both surjective and injective. **(2025)**

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

37. A function $f : R_+ \to R$ (where R_+ is the set of all nonnegative real numbers) defined by $f(x) = 4x + 3$ is :

(A) one-one but not onto **(2024)**

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

38. A function f: ??→ ?? defined as $f(x) = x^2 – 4x + 5$ is:

(A) injective but not surjective. **(2024)**

(B) surjective but not injective.

(C) both injective and surjective.

(D) neither injective nor surjective.

39. The function $f : R \to R$ defined by $f(x) = 4 + 3 \cos x$ is.

(a) bijective **(Term I, 2021-22)**

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

40. The number of functions defined from $\{1,2,3,4,5\} \to \{a,b\}$ which are one–one is **(Term I, 2021-22)**

(a) 5 (b) 3 (c) 2 (d) 0

41. Let $f: R \to R$ be defined by $f(x) = 1/x$, for all $x \in R$, Then, f is **(Term I, 2021-22)**

(a) one–one (b) onto

(c) bijective (d) not defined

42. The function $f : N \to N$ is defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{if n is odd} \\ \frac{n}{2}, \text{if n is even} \end{cases}$$ **(Term I, 2021-22)**

The function f is

(a) bijective

(b) one–one but not onto

(c) onto but not one–one

(d) neither one–one nor onto

VSA

43. Show that the function f given by f(x) = sin x + cos x, is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$. **(2024)**

44. If f = {(1,2), (2,4),(3,1),(4,k)} is a one–one function from set A to A, where A = {1,2,3,4} then find the value of k. **(2021)**

SA

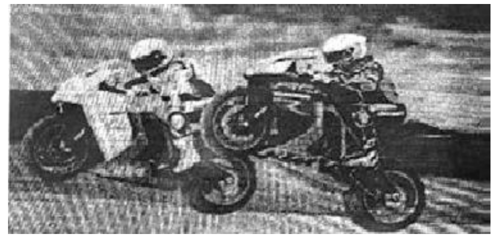
45. A function f is defined from R → R as f(x) = ax + b, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not. **(2024)**

LA

46. Let A = R–(5) and B = R–{1}. Consider the function f: A→ B. defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one–one and onto. **(2024)**

CSBQ:

47. An organization conducted bile race under two different categories – Boys and girls, There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race, Ravi forms two sets B and G with these participants for his college project.
Let B = $\{b_1, b_2, b_3\}$ and G = $\{g_1.g_2\}$, where B represents the set of boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions.
(i) How many relations are possible form B to G?
(ii) Among all the possible relations from B to G?
(iii) Let R : B → B be defined by R = {(x, y) : x and y are students of the same sex}. Check if R is an equivalence relation.

OR

A function f : B → G be defined by f = $\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective, justify your answer. **(2023)**

48. Let f : **?** $-\left\{-\frac{4}{3}\right\}$ → **?** be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one–one function. Also, check whether f is an onto function or not. **(2023)**

49. Show that the function f : (–∞,0) → (–1,0) defined by $f(x) = \frac{x}{1+|x|}$ $x \in (-\infty, 0)$ is one–one and onto. **(2020)**

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (D) 11

2. (D) Assertion (A) is false, but Reason (R) is true.

3. (b) : total number of reflexive relations on a set having n number of elements = 2n2 – n
Here, n = 2
$\therefore$ Required number of reflexive relations = 2^{2^2-1}
$= 2^{4-2} = 2^2 = 4$

4. (b) : Given, R = {(a, b) : a = b – 2, b > 6}
Since, b > 6, so (2, 4) $\in$ R
Also, (3, 8) $\in$ R as $3 \neq 8 - 2$
and (8, 7) $\in$ R as $8 \neq 7 - 2$
Now, for (6, 8), we have
8 > 6 and 6 = 8 – 2, which is true
$\therefore$ (6, 8) $\in$ R

5. (c) : Consider, R = {(x, y) : xy is the square number, x, y $\in$ N}
As, xx = x^2, which is the square of natural number x.
$\Rightarrow$ (x, x) $\in$R. So, R is reflexive.

Concept Referred

- A relation R in a set A is called reflexive, if (a, a) $\in$ R, for all a $\in$ A.

6. (c) : Equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) are
R_1 = {(1
R_2 = {(1
$\therefore$ Number of equivalence relations is 2.

Concept Referred

- A relation *R* in a set *A* is called an equivalence relation, if *R* is reflexive, symmetric and transitive.

7. (d) : Given, aRb, a, b $\in$ Z
Reflexive : for a $\in$ Z, we have
$a^2 - 7a.a + 6a^2 = a^2 - 7a^2 + 6a^2 = 0 \Rightarrow (a, a) \in R$
$\therefore$ relation is reflexive.
Symmetric : Since, (6, 1) $\in$R
As, $6^2 - 7 \times 6 \times 1 + 6 \times 12 = 36 - 42 + 6 = 0$
But (1, 6) $\notin$ R. $\therefore$ Relation is not symmetric.

8. (b) : Equivalence relations in the set containing the element (1, 3) are
R_1 = {(1
R_2 = {(1,1),(3,3),(5,5),(1,5),(5,1),(3,5),(5,3),(1,3),(3,1)}
$\therefore$ There are 2 possible equivalence relations.

9. (c) : Given R = {(1, 2), (2, 1), (1, 1)} is a relation on set {1, 2, 3}
Reflexive : Clearly (2,2), (3,3) $\notin$ R
$\therefore$ R is not a reflexive relation.
Symmetric : Now, (1, 2) $\in$ R and (2, 1) $\in$ R $\therefore$ R is symmetric.
Transitive : Now, (2, 1) $\in$ R and (1, 2) $\in$ R but (2, 2) $\notin$ R
$\therefore$ R is not transitive relation.
R is symmetric : Out neither reflexive nor transitive.

10. We have, A = {a, b, c}
A relation R on the set A is said to be reflexive if (a, a) $\in$ R,
$\therefore$ R = {(a, a),(b, b), (c, c)} is the required smallest reflexive relation on A.

11. A relation R in a set A is called symmetric, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

12. A relation in a set A is called reflexive relation, if each element of A is related to itself.

13. Here, R = {(x, y) : x + 2y = 8x, y $\in$N}.
For x = 1, 3, 5, ...
x + 2y = 8 has no solution in N.
For x = 2, we have 2 + 2y = 8 $\Rightarrow$ y = 3
For x = 4, we have 4 + 2y = 8 $\Rightarrow$ y = 2
For x = 6 , we have 6 + 2y = 8 $\Rightarrow$ y = 1
For x = 8, 10, ...
x + 2y = 8 has no solution in N.
$\therefore$ Range of R = {y : (x, y) $\in$R} = { 1, 2, 3}

14. Given relation is
R = {(a, a^3) : a is a prime number less than 5}.
$\therefore$ R = {(2, 8), (3, 27)}. So, the range of R is {8, 27}.

15. Here, R = {(a, b) : 2 divides (a – b)}
$\therefore$ Equivalence class of [0] = {a $\in$ A : (a, 0) $\in$ R}.
$\Rightarrow$ (a – 0) is divisible by 2 and a $\in$ A $\Rightarrow$ a = 0, 2, 4
Thus [0] = {0, 2, 4}.

16. Let x ∈ ℝ. Then we know that $x - x + \sqrt{3} = \sqrt{3}$, which is an irrational number.
⇒ (x, x) ∈ R
Hence, R is reflexive.
We have $\sqrt{3}, 2 \in$ **?** such that $\sqrt{3} - 2 + \sqrt{3} = 2(\sqrt{3} - 1)$, which is an irrational number.
$\Rightarrow (\sqrt{3}, 2) \in R$.
But $2 - \sqrt{3} + \sqrt{3} = 2$, which is a rational number
Hence, $\Rightarrow (2, \sqrt{3}) \notin R$
Therefore, R is not symmetric
Let $-\sqrt{3}, \sqrt{3}, 2 \in$ **?** such that $(-\sqrt{3}, \sqrt{3}), (\sqrt{3}, 2) \in R$
But $(-\sqrt{3}, 2) \notin R$
Therefore, R is not transitive.

17. Reflexive: yes $-(x, x)$ gives $\left|x^2 - x^2\right| = 0 < 8$ for all $x \in A$.
Symmetric: yes – if $\left|x^2 - y^2\right| < 8$ then $\left|y^2 - x^2\right| < 8$, so $(x, y) \in R \Rightarrow (y, x) \in R$.
Transitive: No–e.g. $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$ since $\left|1^2 - 3^2\right| = 8 \quad 8$.

18. We have, R = {(a, b) : a < b}, where a, b **? ?**
(i) Symmetric : Let (x, y) ∈ R, i.e., x R y ⇒ x < y
so (x, y) ∈ R ⇒ (y, x) ∈ R
Thus, R is not symmetric.
(ii) Transitive : Let (x, y), (y, z) ∈ R
⇒ x < y and y < z ⇒ x < z
⇒ (x, z) ∈ R. Thus, R is transitive.

19. We have, R = {(x, y) ∈ W × W : x and y have at least one letter in common}
Reflexive : Clearly (x, x) ∈ R, because same words will contains all common letters.
⇒ R is reflexive.
Symmetric : Let (x, y) ∈ R i.e., x and y have at least one letter in common.
⇒ y and x will also have at least one letter in common.
⇒ (y, x) ∈ R
⇒ R is symmetric.
Transitive : Let, x = LAND, y = NOT and z = HOT
Clearly (x, y) ∈ R as x and y have a common letter and (y, z) ∈ R as y and z have 2 common letters.
but (x, z) ∉ R as x and z have no letter in common.
Hence, R is not transitive.

Concept Referred

- A relation R in a set A is not transitive if for (a, b) ∈R and (b, c) ∈R but (a, c) ∉R

20. For a relation to be reflexive aRa
For real a
$aRa \Rightarrow a - a + \sqrt{2} = \sqrt{2}$
$\sqrt{2}$ is an irrational number
aRa is Reflexive
For a Relation to be symmetric
aRb ⇒ bRa
For real number a and b
$aRb \Rightarrow a - b + \sqrt{2} \Rightarrow aRb \neq bRa$
$aRb \Rightarrow b - a + \sqrt{2}$ It is not symmetric
For Transitive
aRb = bRc = aRc
For real number a, b and c
Let $a = -\sqrt{2}$
$b = 3\sqrt{2}$
c = 2
$aRb \Rightarrow a - b + \sqrt{2} = -\sqrt{2} - 3\sqrt{2} + \sqrt{2}$
$= -3\sqrt{2}$ is an irrational
$bRc \Rightarrow 3\sqrt{2} - 2 + \sqrt{2}$
$= 4\sqrt{2} - 2$ is an irrational
$aRc \Rightarrow -\sqrt{2} - 2 + \sqrt{2}$
= –2 is not an irrational
aRb, bRc then a is not related to b.
The Relation is not transitive.

21. We have, A = {1, 2, 3, 4, 5, 6} and R = {(a, b) : |a – b| is divisible by 2}
(i) Reflexive : for and a ∈ A
|a – a| = 0, which is divisible by 2.
Thus, (a, a) ∈ R. So, R is reflexive.
(ii) Symmetric : For any a, b ∈ A
Let (a, b) ∈ R
⇒ |a – b| is divisible by 2 ⇒ |b – a| is divisible by 2
⇒ (b, a) ∈ R ⇒ (a, b)∈ R ⇒ (b, a)∈R ∴R is symmetric.
(iii) Transitive : For any a, b, c ∈ A
Let (a, b) ∈ R and (b, c) ∈ R
⇒ |a – b| is divisible by 2 and |b – c| is divisible by 2.
⇒ $a - b = \pm 2\,k_1$ and $b - c = \pm\, 2k_2\ \forall\ k_1, k_2 \in N$
⇒ $a - b + b - c = \pm 2(k_1 + k_2) \Rightarrow a - c = \pm 2\,k_3\ \forall\ k_3 \in N$
⇒ |a – c| is divisible by 2 ⇒ (a, c)∈ R ∴ R is transitive.
Hence, R is an equivalence relation.

22. We have, A = {1, 2, 3, 4, 5, 6} and a relation R on A defined as R = {(a, b) : b = a + 1}

Reflexive : Let $(a,a) \in R$

$\Rightarrow a = a + 1 \Rightarrow a - a = 1 \Rightarrow 0 = 1$, which is not possible.

$\therefore (a, a) \notin R \Rightarrow$ R is not reflexive.

Symmetric : Let $(a, b) \in R \Rightarrow b = a + 1$ ...(i)

Now, if $(b,a) \in R$

$\Rightarrow a = b + 1 \Rightarrow b = b + 1 + 1$ (using(i))

$\Rightarrow b = b + 2 \Rightarrow b - b = 2 \Rightarrow 0 = 2$, which is not possible

$\Rightarrow (b, a) \notin R \Rightarrow$ R is not symmetric.

Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow b = a + 1$ and $c = b + 1 \Rightarrow c = a + 1 + 1$

$\Rightarrow c = a + 2 \Rightarrow a + 1 \Rightarrow (a,c) \notin R \Rightarrow$ R is not transitive.

23. We have, R = {(a, b) : 2 divides (a – b)}

Reflexive: a∈ or and a to Z , a – a = 0 and 2 divides 0.

$\Rightarrow (a, a) \in R$ for every $a \in Z \therefore$ R is a reflexive.

Symmetric : Let $(a, b) \in R$

$\Rightarrow$ 2 divides (a – b)

$\Rightarrow a - b = 2m$, for some $m \in Z$

$\Rightarrow b - a = 2m$

$\Rightarrow$ 2 divides b – a

$\Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ 2 divides (a – b) and 2 divides (b – c)

$\Rightarrow a - b = 2m$ and $b - c = 2n$ for some $m, n \in Z$

$\Rightarrow a - b + b - c = 2m + 2n$

$\Rightarrow a - c = 2(m + n)$

$\Rightarrow$ 2 divides a – c

$\Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive.

Hence, R is an equivalence relation.

24. We have, R= {(a,b):a - b, a,b ∈ **?** }

(i) reflexive : Since $a \le a \therefore aRa\ \forall a \in R$

Hence, R is reflexive.

(ii) Symmetric :$(a,b) \in R$ such that $aRb \Rightarrow a \le b \not\Rightarrow b \le a$

So, $(b, a) \notin R$.

Hence, R is not symmetric.

(iii) Transitive : Let $a, b, c \in R$ such that aRb and bRc

Now, aRb a ≤ b ... (i) and bRc b ∉ c ... (ii)

From (i) and (ii), we have a b c ≤ a ≤ c ∴aRc

Hence, relation R is transitive.

25. We have, $A = \{x \in Z : 0 \le x \le 12\}$

$\therefore A = \{0, 1, 2, 3, ..., 12\}$

Also, S = {(a,b):a,b-Z,|a−b|isdivisibleby3}

(i) Reflexive : for and $a \in A$,

|a – a| = 0, which is divisible by 3

Thus, $(a, a)S \in S$ is reflexive.

(ii) Symmetric : Let $(a, b) \in S$

|a−b|isdivisibleby3.

|b −a|isdivisibleby3 $\Rightarrow (b, a) \in S$ i.e. $(a,b) \in S \Rightarrow (b, a)$

$\therefore$ S is symmetric.

(iii) Transitive :

Let $(a, b) \in S$ and $(b, c) \in S$

$\Rightarrow |a - b|$ is divisible by 3 and $|b - c|$ is divisible by 3.

$\Rightarrow (a - b) = \pm 3k_1$ and $(b - c) = \pm 3k_2$; $\forall k_1, k_2 \in N$

$\Rightarrow (a - b) + (b - c) = \pm 3(k_1 + k_2)$

$\Rightarrow (a - c) = \pm 3(k_1 + k_2)$; $\forall k_1, k_2 \in N$

$\Rightarrow |a - c|$ is divisible by 3 $\Rightarrow (a, c) \in S \therefore$ S is Transitive.

Hence, S is an equivalence relation.

Concept Referred

- A relation R in a set A is called
  (i) reflexive, if $(a, a) \in R$, for all $a \in A$
  (ii) symmetric, if $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$
  (iii) transitive, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$, for all $a, b, c \in A$

26. Given A = {1, 2, 3, 4,...,9}

To show : R is an equivalence relation.

(i) Reflexive: Let (a, b) be an arbitrary element of A × A.

Then, we have $(a, b) \in A \times A \Rightarrow a, b \in A$

$\Rightarrow a + b = b + a$ (by commutativity of addition on $A \subset N$)

$\Rightarrow$ (a, b) R (a, b)

Thus, (a,b) R (a,b) for all $(a,b) \in A \times A$. So, R is reflexive.

(ii) Symmetric : Let $(a, b), (c, d) \in A \times A$ such that (a, b) R (c, d)

$\Rightarrow a + d = b + c \Rightarrow b + c = a + d$

$\Rightarrow c + b = d + a$ (by commutativity of addition on $A \subset N$)

$\Rightarrow$ (c, d) R (a, b).

Thus, (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in A \times A$.

So, R is symmetric.

(iii) Transitive : Let $(a, b), (c, d), (e, f) \in A \times A$ such that (a, b) R (c, d) and (c, d) R (e, f)

Now, (a, b) R (c, d) $\Rightarrow a + d = b + c$ ...(i)

and (c, d) R (e, f) $\Rightarrow c + f = d + e$ ...(ii)

Adding (i) and (ii), we get (a+d) + (c+f) = (b+c) + (d + e)

$\Rightarrow a + f = b + e \Rightarrow$ (a, b) R (e, f)

Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f).

So, R is transitive. $\therefore$ R is an equivalence relation.

Equivalence class of $[(2,5)] = \{(x,y) \in N \times N : (x, y)R(2, 5)\}$

$= \{(x, y) \in N \times N : x + 5 = y + 2\}$

$= \{(x, y) \in N \times N : y = x + 3\} = \{(x, x + 3) : x \times A\}$

= {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}.

Concept Referred

- First, prove the given relation is an equivalence relation and then find the equivalence class by using the given relation.

27. Here, R = {(x, y) | x∈N, y∈N and 2x + y = 24}
R = {(1, 22), (2, 20), (3, 18), ..., (11, 2)}
Domain of R = {1, 2, 3, 4, ..., 11}
Range of R = {2, 4, 6, 8, 10, 12 ,..., 22}
R is not reflexive as if (2,2)∈ R ⇒ 2 × 2 + 2 = 6 ≠ 24
In fact R is neither symmetric nor transitive.
⇒ R is not an equivalence relation.

28. (i) Reflexcive : Let (a, b) be an arbitrary element of N × N. Then, (a, b) ∈ N × N
⇒ ab(b + a) = ba(a + b)
[by commutativity of addition and multiplication on N]
⇒ (a, b) R (a, b)
So, R is reflexive on N × N.
(ii) Symmetric : Let (a, b), (c, d) ∈N × N such that (a, b) R (c, d).
⇒ ad(b + c) = bc(a + d) ⇒cb(d + a) = da(c + b)
[by commutativity of addition and multiplication on N]
Thus, (a, b) R (c, d) ⇒ (c, d) R (a, b) for all (a, b), (c, d) ∈ N × N.
So, R is symmetric on N × N.
(iii) Transitive : Let (a, b), (c, d), (e, f) ∈N × N such that (a, b) R (c, d) and (c, d) R (e, f). Then,
(a, b) R (c, d) ⇒ ad(b + c) = bc(a + d)

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{b}+\frac{1}{c} = \frac{1}{a}+\frac{1}{d} \qquad \text{(i)}$$

and (c, d) R (e, f) ⇒ cf(d + e) = de(c + f)

$$\Rightarrow \frac{d+c}{de} = \frac{a+d}{ad} \Rightarrow \frac{1}{b}+\frac{1}{c} = \frac{1}{a}+\frac{1}{d} \qquad \text{....(ii)}$$

Adding (i) and (ii), we get

$$\left(\frac{1}{b}+\frac{1}{c}\right)+\left(\frac{1}{d}+\frac{1}{e}\right)=\left(\frac{1}{a}+\frac{1}{d}\right)+\left(\frac{1}{c}+\frac{1}{f}\right)$$

$$\frac{1}{b}+\frac{1}{e}=\frac{1}{a}+\frac{1}{f} \Rightarrow \frac{b+e}{be}=\frac{a+f}{af}$$

⇒ af(b + e) = be(a + f) ⇒ (a, b) R (e, f)
So, R is transitive on N × N.
Hence, R is an equivalence relation.

29. We have, A = {x ∈ Z : 0 ≤ x ≤ 12}
∴ A = {0, 1,
and S = {(a, b) : |a – b| is divisible by 4}
(i) Reflexive : for and a ∈ A, |a – a| = 0, which is divisible
by 4. Thus,(a,a) ∈ R ∴ R is reflexive.
(ii) Symmetric : Let (a,b) ∈ R
⇒ |a – b| is divisible by 4
⇒ |b – a| is divisible by 4 ⇒ (b, a) ∈ R
i.e., (a, b) ∈ R ⇒ (b, a) ∈ R ∴ R is symmetric.
(iii) Transitive : Let (a,b) ∈ R and (b,c) ∈ R
⇒ |a – b| is divisible by 4 and |b – c| is divisible by 4
⇒ a – b = ± 4k1 and b – c = ± $4k_2$; ∀ k_1, k_2 ∈N
⇒ (a – b) + (b – c) = ± 4 ($k_1 + k_2$) ; ∀ k_1, k_2 ∈ N
⇒ a – c = ± 4 ($k_1 + k_2$) ; ∀ k_1, k_2 ∈ N
⇒ |a – c| is divisible by 4 ⇒ (a,c) ∈ R
∴ R is transitive.
Hence R is an equivalence relation.
The set of elements related to 1 is {1,5,9}.
Equivalence class for [2] is {2,6,10}

Concept Referred

- In a relation R in a set A, the set of all elements related to any element a ∈ A is denoted by [a]
  i.e.,[a] = {x ∈ A :(x,a)∈ R}
  Here,[a] is called an equivalence class of a ∈ A.

30. We have, A = {1,2, 3, 4, 5}
and R = {(a, b) : |a – b| is divisible by 2}
(i) Reflexive : For and a ∈ A,
|a – a| = 0, which is divisible by 2
Thus, (a, a) ∈ R ∴ R is reflexive.
(ii) Symmetric : Let (a,b)∈R
⇒ |a – b| is divisible by 2
⇒ |b – a| is divisible by 2 ⇒ (b,a)∈R
i.e., (a,b) ∈R ⇒ (b,a) ∈R ∴ R is symmetric.
(iii) Transitive : Let (a, b) ∈R and (b,c) ∈R
⇒ |a –b| is divisible by 2 and (b,c) ∈R
⇒ a –b ± 2 and b – c = ± $2k_2$; ∀k_1,k_2 ∈N
⇒ (a –b) + (b – c) = ± 2(k_1,k_2) ; ∀k_1,k_2 ∈N
⇒ (a – c) = ± 2 ($k_1 + k_2$) ; ∀ k_1, k_2 ∈N
⇒ |a – c| is divisible by 2 ⇒ (a,c) ∈ R ∴ R is transitive.
Hence, R is an equivalence relation.
Further R has only two equivalence classes, namely [1] = [3] = [5] = {1,3,5} and [2] = [4] = {2,4}.

31. (i) The number of relations = $2^{4\times3} = 2^{12}$
(ii) Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective.
(iii) There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0.

OR

(iii) To make *R*1 reflexive and not symmetric we need to add the following ordered pairs:
(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)

32. (i) R_4
(ii) R_5
(iii) (a) R_1 and R_3
(iii) (b) Required pairs to be added to make the relation R_2 as an equivalence relation are:
(1,1), (2,2), (3,3), (2,1), (3,1) and (2,3)

33. (a) (i) Given Relation, R = $\{l_1, l_2\}$: d_1 is parallel to l_2

? $l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1$

Hence, given relation is symmetric.

(ii) Let $l_1 \parallel l_2$ and $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3$

[As parallel lines are parallel to each other]

(iii) Given equation of line : y = 3x + 2

Comparing with y = mx + C

Slope of parallel, lines is same

$\therefore$ Equation of set of rall line is given by

$y = 3x + \lambda \quad \lambda \in R$

OR

(b) Given relation, $S = \{(l_1, l_2) : l_1$ is perpendicular to $l_2\}$

Symmetric $l_1 \perp l_2 \Rightarrow l_2 \perp l_1$

(Two lines are perpendicular to each other)

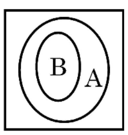
Hence, S is symmetric.

Transitive:

Let $L_1 \perp L_2$ and $l_2 \perp l_3 \Rightarrow l_1 \parallel l_3$

Thus, relation S is not transitive.

34. (B)



35. (D) tan x - sec y

36. (D) Assertion (A) is false, but Reason (R) is true.

37. (A) one-one but not onto

38. (D) neither injective nor surjective.

39. (d) : We have, f(x) = 4 + 3 cosx, $\forall$ x $\in$ R

At

$x = \frac{\pi}{2}, f\left(\frac{\pi}{2}\right) = 4 + 3\cos\frac{\pi}{2} = 4 \Rightarrow f\left(-\frac{\pi}{2}\right) = 4 + 3\cos\left(-\frac{\pi}{2}\right) = 4$

Since, $f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$, But $\frac{\pi}{2} \neq -\frac{\pi}{2}$

Therefore, f is not one–one.

As$-1 \le \cos x \le 1 \ \forall \ x \in R \Rightarrow 1 \le 7 \ \forall \ x \in R$

$\Rightarrow f(x) \in [1,7]$, where [1,7] is subset of R.

$\therefore$ f is not onto.

Concept Referred

- Range of cos x is [–1, 1]

40. (d) : $\therefore$ f : X $\to$ Y is one-one, if different element of X have different image in Y under f. But here, no such situation is possible.

41. (d) : Given f(x) = $\frac{1}{x}$, for all x $\in$ R

At x = 0$\in$R, f(x) is not defined.

42. (c) Given, $f(x) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$

Now, $f(1) = \frac{1+1}{2} = 1, f(2) = \frac{2}{2} = 1$

$\Rightarrow$ f(1) = f(2) but 1 $\neq$ 2 $\therefore$ f is not one-one.

But f is onto (**?** range of f is N.)

43. Given f(x) = sin x + cos x

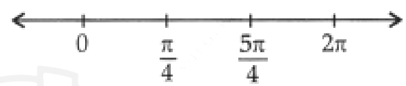
f'(x) = cos x – sin x

putting f'(x) = 0

cos x= sin x

$x = \frac{\pi}{4}, \frac{5\pi}{4} \quad \ldots(\text{ for } x \in [0, 2\pi])$

plotting points



Here, where $x \in \frac{\pi}{4}, \frac{5\pi}{4}$

putting f'(x) = cos x – sin x

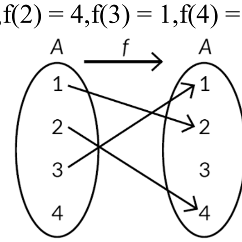
at $x = \frac{\pi}{2} \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} - \sin\frac{\pi}{2} = -1 < 0$

thus $f'(x) < 0$ for $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\Rightarrow$ f is strictly decreasing in $x \in \left(\frac{\pi}{4}, \frac{0\pi}{4}\right)$

44. We have, A = {1,2,3,4} function f : A $\to$ A is one–one and f(1) = 2,f(2) = 4,f(3) = 1,f(4) = k.



As f is one–one, so no two element of A has same image in A.

$\therefore$ f(4) = 3$\Rightarrow$ k = 3

Concept Referred

- For a function to be one-one, no two elements should have the same image in A.

45. f(1) = 1 gives
a + b = 1 - - - - - - (1)
f(2) = 3 gives
2a + b = 3 - - - - - (2)
Equation 2 - Equation 1 gives
a = 2
From Equation 1 we get
2 + b = 1
$\Rightarrow b = 1 - 2$
$\Rightarrow b = -1$
Putting the value of a and b in f(x) = ax + b we get the function as f(x) = 2x - 1
Determine whether the function is one one and onto or not
Let a , b ∈ R such that f(a) = f(b)
f(a) = f(b)
⇒ 2a –1 = 2b –1
⇒ 2a = 2b
⇒ a = b
∴ f(a) = f(b) gives a = b
So f is one to one
Onto :
Let y ∈ R
If possible let us assume that x ∈ R such that f(x) = y
$f(x) = y$
$\Rightarrow 2x - 1 = y$
$\Rightarrow x = \frac{y+1}{2}$
So every element y in codomain set contains a pre - image (y + 1)/2 in domain set
∴ f is onto
Hence the function f defined on R is both one one and onto

46. Given, $A = R - \{5\}, B = R - \{1\}$
$f : A \to B$
$f(x) = \frac{x-3}{x-5}$
for f is one – one
$f(x_1) = f(x_2)$
$\frac{x_1 - 3}{x_1 - 5} = \frac{x_2 - 3}{x_2 - 5}$
$(x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5)$
$x_1x_2 - 5x_1 - 3x_2 + 15 = x_1x_2 - 5x_2 - 3x_1 + 15$
$-5x_1 + 3x_1 = -5x_2 + 3x_2$
$-2x_1 = -2x_2$
$\therefore x_1 = x_2$
f is one–one.
For if is onto
Let y be any element of R
$y = f(x)$
$\Rightarrow y = \frac{x-3}{x-5}$
$\Rightarrow y(x-5) = x - 3$
$\Rightarrow yx - 5y = x - 3$
$\Rightarrow yx - x = 5y - 3$
$\Rightarrow x = \frac{5y-3}{y-1}$
$f(x) = \frac{x-3}{x-5}$
$$f\left(\frac{5y-3}{y-1}\right) = \frac{\frac{5y-3}{y-1} - 3}{\frac{5y-3}{y-1} - 5}$$
$$\Rightarrow f\left(\frac{5y-3}{y-1}\right) = \frac{\frac{5y-3-3y+3}{y-1}}{\frac{5y-3-5y+5}{y-1}}$$
$$f\left(\frac{5y-3}{y-1}\right) = \frac{2y}{2} = y$$
f is into

47. (i) Here n(B) = 3 and n(G) = 2
∴ Number of relation from B to G = $2^{3\times2} = 2^6$
(ii) Number of functions formed from B to G = $2^3 = 8$
(iii) We have, R = {(x,y) =x and y are student of the same sex}
∴ R is reflexive as (x,x) ∈R.
Since,(x,y) ∈R and (y,z) ∈R ⇒ (x,z) ∈R
Hence, R is an equivalence relations.

OR

We have f : B → G be defined by f = {(b_1, g_1), (b_2, g_2), (b_3, g_1)}
Since, elements b_1 and b_3 have the same image, therefore, the functions is not one-one but it is many one functions.
Since, every element in G has its pre-image in B, so the functions is onto.
For bijection, function should be one-one and onto both.
Hence, the function is subjective but not injective.

48. The function $f : R - \left\{-\frac{4}{3}\right\} \to R$ is given by $f(x) = \frac{4x}{3x+4}$.
One–one : Let x,y $\in R - \left\{-\frac{4}{3}\right\}$ such that f(x) = f(y)
$\Rightarrow \frac{4x}{3x+4} = \frac{4y}{3y+4}$
⇒ 4x(3y + 4) = 4y(3x + 4)

$\Rightarrow 12xy + 16x = 12xy + 16y$

$\Rightarrow 16x = 16y \Rightarrow x = y$

$\therefore$ f is one–one

Onto : Let y be an arbitrary element of R. Then f(x) = y

$\Rightarrow \dfrac{4x}{3x+4} = y \Rightarrow 3xy + 4y \Rightarrow 4x - 3xy = 4y \Rightarrow x = \dfrac{4y}{4-3y}$

As $y \in R \ R - \left\{\dfrac{4}{3}\right\}, \dfrac{4y}{4-3y} \in R$

Also, $\dfrac{4y}{4-3y} \neq \dfrac{-4}{3}$ as if

$\dfrac{4y}{4-3y} = -\dfrac{-4}{3} \Rightarrow 12y = 12y - 16$, which is not possible.

Thus, $x = \dfrac{4y}{4-3y} \in R - \left\{-\dfrac{4}{3}\right\}$ such that

$$f(x) = f\left(\frac{4x}{3x+4}\right) = \frac{4\left(\frac{4x}{4-3y}\right)}{3\left(\frac{4x}{4-3y}\right)+4} = \frac{16y}{12y+16-12y} = \frac{16y}{16} = y$$

So, every element in $R - \left\{\dfrac{4}{3}\right\}$ has pre–image in $R - \left\{-\dfrac{4}{3}\right\}$

$\therefore$ f is not onto.

49. Given, $f(x) = \dfrac{x}{1+|x|}, x \in (-\infty, 0)$

$= \dfrac{x}{1-x}$ $\quad (\therefore x \in (-\neq, 0), |x| = -x)$

For one-one : Let $f(x_1) = f(x_2)$, $x_1, x_2 \in (-\infty, 0)$

$\Rightarrow \dfrac{x_1}{1-x_1} = \dfrac{x_2}{1-x_2} \Rightarrow x_1(1-x_2) = x_2(1-x_1)$

$\Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$

Thus, $f(x_1) = f(x_2), \Rightarrow x_1 = x_2$

$\therefore$ f is one-one

For onto : Let f(x) = y

$\Rightarrow y = \dfrac{x}{1-x} \Rightarrow y(1-x) = x \Rightarrow y - xy = x$

$\Rightarrow x + xy = y \Rightarrow x(1+y) = y \Rightarrow x = \dfrac{y}{1+y}$

Here, $y \in (-1, 0)$

So, x is defined for all values of y in co–domain.

$\therefore$ f is onto.

Concept Referred

- A function $f : A \to B$ is called
  (i) one-one or injective function, if distinct elements of A have distinct images in B.
  i.e., for $a, b \in A$, $f(a) = f(b) \Rightarrow a = b$
  (ii) onto or surjective function, if for every element $b \in B$, there exists some $a \in A$ such that $f(a) = b$.

Previous Year Questions CBSE

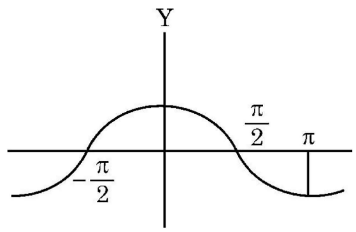
Basic Concepts

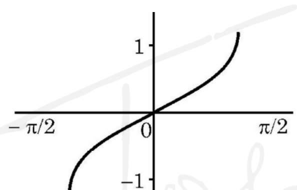
MCQ

1. The value of $\cos\left(\frac{\pi}{6}+\cot^{-1}(-\sqrt{3})\right)$ is **(2025)**

(A) –1 (B) $\frac{-\sqrt{3}}{2}$ (C) 0 (D) 1

2. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ? **(2025)**



(A)


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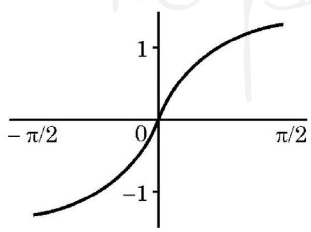
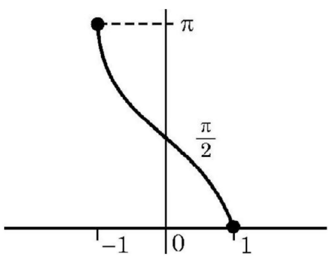


(C)


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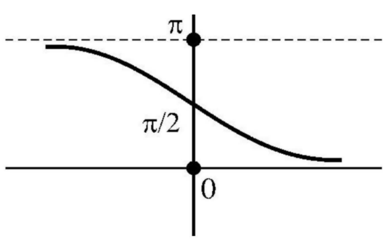


3. **Assertion (A) :** Domain of $y = \cos^{-1}(x)$ is [–1, 1].
Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$. **(2024)**

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

4. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to. **(2023)**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

5. If f(x) = |cosx|, then $f\left(\frac{3\pi}{4}\right)$ is **(2023)**

(a) 1 (b) –1 (c) $\frac{-1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

6. Two statements are given, one labeled Assertion (A) and the other labeled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below. **(2023)**
Assertion (A) : All trigonometric functions have their inverses over their respective domains.
Reason (R) : The inverse of $\tan^{-1}x$ exists for some x-.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

7. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is. **(Term I, 2021-22)**

(a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

8. The principal value of $\cot^{-1}(-\sqrt{3})$ is **(2020)**

(a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

9. $\tan^{-1}3 + \tan^{-1}\lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ ? **(2020)**

(a) $\lambda \in \left(-\frac{1}{3},\frac{1}{3}\right)$ (b) $\lambda > \frac{1}{3}$

(c) $\lambda < \frac{1}{3}$ (d) All real values of λ

10. The principal value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ is **(2020)**

(a) $\frac{2\pi}{5}$ (b) $\frac{-2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) $\frac{-3\pi}{5}$

VSA

11. Evaluate: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$: **(2025)**

12. Evaluate: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ **(2025)**

13. The range of the principal value branch of the function $y = \sec^{-1}x$ is ___________ . **(2020)**

14. The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is _______. **(2020)**

15. Write the value of $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$ **(Foreign 2014)**

16. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ **(AI 2014C)**

17. Find the value of the following :

$\cot\left(\frac{\pi}{2}-2\cot^{-1}\sqrt{3}\right)$ **(AI 2014C)**

SAI

18. (a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. **(2024)**

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

19. Find value of k if $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$. **(2024)**

20. If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$ and $b = \tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ then find the value of a + b.**(2024)**

21. Simplify : $\cos^{-1}x + \cos{-1}\left[\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right];\frac{1}{2}\le x\le 1$ **(2024)**

22. Write the domain and range (principle value branch) of the following functions:
$f(x) = \tan^{-1}x$. **(2023)**

23. Evaluate : $\cos^{-1}\left[\cos\left(-\frac{7p}{3}\right)\right]$ **(2023)**

24. Simplify $\sec{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$. **(2021C)**

25. Prove that : $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ **(2020)**

26. Prove that : $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\cos^{-1}x, \frac{1}{\sqrt{2}}\le x\le 1$

LAI

27. Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$. **(2020C)**

28. Prove that : $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$. **(2020C)**

29. Prove that : $\tan^{-1}\sqrt{x}=\frac{1}{2}+\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in[0,1]$ **(2020, 2019 C)**

30. Prove that : $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$ **(AI 2019)**

31. Prove that : $\sin^{-1}\frac{4}{5}+\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{63}{65}=\frac{\pi}{2}$ **(2019)**

32. If $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$, find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$. **(2019)**

33. Find the value of $\sin\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$. **(2019)**

34. Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x; -\frac{1}{\sqrt{2}}\le x\le 1$$

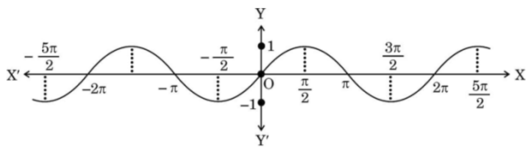
(2019C)

CSBQ

35. If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. **(2024)**

Function g is called the inverse of function f.

The domain of sine function is R and function sine : R $\to$ R is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be defined from set A to [– 1, 1] such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from [– 1, 1] to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval.

(ii) If $\sin^{-1}(x)$ is defined from [– 1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$

(iii) (a) Draw the graph of $\sin^{-1} x$ from [– 1, 1] to its principal value branch.

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$.

Detailed SOLUTIONS

1. (A) –1

2. (C)

3. (C)
Range of $\cos^{-1} x$ is $[0, \pi]$.

4. (a) : We have,
$= \sin\left(\frac{\pi}{2}\right) = 1$

5. (d) : f(x) = |cosx|
At $\frac{\pi}{2} < x < \pi, \cos x < 0 \therefore |\cos x| = -\cos x \Rightarrow f(x) = -\cos x$
$\therefore\ f\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{3\pi}{4}\right) = -\cos\left(\pi - \frac{\pi}{4}\right)$
$= \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ [? $\cos(\pi - \theta) = -\cos\theta$]

6. (d) : All trigonometric functions are periodic and hence not invertible over their respective domains but all trigonometric functions have inverse over their restricted domains.
Inverse of $\tan^{-1}x$ is tanx which is defined for
$x \in R - (2n+1)\frac{\pi}{2}, n \in Z$
$\therefore$ Assertion is false and reason is true.

7. (b) : We have,
$\sin^{-1}\left(\cos\frac{13\pi}{5}\right) = \sin^{-1}\left[\cos\left(2\pi + \frac{3\pi}{5}\right)\right]$
$= \sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$
$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) = -\sin^{-1}\left(\sin\frac{\pi}{10}\right) = -\frac{\pi}{10}$

Quick Move
- $\cos(2\pi + \theta) = \cos\theta, \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

8. (d) : We know that $\cot^{-1}(x) \in (0, \pi)$
$\cot^{-1}\left(-\sqrt{3}\right) = \cot^{-1}\left(-\cot\frac{\pi}{6}\right)$
$= \cot^{-1}\left[\cot\left(\pi - \frac{\pi}{6}\right)\right]$ [? $\cot(\pi - \theta) = -\cot\theta$]
$= \cot^{-1}\left[\cot\left(\frac{5\pi}{6}\right)\right] = \frac{5\pi}{6}$ [? $\cot^{-1}[\cot\theta] = \theta$]
Thus, the principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is $\frac{5\pi}{6}$.

9. (c) : Given, $\tan^{-1} 3 + \tan^{-1}\lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$
$\tan^{-1} 3 + \tan^{-1}\lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$
for $3\lambda < 1 \therefore 3\lambda < 1 \Rightarrow \lambda < \frac{1}{3}$.

Concept Referred
- $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$

10. (b) : We have, $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$
We know that the range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\therefore\ \tan^{-1}\left(\tan\frac{3\pi}{5}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{2\pi}{5}\right)\right)$
$= \tan^{-1}\left[-\tan\left(\frac{2\pi}{5}\right)\right]$ [? $\tan(\pi - \theta) = \tan\theta$]
$= \tan^{-1}\left[-\tan\left(\frac{2\pi}{5}\right)\right] = -\frac{2\pi}{5}$ [? $\tan^{-1}(\pi - \theta) = \tan\theta$]

11. $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$
$= \tan^{-1}\left[2\sin\left(2 \times \frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\sin\frac{\pi}{3}\right]$
$= \tan^{-1}\left[2 \times \frac{\sqrt{3}}{2}\right] = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

12. $\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{2\pi}{5}\right)\right)$
$= \sin^{-1}\left(\sin\left(\frac{2\pi}{5}\right)\right) = \frac{2\pi}{5}$

13. The range of the principal value branch of the function $y = \sec^{-1}x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

14. Let y = cos–1
$\left(\frac{-1}{2}\right) \Rightarrow \cos y = \frac{-1}{2} \Rightarrow \cos y = -\cos\left(\frac{\pi}{3}\right)$
Since, the range of $\cos^{-1}x$ is $[0, \pi]$
$\Rightarrow\ \cos y = -\cos\left(\pi - \frac{\pi}{3}\right)$ [? $\cos(\pi - \theta) = -\cos\theta$]
$\Rightarrow y = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$
hence, the prinicipal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

15. Given $\cos^{-1}\left(1\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

$=\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+2\sin^{-1}\left(\sin\frac{\pi}{6}\right)=\frac{2\pi}{3}+2\times\frac{\pi}{6}=\pi$

[? Range of $\cos^{-1}x$ is $[0,\pi]$ & of $\sin^{-1}x$ is $[-\pi/2,\pi/2]$]

16. Here, $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]=\tan^{-1}(-1)=-\frac{\pi}{4}$

This is the required principal value as it is lie in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

NCERT Core

- The range of $y=\tan^{-1}x$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

17. $\cot\left(\frac{\pi}{2}-2\cot^{-1}\sqrt{3}\right)$.

$=\cot\left(\frac{\pi}{2}-2\cot^{-1}\left(\cot\frac{\pi}{6}\right)\right)=\cot\left(\frac{\pi}{2}-2.\frac{\pi}{6}\right)$

$=\cot\left(\frac{\pi}{2}-\frac{\pi}{3}\right)=\cot\frac{\pi}{6}=\sqrt{3}$

18. **(a)**

$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ ...$\left[\therefore \cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right]$

$=\tan^{-1}\left[\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$ ...$\left[a^2-b^2=(a+b)(a-b)\right]$

$=\tan^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$

$=\tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}\right]$ ...$\left[\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\cdot\tan B}\right]$

$=\tan^{-1}\left(\tan\frac{\pi}{4}+\frac{x}{2}\right)=\frac{\pi}{4}+\frac{x}{2}$

(b)

19. We have, $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$

$k\left[\tan\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)\right]=\sin\frac{\pi}{3}$

$k\tan\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

$k\sqrt{3}=\frac{\sqrt{3}}{2}$ $\quad k=\frac{1}{2}$

20. Let $\sin^{-1}\left(-\frac{1}{2}\right)=y$

Then,

$\sin y=-\frac{1}{2}$

We know that the range of the principal value branch is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Thus,

$\sin y=-\frac{1}{2}=\sin\left(-\frac{\pi}{4}\right)$

$\Rightarrow y=-\frac{\pi}{4}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Now,

Let $\cos\hat{}-1(-1/2)=z$

Thus,

$\cos z=-\frac{1}{2}$

We know that the range of the principal value branch is $[0,\pi]$.

$\cos z=-\frac{1}{2}=\cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow z=\frac{2\pi}{3}\in[0,\pi]$

so,

$\tan^{-1}1+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{4}=\frac{2\pi}{3}$

21. We have

$\cos^{-1}x+\cos^{-1}\left(x/2+\left(\sqrt{3}-3x^2\right)/2\right),\forall x\in(1/2,1)$

$=\cos^{-1}(x)+\cos^{-1}\left(x\cdot\frac{1}{2}+\sqrt{1-\frac{1}{4}}\sqrt{1-x^2}\right)$

$=\cos^{-1}(x)+\cos^{-1}\left(\frac{1}{2}\right)-\cos^{-1}(x)$

$=\cos^{-1}\left(\frac{1}{2}\right)\quad=\frac{\pi}{3}$

22. Domain of $\tan^{-1}x=(-\infty,\infty)$ and range of $\tan^{-1}x$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

23. $=\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]=\cos^{-1}\left[\cos\left(\frac{7\pi}{3}\right)\right]$ (? $\cos(-\theta)=\cos\theta$)

$=\cos^{-1}\left(\cos\left(2\pi+\frac{\pi}{3}\right)\right)=\cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$

$=\frac{\pi}{3}$ (? $\cos^{-1}(\cos x)=x\ \forall\ 0\le x\le\pi$)

24. Let

$x = \cos\theta \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$= \sec^{-1}(\sec 2\theta) = 2\theta$

Hence, $\sec^{-1}\left(\frac{1}{2x^2-1}\right) = 2\cos^{-1}x$.

Concept Referred

- $\cos 2\theta = 2\cos^2\theta - 1, \sec\theta = \frac{1}{\cos\theta}$

25. Consider L.H.S. =

$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)\right]$

$\frac{9}{4}\cos^{-1}\left(\frac{1}{3}\right)$ ....(i) $\left[? \ \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

Let $a = \cos^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \cos a = \frac{1}{3} \Rightarrow \sin a = \sqrt{1-\cos^2 a}$ [? $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow \sin a = \sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3} \Rightarrow a = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

So, L.H.S. $= \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ = R.H.S

26. Consider L.H.S. $= \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \cos\theta$, we get

$= \sin^{-1}\left(2\cos\theta\sqrt{\sin^2\theta}\right)$ $\left[? \ \sin^2\theta + \cos^2\theta = 1\right]$

$= \sin^{-1}(2\cos\theta \sin\theta) = \sin^{-1}(\sin 2\theta)$

$= 2\theta$ $\left[? \ \sin^{-1}(\sin\theta) = \theta\right]$

Since, $x = \cos\theta$

$\Rightarrow \theta = \cos^{-1}x$

$\therefore 2\theta = 2\cos^{-1}x$ = R.H.S

27. Given, $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

Put $x = \sin y$

$\Rightarrow \sin^{-1}(1-\sin y) - 2\sin^{-1}(\sin y) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-\sin y) - 2y = \frac{\pi}{2}$ $[\sin^{-1}(\sin\theta) = \theta]$

$\Rightarrow \sin^{-1}(1-\sin y) = \frac{\pi}{2} + 2y \Rightarrow 1-\sin y = \sin\left(\frac{\pi}{2}+2y\right)$

$\Rightarrow 1 - \sin y = \cos^2 y$ $[\sin(\pi/2 + \theta) = \cos\theta]$

$\Rightarrow 1 - \sin y = 1 - 2\sin^2 y$ [? $\cos^2\theta = 1 - 2\sin^2\theta$]

$\Rightarrow 2\sin^2 y - \sin y = 0$

Replace $\sin y = x$

$\Rightarrow 2x^2 - x = 0 \Rightarrow x(2x-1) = 0 \Rightarrow x = 0, \frac{1}{2}$

28. Consider, L.H.S. $= 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{1}{\frac{3}{4}} + \tan^{-1}\frac{1}{7}$

$\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{\left(\frac{4}{3}+\frac{1}{7}\right)}{1-\frac{4}{21}} = \tan^{-1}\frac{\frac{31}{21}}{\frac{17}{21}}$

$\left[? \ \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{(x+y)}{(1-xy)}\right)\right]$

$= \tan^{-1}\frac{31}{17}$ = R.H.S

Hence proved.

Concept Referred

- $2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right)$

29. Consider, R.H.S. $= \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan^2\theta$ ....(i)

? R.H.S $= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}(2\theta) = \theta$

From equation (i), we get

$\tan\theta = \sqrt{x} \Rightarrow \theta = \tan^{-1}\sqrt{x}$ = L.H.S

$\therefore$ L.H.S. = R.H.S

Hence proved.

30. Let $x = \cos^{-1}\left(\frac{12}{13}\right)$ and $y = \sin^{-1}\left(\frac{3}{5}\right)$.

or $\cos x = \frac{12}{13}$ and $\sin y = \frac{3}{5}$

Now, $\sin x = \sqrt{1-\cos^2 x}$ and $\cos y = \sqrt{1-\sin^2 y}$

$\Rightarrow \sin x = \sqrt{1\frac{144}{169}}$ and $\cos y = \sqrt{1-\frac{9}{25}}$

$\Rightarrow \sin x = \frac{5}{13}$ and $\cos y = \frac{4}{5}$

We know that,

$\sin(x+y) =$

$\frac{5}{13}\times\frac{4}{5} + \frac{12}{13}\times\frac{3}{5} = \frac{20}{65} + \frac{36}{65} = \frac{56}{65} \Rightarrow x + y = \sin^{-1}\left(\frac{56}{65}\right)$

or, $\cos{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$